

THE
P R I N C I P L E S
OF THE
D O C T R I N E
OF
L I F E - A N N U I T I E S .

THE AMERICANS

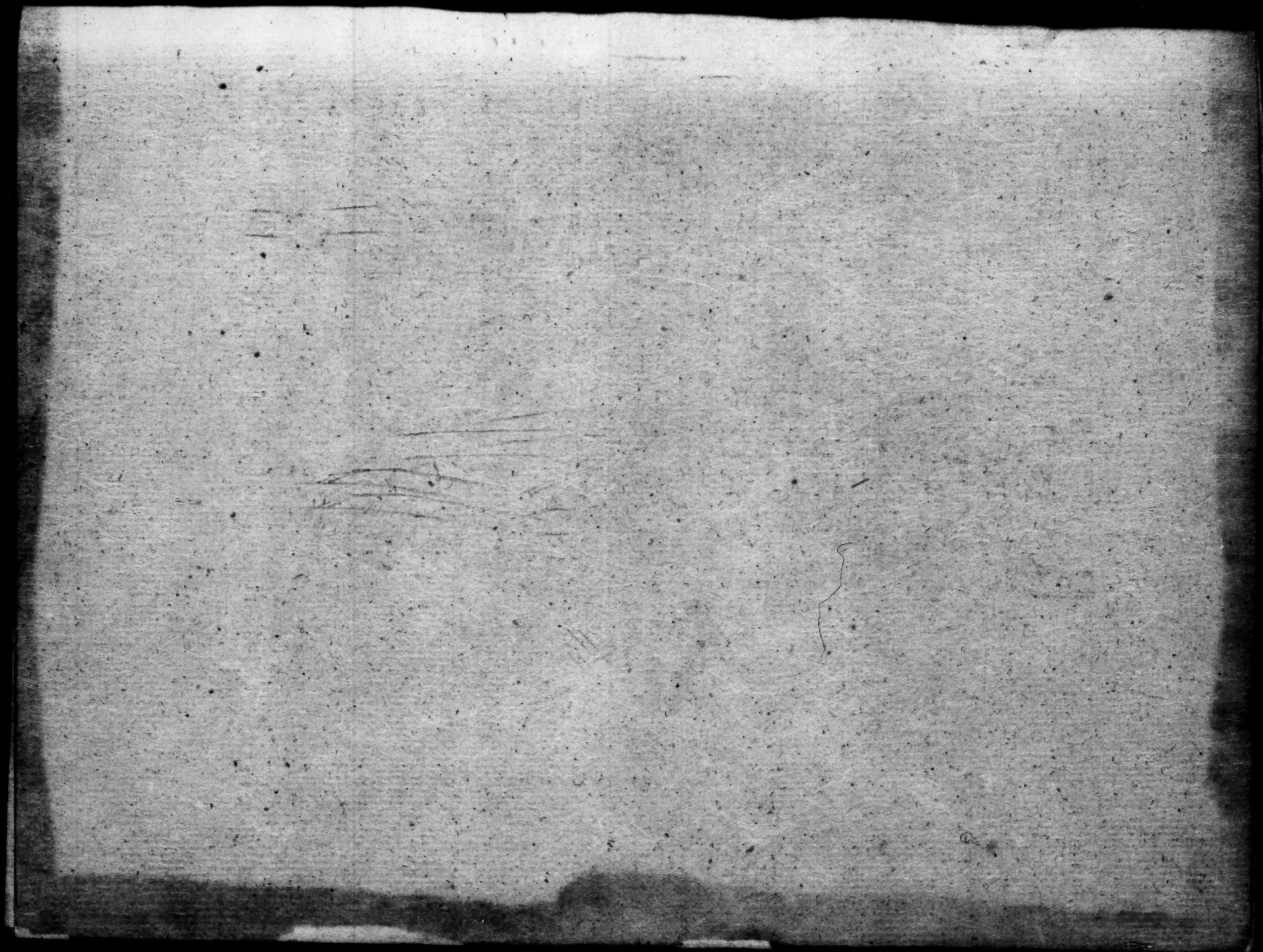
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PREFACE

THE following work is grown to so much greater a size than was at first intended, that it seems to be necessary to give my readers some previous information of its contents, that they may be able to distinguish between the different parts of it, (which are by no means all equally interesting) and to select those which shall most excite their curiosity, and be thought most deserving of their perusal.

The principles of the whole doctrine are contained in the first 90 pages, which I would therefore recommend to the attentive perusal of every reader. Of these the two first pages contain an explanation of the *data*, or grounds, upon which the computations of the values of annuities for lives are built. These are, first, the decrease of the present value of a future sum of money arising from the mere distance of the time at which it is to be paid, and the consequent discount that is to be allowed to the purchaser of it for prompt payment, (the quantity of which discount, it is evident, will depend on the rate of the interest of money;) and, secondly, the chance which, when the payment of such future sum is not made certain, but is to depend on the continuance of the life of a person of a given age, the grantor of it has of escaping the necessity of paying it at all by means of the death of the said person before it becomes due; in order to determine which chance, it is necessary to have recourse to certain tables of the several probabilities of the duration of human life at every different year of age, which have been formed from observations of the numbers of persons who have died every year, in the course of a long series of years, at different ages, in divers cities and parishes, and other numerous bodies of men,

On the pages 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000.

In page 15 a preference is given to Monsieur de Parcieux's table above that of Monsieur Kerseboom; and the reason of the said preference is stated.

In page 16 mention is made of the *Brethno* table of the probabilities of the duration of human life, which was formed and published by the celebrated Dr. Edmund Halley towards the end of the last century, (and which, I believe, was the first table of the kind ever published), and of the *London* table of those probabilities published by Mr. Smart and adopted by the learned Mr. Thomas Simpson, of Woolwich, and likewise of two other tables of those probabilities published by Mr. Richard Price, of Newington-Green, which were derived from observations made at Norwich and Northampton. And in page 17 the reader is referred to the most celebrated writers on this subject for an explanation of the manner in which these tables of probabilities are formed from parish registers, or other memorial, of the births and burials of mankind. And in pages 18 and 19, I have inserted some very useful remarks of the late very learned Mr. de Moivre on the several different tables of the four tables before-mentioned, of Mr. Halley, Monsieur Kerseboom, Monsieur de Parcieux, and Messieurs Smart and Simpson. And with these remarks I conclude the account of the data, or grounds, upon which the computations of the values of life-annuities are to be founded.

I then

THE
PRINCIPLES
OF THE
DOCTRINE
OF
LIFE-ANNUITIES;
EXPLAINED IN A FAMILIAR MANNER,

SO AS TO BE
Intelligible to Persons not acquainted with the Doctrine of CHANCES;

AND ACCOMPANIED WITH

With a VARIETY of NEW TABLES

Of the Values of such ANNUITIES at several different Rates of
Interest, both for Single Lives and for Two Joint Lives,

ACCURATELY COMPUTED FROM OBSERVATIONS.

By FRANCIS MASERES, Esq; F.R.S.
CURSITOR-BARON OF HIS MAJESTY'S COURT OF EXCHEQUER.

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I then proceed to lay down the fundamental doctrine of the doctrine of life annuities, at so great length, as to give a full and complete idea of the doctrine, by the *simpliciter* of a life annuity, which I have done with as much care and exactness as I was able, in pages 20 and 21, articles XXI and XXII.

And, having thus settled the grounds of the ensuing methods of computing the values of annuities either for terms of years, or for lives, I then proceed to deliver those methods themselves by the solution of four problems that contain them.

The first problem is, "To find the present value of a future sum of money, which is certainly to be paid at the end of one or more years, according to any given rate of interest." The solution of this problem, with an illustration of it by an example, and a corollary to it, is contained in pages 21 and 22. And in page 23 an account is given of Mr. Smart's most valuable tables of interest, which are computed upon the principles explained in this problem.

The second problem begins in page 24, and shows how we may find the present value of a future sum of money that is to be received at the end of a given number of years in case a person of a given age shall then be living, but not otherwise, according to any given rate of interest. Of this problem two solutions are given; the first, a particular one, in which the given number of years, at the end of which the money is to be paid, and the number of years in the age of the person on whose life it depends, are both specified; the other generally, in which the said numbers are represented, (as is usual in Algebra,) by two letters of the alphabet. The former of these solutions, it is hoped, will make this subject familiar to the reader, and serve to facilitate to him the perusal of the latter solution, which will be necessary for his more perfect satisfaction. These two solutions take up pages 24, 25, 26, and 27. And in corollary to this problem, the solution is extended to the determination of the present value of a number of years future payments of a given sum of money, to be made at the ends of several successive years, in case of the continuance of the life of a person of a given age, or, in other words, to the determination of the value

After the foregoing, in which this Project for establishing life-annuities in parishes is set forth and defended, there follow a 2nd and a 3rd corollary to this 2nd problem, showing how the value of an annuity shall be determined upon the principles explained in the solution of the problem. These corollaries, with an example to each of them, are contained in pages 40 and 41, and conclude the whole doctrine of the computation of the values of life-annuities depending upon only one life.

Problem 3^d relates to the computation of the present value of a future sum of one pound, sterling, that is to take place at the end of a certain number of years, provided two persons of given ages shall then be living, and upon the supposition of a given rate of the interest of money. This problem is solved in a double manner, as well as the former, to wit, 1st. in the case of a particular example, and, 2^{ndly}, in general terms. These solutions are contained in pages 42, 43, 44, 45, 46, and 47. And from them are deduced, in pages 47 and 48, two corollaries which extend them to the determination of the present values of any future sums (that are greater or less than one pound, sterling,) and of a number of such sums to be paid at the end of every year during the continuance of the lives of both the persons of given ages, or, in other words, of an annuity for their joint lives. And in pages 48, 49, 50, 51, and 52, the method of computing the value of an annuity for two joint lives, prescribed in corollary 2, is illustrated by an example.

After the example to corollary 2, follow three more corollaries relating to annuities for two joint lives, to wit, coroll. 3, which relates to remote annuities for two joint lives; and coroll. 4, which relates to immediate, but imperfect, annuities for two joint lives; and coroll. 5, which relates to remote and imperfect annuities for two joint lives. These three corollaries are contained in pages 52 and 53.

After these five corollaries the principles of the solution of this problem are extended, in coroll. 6, to the case of a future payment depending on the joint continuance of three lives. This, as well

as the former solutions, is done in a two-fold manner, to wit, first, in the case of a particular example, and afterwards in general terms. It takes up part of page 53, and all the 54th, 55th, and 56th pages, and part of page 57; it being rather a complex and difficult business, and much pains having been bestowed upon it to make it as plain as possible. And in coroll. 7 the conclusions obtained in coroll. 6 are applied to the determination of the value of a number of successive future payments of one pound each that are to be made at the end of every successive year during the joint lives of three persons of given ages, or, to the determination of the value of an annuity of one pound *per annum* for three joint lives. And this concludes the whole doctrine of the computation of the values of annuities for two, or more, joint lives.

The fourth problem relates to the value of a future sum of money, the payment of which depends not upon the joint continuance of the lives of two persons of given ages, but upon the continuance of the life of *either* of them; which value, it is evident, will be very different from the other, and will greatly exceed it. This problem is rather more difficult than the foregoing one, which relates to the joint continuance of the two lives. It is solved (as all the former ones,) in a two-fold manner, to wit, first, in the case of a particular example; and, secondly, in general terms. The former of these solutions is contained in pages 58 and 59; and the latter in pages 59, 60, and 61. From these solutions several corollaries are deduced.

In the 1st corollary (which is in page 62) it is shewn that the value of such future payment of a given sum of money, depending upon the longest of two given lives, has a remarkable relation to the value of the future payment of the same sum of money depending upon the joint continuance of the same lives, and may be easily deduced from the said latter value together with the values of the future payment of the same sum of money depending upon the same two lives separately; for that the value of such future payment of a given sum of money depending on the longest of two given lives, is equal to the excess of the sum of the two values of the future payments of the same sum of money depending upon the separate continuance of the same two lives, above the value of the future payment

ment of the same sum of money depending upon the joint continuance of the same two lives, and the value of a future payment of one pound, to the value of a future of any other sum, greater, or less, than one pound.

The 3^d corollary extends the solution of the problem to the determination of the value of a number of equal future payments of one pound each, to be received at the end of every year during the life of either of two persons of given ages; or, in other words, to the determination of the value of an annuity of one pound a year during the longest of two given lives.

The 4th corollary shews that there is the same relation between the value of a life-annuity for the longest of two given lives, and that of the same annuity for the joint continuance of the same lives, together with those of the same annuity for the two separate lives, as there is between the value of a single future payment depending on the continuance of either of the two lives, and the value of the same future payment in case of the continuance of both lives, together with the values of it in case of the continuance of each of the two lives separately; or, that the value of an annuity of one pound a year for the longest of two given lives is equal to the excess of the sum of the values of two separate annuities of one pound a year for the same single lives, above the value of an annuity of one pound a year for the joint continuance of both lives. This corollary is contained in pages 63, 64, 65, and 66, and is evidently of very great importance, inasmuch as it enables us, when we have tables of the values of annuities for single lives and for two joint lives, ready computed to our hands, to derive from them the values of annuities for the longest of two given lives by the easy operations of addition and subtraction.

In pages 67 and 68 are three more corollaries, to wit, corollaries 5th, 6th, and 7th, which relate to the values of remote life-annuities for the longest of two given lives, and of immediate, but imperfect, life-annuities depending on the continuance of either of two given lives, and of remote and imperfect life-annuities depending likewise on the continuance of either of two given lives.

After

After these three corollaries (which are short and easy) comes the 8th corollary to this fourth problem, which is much longer and more difficult to understand than any preceding part of the book. It contains an extension of the solution of this problem to the case of a future sum of money depending on the continuance of *any one of three* lives of given ages; which, though it is determined by the same principles as the former case of a future sum of money depending on the continuance of *either of two given lives*, is greatly more complicated than that case, and consequently requires a much longer investigation. This investigation is carried on (like all the former solutions,) in two different manners, namely, first, in the case of a particular example, and afterwards in general terms. The particular investigation is contained in pages 68, 69, 70, 71, 72, 73, 74, and 75; and the general investigation is contained in pages 76, 77, 78, and part of page 79. And, after these two investigations, certain difficulties, that will probably arise in the attentive reader's mind after he has perused them, are examined and removed in pages 79 and 80, and the beginning of page 81.

These investigations are followed by three corollaries, namely, corollaries 9th, 10th, and 11th. The first of these corollaries is contained in pages 81 and 82, and is employed in shewing the relation of the value of a future payment of one pound depending on the longest of three lives of given ages to the value of a future payment of the same sum of money depending on the joint continuance of all the three lives, and to the three different values of a future payment of the same sum of money depending on the joint continuance of every two of the same three lives, and to the three different values of a future payment of the same sum of money depending on the continuance of each of the same three lives taken separately; the said first value being equal to the excess of the sum of the said second value (which relates to the joint continuance of all the three lives) and the said three last values (which relate to the three single lives) above the sum of the said third, fourth, and fifth values, which relate to the joint continuance of every two of the said lives. The 10th corollary extends the foregoing investigations to the case of a number of future payments of a sum of one pound that are to be made at the end of every year during the continuance of any one of three lives of given ages, or, in other words, to the case of a life-annuity depending

depending on the longest of three lives of given ages. This corollary is very short, and is contained in the first part of page 83. But the 11th corollary is a very long one, and takes up the remainder of page 83, and all the 84th, 85th, 86th, 87th, 88th, and 89th pages, together with a part of page 90. But it is also very important and useful, and for that reason is so much enlarged upon. For it shews, that the value of an annuity of one pound a year for the longest of three lives of given ages, is equal to the excess of the sum of the value of a like annuity for the joint continuance of the same three lives and the three values of the like annuity for the same three lives taken separately, above the sum of the three values of the like annuity for the joint continuance of every two of the said lives; and consequently that, whenever we have tables of the values of annuities for single lives and for two and three joint lives ready calculated, we may easily deduce from them the values of annuities for the longest of three lives by mere addition and subtraction.

And here ends the fundamental part of the whole work, or the explanation of the principles of the doctrine of life-annuities. The remainder of the book, long as it is, is taken up in applications of these principles, and illustrations of them by numerous examples, in order to render these computations familiar to the reader, and in contrivances to abridge the labour of them, and in other such matters, which are much less curious and important than the explanation of the principles themselves of this useful species of computation, which are contained in the foregoing 90 pages. And therefore I expect that many of my readers will wholly pass over many large parts of this remainder, which they will esteem, and perhaps justly, not worth the trouble of perusing them; though others of my readers, (who may have more leisure and a greater liking to the subject,) will, I imagine, be inclined to go through every page of it. I shall therefore here continue the account of the contents of the book throughout this long and less interesting remainder of it, in the same manner as I have done already with respect to the first and more important 90 pages of it; to the end that the former set of readers may easily determine before-hand which parts of it they will pass over, and that even the latter class of readers may be the better able to judge which parts of it they will chuse to read first, and which they will read with the most attention.

Pages 90, 91, 92, 93, 94, and 95 relate to the computation of the values of annuities for terms of years certain, and contain short and convenient, algebraic, expressions of the said values. And here I have inserted a geometrical demonstration of the rule for finding the sum of the terms of a decreasing geometrical progression, consisting of a finite number of terms, as *A, B, C, D, E*, to wit, that it is equal to $\frac{AA - BE}{A - B}$, or to the quotient that arises by dividing the excess of the square of the first, or greatest, term above the rectangle, or product, contained under the second term and the last, or least, term, by the excess of the first, or greatest, term above the second term.

Page 96 contains a like short and convenient, algebraic, expression of the value of an immediate annuity of one pound a year for a given number of years depending on a single life of a given age; and page 97 contains a like expression for the value of such an immediate annuity during the whole life of a person of a given age, which is only a particular case of the former expression. And in the remainder of page 97, and in page 98 and part of page 99, are contained examples of the computation of the values of such life-annuities by means of the said algebraic expressions.

Then follows in pages 99 and 100 a like algebraic expression for the value of a *remote* annuity for a given number of years, depending on a life of a given age; and in pages 100, 101, and 102, is contained an example of the computation of the value of such a remote life-annuity by means of such algebraic expression.

All these examples of the computations of the values of life-annuities are the very same which were given in the former part of the work after the scholium of Prob. 2nd; only they are here performed by the help of the algebraic expressions here given of those values, in order to make the use of those expressions familiar to the reader. And it was in performing the computation of the last of these examples that I discovered the slip in an arithmetical operation which has been already mentioned in this preface to have been made in page 30, by making the quotient of the division of 31.8212 by 118 be .2612 instead of .2696. See the note in page 102, Art. XCII.

After

After these first examples, which relate to the values of annuities for the lives of very old persons, I have, in Pages 103, 104, 105, 106, and 107, inserted the calculation of the value of an annuity of one pound a year for the life of a person only 10 years old; which exhibits a clear view of the labour that is necessary to be employed in making these computations exactly, when the lives are young, and, consequently, of the reason which, probably, induced Mr. De Moivre, and some other calculators of the values of life-annuities, to decline the foregoing accurate method of computing them, and resort to hypotheses concerning the probabilities of the duration of human life, which they knew to be not perfectly true, in order to facilitate the computation of them. The principal hypothesis used by Mr. De Moivre for this purpose is described in a subsequent part of the work.

After the above-mentioned long example of the computation of the value of a life-annuity of one pound a year for the whole life of a child of 10 years of age, there is in Art. xcvi, pages 107, 108, an example of the computation of the value of an annuity of one pound a year for the first 30 years of the said life; which is involved in, or makes only a part of, the foregoing computation.

Art. xcvi, pages 108, 109, contains some remarks on the fluctuation of the price of the public funds, and of the interest of money in general; which makes it expedient to have the values of life-annuities calculated according to several different rates of interest.

In pages 109, 110, 111, and 112, an account is given of a very easy and convenient method of deducing the value of a life-annuity of one pound a year for a life of any given age from the value of the same annuity for a life that is older than the former by one year; by the help of which method a whole table of the values of a life-annuity of one pound a year for every age of human life, proceeding from the older ages to the younger by the constant difference of a year, may be computed with nearly the same labour as is necessary to obtain the value of the same annuity for the first, or youngest, life in the table. This method was first communicated to me by Dr. Price: but it was published in the year 1779 by Mr. William Morgan, the actuary to the Society for Equitable Assurances near

Black-friars Bridge, in his *Treatise on the Doctrine of Annuities and Assurances on Lives*, pages 56, 57; and it had been published before by Dr. Price himself in his *Treatise on Reverſionary Payments*, Note O of the Appendix, and likewise by Mr. Thomas Simpson in his book on *Life-annuities*, Prob. 1, Coroll. 7; which last book was published ſo long ago as the year 1742. But I ſhould ſuſpect that it was not known to Mr. De Moivre, when he calculated his tables of the values of life-annuities. For, if it had, I ſhould imagine he would hardly have thought it neceſſary to have recourſe to a certain inaccurate hypotheſis concerning the probabilities of life, in order to diminiſh the labour of his computations, which would have been almoſt equally facilitated by the uſe of this excellent method. This method I have not only deſcribed, but explained and demonſtrated in the ſulleſt and cleareſt manner I was able, in the ſaid pages 109, 110, 111, and 112.

In pages 112, 113, and 114 I have applied this method to the computation of the values of a life-annuity of one pound a year for lives of the ages of 8 years, 7 years, 6 years, 5 years, 4 years, and 3 years, from the value of a like annuity for a life of the age of 9 years (which had been computed by it in pages 110 111, from the value of a like annuity for a life of the age of 10 years,) and from each other. And afterwards, in pages 115, 116, 117, 118, 119, 120, &c. — 130, in order to make this method ſtill more familiar to the curious and induſtrious reader (for all others, I ſuppoſe, will paſs over this part of the book,) I have applied it to the computation of a compleat table of the values of a life-annuity of one pound a year for every age of life, from the age of 93 years up to the age of 3 years, according to Monſieur de Parcieux's table of the probabilities of the duration of human life, and upon a ſuppoſition that the intereſt of money is $3\frac{1}{4}$ per cent. And here I have diſcovered another error in one of the arithmetical operations, which pervades all the ſubſequent computations, and renders them in a ſmall degree erroneous. For at the bottom of page 123, in deducting the value of an annuity of one pound a year for a life of the age of 41 years from £15.580,939, or the value of the ſame annuity for a life of the age of 42 years, I have made the product of the multiplication of 16.580,939 by the fraction

$$\frac{643}{650} \text{ amount to } \pounds 16.448,528,$$

£16.448,528; whereas in truth it amounts to only £16.402,375; in consequence of which mistake the quantity $\frac{1}{1.035} \times \frac{643}{650}$ £16.580,939, (which is equal to the value of an annuity of one pound a year for a life of the age of 41 years,) appears to be equal to $\frac{1}{1.035} \times$ £16.448,528, or £15.892,297, or 15/. 17s. 10d. instead of $\frac{1}{1.035} \times$ £16.402,375, or £15.847,705, or 15/. 16s. 11½d. which is less than the former sum, 15/. 17s. 10d. by a difference of 10½d. This error necessarily affects all the following values of a life-annuity of one pound a year for lives of younger ages than 41 years, which are set down in pages 124, 125, 126, 127, 128, 129, and 130. But it is not of much importance with respect to the value of the annuity for a life of the age of 41 years, in which it occurs; the difference of 10½d. being but a trifling sum of money in comparison of 15l. 16s. 11½d. which is the true value of that annuity. And in all the following values of a life-annuity of one pound a year, for lives of younger ages than 41 years, the error, or difference of the false and true values, grows continually less and less. For the erroneous value of this annuity for a life of the age of 10 years, obtained in page 128, is only £20.752,981, or 20l. 15s. 0½d. which exceeds the true value of it obtained before in Art. xciv, page 106, to wit, £20.739,25, or 20l. 14s. 9½d. by only 3½d. and the erroneous value of the same annuity for a life of the age of 3 years, obtained in page 130, is only £19.987,654, or 19l. 19s. 9d. which exceeds the true value of it, obtained before in Art. cv, page 114, to wit, £19.978,21, or 19l. 19s. 6½d. by only 2½d. I therefore hope the reader will be of opinion that it was hardly worth while, on account of this mistake, to be at the trouble and expense of computing all these numbers over again, and of causing so many pages to be reprinted; more especially as the true numbers, which express the values of all these annuities, are all set down in two different places in the subsequent part of this work, namely, in Table XI, pages 214, 215, 216, 217, and in Table XV, page 224. See more concerning this mistake in Art. cc, pages 218, 219.

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After this computation of a compleat set of the values of a life-annuity of one pound a year for every year of human life in pages 115, 116, &c. — 130, the said values are all set down in regular order in Table III, pages 131, 132, 133, 134, 135, and are expressed there in two different manners, to wit, in decimal parts of a pound in the second column, and in pounds, shillings, and pence in the fourth column. And the differences of the said values are likewise set down in the same table, and are expressed also in two different manners, namely, in decimal parts of a pound in the third column, and in pounds, shillings, and pence in the fifth column.

In this table, it is evident, the numbers relating to ages younger than 42 years are in a small degree greater than they should be, being the same with those obtained in the foregoing computation. But these numbers the reader may correct, if he thinks fit, by means of Table XI, pages 214, 215, &c. or Table XV. page 224.

In pages 136, 137, 138, 139, 140, 141, are some observations which will be of use to a calculator of such a table in discovering any arithmetical errors he may happen to have fallen into in the course of his computations; though it does not answer that purpose so compleatly as another method given us for that purpose by the above-mentioned ingenious Mr. Morgan, in his *Doctrine of Annuities and Assurances on Lives*, and which I have afterwards inserted in this work and explained in as ample a manner as I could, in pages 208, 209, 210, 211, 212, and 213.

Having in these first 142 pages of the book treated only of the true and exact methods of computing the values of life-annuities from tables of the probabilities of the duration of human life, I next proceed to give some account of what Mr. De Moivre and other writers on this subject have called *the decrements of human life*, and of the observations which they have made on the course, or variations, of the said decrements, as the ages of life increase; by which observations Mr. De Moivre was induced to adopt the hypothesis concerning the probabilities of human life which has been already alluded to, (though not perfectly agreeable to the truth,) for the
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sake of facilitating his computations. These *decrements of life* are the numbers of persons who, out of a given original number of persons, all of the same age, supposed to be living at a very early age, are represented, in tables of the probabilities of the duration of human life, as dying in the several succeeding years. In order to make the course of these decrements the more apparent, I have again inserted the above-mentioned tables of the probabilities of life of Mr. Kerseboom and Monsieur de Parcieux, under the titles of Table IV and Table V, with an additional column in each, containing the decrements of life corresponding to every year of life, or the number of persons who have died in the course of it. These tables are contained in pages 144, 145, 146, 147, 148, and 149. And from them it appears that in several parts of human life, the decrements of life, or the numbers of persons dying every year, are equal for seven or eight, and, in some parts, for twelve, or more, years together. Now, whenever this happens, the terms that express the present values of the future payments of one pound each, which are to be received at the ends of those years, will be a set of fractions, of which the numerators will form a decreasing arithmetical progression, and the denominators will form a decreasing geometrical progression; and therefore the sum of all those terms may be found at once by a single short expression, without taking the pains to compute each of the terms separately and then add them up into one sum. The method of finding a fit expression for this purpose is explained in the following 18 pages, to wit, pages 151, 152, 153, &c. — 168. And then in pages 169, 170, an account is given of Mr. De Moivre's hypothesis concerning the decrements of human life, and of the grounds upon which he conceived that the values of life-annuities computed from it would differ but in a small degree from their true values.

Page 171 contains Table VI, which is a small table of the values of a life-annuity of one pound a year for every fifth year of human life, computed by Dr. Halley from the Brewster table of the probabilities of the duration of human life, upon a supposition that the interest of money is 6 per cent. This was, probably, the first table of the values of life-annuities that ever was published. And with one of these values, but he does not say which, Mr. De Moivre tells us that he compared the value of the same annuity for a life of
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the same age computed from his own hypothesis, and found the two values to be so very little different from each other that, for all useful and practical purposes, they might well be considered as the same. And from hence he concluded (but, I think, too hastily,) that, in the business of computing the values of life annuities, he might safely neglect the tables of probabilities deduced from observations, and proceed upon the ground of his own hypothesis.

Pages 172, 173, are employed in shewing how the conjectural probabilities of the duration of human life, resulting from Mr. De Moivre's hypothesis, may be compared with the real probabilities of the same, as exhibited in Monsieur de Parcieux's table of them. And then in pages 174, 175, a comparison is made between the said probabilities, by setting down in Table VII, in two columns adjoining to each other, the numbers that express the probabilities of life according to Monsieur de Parcieux's table, and likewise those which express the same probabilities according to Mr. De Moivre's hypothesis: and at the end of this table of comparison between the said probabilities, some observations are made, in pages 176, 177, and 178, concerning the differences between them and the manner in which the values of life-annuities, computed from them, would be affected by them.

Afterwards in page 178, Art. CLIX, it is shewn that the numbers exhibiting the probabilities of human life according to Mr. De Moivre's hypothesis may be reduced to much smaller numbers, and even to such as are only equal to those which are necessary to compute the numbers of years in the several ages of life to the number contained in the utmost supposed extent of life, and which, from this circumstance, Mr. De Moivre calls the *complements of life*. And then in Table VIII, page 179, those lowest numbers expressing the probabilities of human life according to Mr. De Moivre's hypothesis, and which are the complements of life to its greatest possible extent, are set down in order, so as to enable the reader to compute the value of any life-annuity according to Mr. De Moivre's hypothesis with the greatest ease and readiness.

After Table VIII, I have added the computations of the values of a life-annuity of one pound a year for the several ages of 1 year, 3 years,

3 years, 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35 years, 40 years, 45 years, 50 years, 55 years, 60 years, 65 years, and 70 years, from Table VIII. or Mr. De Moivre's hypothesis, upon a supposition that the interest of money is $3\frac{1}{4}$ per cent. in order to compare them with the true values of the same annuity for the same lives obtained before from Monsieur de Parcieux's table of probabilities. These computations are contained in pages 180, 181, 182, &c. — 193. And the results of them, or the values of an annuity of one pound a year for lives of the said ages obtained by them, are set down in Table IX, page 194, together with the corresponding true values of the same annuity for the same lives according to Monsieur de Parcieux's table of probabilities, and with the differences between the said true values and the said conjectural values set down in an adjoining column.

By means of this table the reader is fully enabled to judge of the degree of exactness with which Mr. De Moivre's hypothesis enables us to find the values of life-annuities. And it appears that these conjectural values of life-annuities differ from their true values in many ages of life by more than a whole year's purchase, and after the age of 65 years by more than two years purchase, though about the age of 45 years they are very nearly equal to each other. These observations, with a few more of the same kind, are made in page 195, and a general conclusion is drawn from them against the expediency of ever making use of Mr. De Moivre's hypothesis in computing the values of life-annuities.

In pages 196, 197, 198, an account is given of another method of computing the values of life-annuities, which is different both from that above-explained in Prob. II. and its corollaries, and likewise from Mr. De Moivre's method by means of his hypothesis. This method was given by Mr. Weyman Lee, a Barrister at Law and Benchet of the Inner Temple, in a book he published on life-annuities in the year 1738. It is exceedingly erroneous, and gives the values of life-annuities, throughout the greatest part of human life, much greater than they should be. In the younger ages of life the difference of the erroneous value from the true one amounts to about 3 years purchase. Yet the principle, upon which Mr. Lee grounds this method, has something in it that is plausible at first sight, and

is apt to mislead the understanding with an appearance of truth and simplicity, unless it be examined with a great degree of attention. And for this reason I have thought it worth mentioning and examining at considerable length. The method itself is described in page 197; and some remarks on the difference of the values of life-annuities, resulting from it, from the true values of the same annuities, are made in page 198. And in pages 199, 200, 201, I have inserted a whole table of the values of a life-annuity of one pound a year for every age of life, computed by this method, upon a supposition that the interest of money is $3\frac{1}{4}$ per cent. This table is called Table X; and in it are set down, besides these erroneous values of a life-annuity of one pound a year for the different ages of life, computed according to Mr. Lee's method, the true values of the same annuity for the same ages computed in the manner above-explained in Prob. II, and its corollaries, and likewise the differences between the said erroneous values and true values, in an adjoining column.

In page 202 some remarks are made on this table; and in the five following pages, to wit, pages 203, 204, 205, 206, and 207, the reasons alledged by Mr. Lee in support of his method of determining the values of life-annuities are stated in his own words, and examined, and the fallacy, or insufficiency, of them is pointed out.

From this account of what has been delivered concerning Mr. De Moivre's hypothesis and Mr. Weyman Lee's erroneous method of valuing life-annuities, the reader must see that all the last 66 pages, from page 142 to page 207, inclusively, are not necessary to the right understanding of the true method of computing the values of life-annuities, but only to the knowledge of the most remarkable methods and opinions that have been adopted for this purpose by writers of note. And therefore such of my readers as have no curiosity for this historical kind of information, will do well to pass over all these pages, and go at once from page 142 to page 208. And such of them as are only desirous of understanding the right method of computing the values of these annuities, without wishing to make these computations familiar to them, will do well to pass over in like manner the foregoing 28 pages from the end of page 114 to the end of Art. cxxiv in page 142.

In page 208 I return to the more useful part of the subject, to wit, the true method of computing the values of life-annuities according to the principles of Prob. II, and its corollaries, and have employed that and the five following pages, to wit, pages 209, 210, 211, 212, and 213, in explaining a method invented by the ingenious Mr. Morgan above-mentioned for proving the truth of the computations of the values of life-annuities, (when they are deduced one from the other in a regular succession proceeding from the older lives to the next younger,) as fast as they are made. This method is so satisfactory, and answers the purpose, for which it was invented, so completely, that nothing further need be wished for on the subject. And therefore I thought it would be proper, not only to explain it and demonstrate the truth of it, but also to illustrate it by a number of examples. This is done in Table XI, pages 214, 215, 216, and 217; in which table all the values of a life-annuity of one pound a year for every different age of life, are regularly computed one from the other by means of the expression given by Mr. Morgan for that purpose, from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. and set down in the second column of the table over against the corresponding ages in the first column, and three other sets of numbers, which (according to this last-mentioned method of Mr. Morgan, that is explained in pages 208, 209, 210, 211, and 212,) are necessary to prove the truth of the former numbers, (or of the values of the annuities in the second column,) are set down in regular order in a third, a fourth, and a fifth column. These values may be depended upon, as exact: and it was by means of this table, (which carries its own proofs with it) that I found out the mistake I had made in page 123, where the value of an annuity of one pound a year for a life of the age of 41 years is made to be equal to £15.892,297, instead of £15.847,705; of which mistake and its consequences I have already said enough.

Pages 221, 222, 223, &c. ——— 232, contain twelve tables of the values of a life-annuity of one pound a year for all the different ages of human life, from the age of 3 years to the age of 93 years, accurately computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, according to the twelve following rates of interest; to wit, 2 per cent. $2\frac{1}{4}$ per cent.

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3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. This is a greater variety of tables of the values of life-annuities than has ever yet been published. And I will add, for the satisfaction of my readers, and of all such persons as shall be inclined to make use of these tables, that these values have, all of them, been confirmed by the computation of the corresponding proof numbers, in the manner just now mentioned; tho' it has not been thought necessary to insert those numbers in the tables.

These tables are all computed upon a supposition that the annuities are to be paid only once in every year, namely, at the end of every new compleat year after the purchase of them. But it more usually happens that life-annuities are made payable every half-year, and sometimes even every quarter of a year. In these cases the values of them will be somewhat greater than when they are payable only at the end of every year. But to determine exactly *how much* greater the values of them will be in those cases than when they are paid at the end of every year, (as they are supposed to be in the foregoing tables,) is a difficult and intricate question, and hardly worth the trouble of the solution. Yet, for the sake of my more curious and industrious readers, I have taken the first of these cases, to wit, that of annuities paid every half-year, into consideration, and have given a solution of it by approximation, which I find to agree, in the result of it, with that given by Mr. Thomas Simpson, in his doctrine of annuities; the result, or conclusion, of both investigations being, "that a life-annuity payable half-yearly is worth more than the same annuity, when payable only at the end of every year, by somewhat less than a quarter of a year's purchase." The investigation of this question in the following work takes up no less than 27 pages, to wit, pages 233, 234, 235, &c. — 259, and is extremely perplexing and unenterprising. Yet I do not know how to make it less so. I therefore advise the generality of my readers to pass it over, and take mine and Mr. Simpson's word for the truth of the conclusion just now mentioned, to wit, "that the difference of the values of a yearly and half-yearly life-annuity is less than a quarter of a year's purchase." I have followed this advice myself with respect to the values of life-annuities that are payable quarterly; not having taken the trouble of investigating the said values, but believing, upon the authority of the same Mr. Thomas Simpson, that those annuities are worth about three eighth parts of a year's purchase more than those which are paid only once a year, or one eighth part of a year's purchase more than those that are payable every half-year.

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From the subject of the above-mentioned tedious and perplexing investigation, I was led to another inquiry, that bears some analogy to it, but is much easier and more entertaining, namely, concerning the limit of the sum of money to which the interest made of a given sum of money in a year, or in any other given time, may be made to increase, by increasing the number, and diminishing the lengths, of the terms for which the money is lent, so as to improve the money during the said given time, by means of the said repeated loans, at compound interest. This problem and its corollaries are contained in pages 260, 261, 262, &c.—271. And I imagine, the reader will be entertained by them. The conclusion from the whole is, that the advantages that may be made by lending a sum of money for very small portions of a year, (as, for example, for 52nd parts, or weeks,) and then receiving the interest due upon it, and immediately lending both principal and interest at the same rate of interest; I say, the advantages that may be made by this method of proceeding above the interest that will arise from it by lending it at once for the whole year, are so very small as not to be worth attending to. Thus, for example, the interest of £100 in a whole year, if thus lent for only a week at a time, at the interest of 5 per cent. *per annum*, will be less than £5. 2s. 6d. or will exceed the interest of it, when lent at once for a whole year, by less than half a crown; and the utmost possible increase of this interest, or, to speak more properly, the *limit* of the said increase, or the quantity to which the interest of the said sum of money approaches continually, and to which it may be made to come as near as we please, as the number of the short terms, for which the money is lent, is increased *ad infinitum*, is only £5. 2s. 6½d. together with the 15th part of a penny.

In pages 271, 272, a remarkable analogy is pointed out between the infinite series that had been found in pages 268, 269, 270, for the limit of the increase of a sum of money at compound interest in the course of a year, and a certain ordinate of a logarithmick curve. And in pages 272, 273, 274, &c.—278, the said infinite series, expressing the said limit of the increase of a sum of money at compound interest, is further considered and explained.

Pages 278, 279, contain a suggestion that it might be an useful publick measure for the government, when it establishes annuities for a term of 30 years, to enlarge the grant of such annuities to the extent of the lives of the purchasers of them, in consequence of such

such an additional sum of money, to be paid by the said purchasers for such enlargement of their interest in the said annuities, as such enlargement may be fairly worth. Now the value of such an enlargement of a purchaser's interest in an annuity for 30 years certain is evidently the value of a remote life-annuity of the same annual amount for the life of the said purchaser, to commence at the expiration of the said 30 years. And therefore in pages 280, 281, 282, I have shewn how (without having recourse to new computations upon the principles delivered in Prob. II. and its corollaries) the value of such a remote life-annuity may easily be derived from that of an immediate life-annuity for the life of the same person, which, if the interest of money is supposed to be either 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, 8, 9, or 10 per cent. must be contained in one of the preceding twelve tables of the values of life-annuities in pages 221, 222, 223, &c. — 232. And then, in pages 283, 284, 285, 286, and 287, I have inserted four tables of the values of such remote life-annuities, computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, at the four following rates of interest, to wit, $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent. which are the rates that are most likely to be adopted in the establishment of any government securities.

In this part of the work I have digressed from the doctrine of life-annuities, (which is the proper subject of it,) to make some reflections on the National Debt, and the most likely methods of paying off a part of it. These reflections, together with various applications of the foregoing doctrine of life-annuities to different methods which may be taken for shortening the duration of this enormous debt, by converting a part of the perpetual annuities of which it consists, into annuities of a finite duration, such as life-annuities, or annuities for terms of years and likewise for life, and the like,) extend through no less than 102 pages, namely, from page 287 to page 389. And after this very long, but, I hope, not useless, digression, I have reprinted a very valuable pamphlet on the same subject of the National Debt, that was written by Sir Nathaniel Gould, a merchant of eminence, and a Director of the Bank, in the reign of King George the 1st. It was first published in the year 1726, and went through at least four editions: and Dr. Price has told us in his *Appeal to the Publick on the Subject of the National Debt*, that he sets so high a value

value upon it that he wishes he could put it into every hand in the kingdom. This declaration of so eminent a writer induced me first to peruse it, and, afterwards, finding it answer the high character that he had given of it, to cause it to be reprinted. And, to the end that gentlemen may be able to procure it without purchasing this large volume upon a subject that may not happen to interest them, I have caused an additional number of copies of this pamphlet to be printed off, which may be bought separately. This pamphlet ends at page 447. And in the following pages I have added a few reflections connected with the subject of the said pamphlet, concerning the expediency of an equal assessment of the land-tax. These reflections are contained in pages 448, 449, 450, 451, 452, 453, and 454. And with them ends every thing in the book that relates to the values of life-annuities for *single lives*.

In pages 455, 456, a short and convenient, algebraic, expression is given of the value of an annuity of one pound a year for a certain number of years, depending on the *joint continuance of two lives* of given ages; and a like expression is given for the value of such an annuity for the *whole* of the joint continuance of the said lives; which is but a particular case of the former expression. And in pages 457, 458, 459, and 460, examples are given of the computation of the values of such annuities, depending on two joint lives, by means of the said expressions.

In pages 460, 461, a like short and general, algebraic, expression is given for the value of a remote annuity of one pound a year depending on the joint continuance of two lives of given ages; and in pages 462, 463, 464, an example is given of the computation of the value of such a remote annuity by means of the said expression.

Then, in pages 465, 466, 467, and 468, I have set forth and explained another very useful method, with which we have been favoured by the before-mentioned Mr. Morgan, in his *Doctrine of Annuities and Assurances on Lives*; by which the computations of the values of an annuity for two joint lives are as much facilitated as the computations of the values of annuities for single lives are by his former method, which is mentioned and explained in the foregoing

going part of this work, in pages 109, 110, 111, and 112. For by it we are enabled to deduce with great ease (by an expression similar to that obtained in Art. 109, 110, 111, and 112,) the value of an annuity of one pound *per annum* for the joint continuance of any two lives whatever, from the value of the same annuity for the joint continuance of two lives that are respectively older than the two former lives by one year; so that a whole table of the values of such an annuity for two lives, that differ from each other by any given number of years, may be computed with almost as little labour as the two youngest lives possible that differ by the same number of years. Thus, for example, if the difference of the ages is 30 years, it will be almost as easy, by means of this expression of Mr. Morgan, to compute a whole table of the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 93 years and 63 years, 92 years and 62 years, 91 years and 61 years, 90 years and 60 years, and so on up to the ages of 33 years and 3 years, as to compute the value of a single annuity of 1 *l.* a year for the joint continuance of the said last two lives, which are of the ages of 33 years and 3 years. The great usefulness of such a method in facilitating the business of computing compleat tables of the values of annuities for two joint lives, need not be pointed out. I have therefore not only stated it, and demonstrated the truth of it, in the fullest and clearest manner I was able, in pages 464, 465, 466, and 467, but have afterwards illustrated it by a great number of examples in pages 468, 469, 470, 471, 472, 473, 474, and 475; so that I doubt not the attentive reader will have no difficulty in making himself compleatly master of it.

This method of Mr. Morgan for computing the values of annuities for two joint lives one from another, proceeding from the older lives to the younger, is evidently liable to the same inconvenience as his method of computing the values of annuities for single lives one from another, proceeding likewise from the older lives to the younger; namely, that when an error has once crept into the calculation of the value of any one of the annuities, it will affect those of all the remaining ones, that relate to younger lives. But Mr. Morgan has provided a similar and a very compleat remedy for this inconvenience in both cases, by shewing us how, by a certain counter-calculation accompanying that of the values of the annuities themselves,

we may constantly verify those values as fast as we obtain them. This remedy in the case of annuities for single lives I have described and demonstrated in pages 209, 210, 211, 212, and 213, as has been already mentioned. And here, in pages 476, 477, 478, 479, 480, &c.——487, I have explained and illustrated by examples the method prescribed by Mr. Morgan for effecting the same purpose in the case of annuities for two joint lives. And in pages 488, 489, I have inserted a table, to wit, Table XXVIII, containing the values of an annuity of one pound a year for the joint lives of two persons of the ages of 94 years and 84 years, 93 years and 83 years, 92 years and 82 years, 91 years and 81 years, 90 years and 80 years, and the ten next younger ages, up to those of 80 years and 70 years, computed one from the other by the above-mentioned expression of Mr. Morgan, which is explained and illustrated in pages 464, 465, &c.——475; and containing likewise, in three adjoining columns, to wit, in columns 3^d, 4th, and 5th, the proof-numbers which the said last-mentioned method of Mr. Morgan for confirming his calculations of the said values, and which is explained and illustrated in pages 476, 477, &c.——487, directs us to compute: which proof-numbers manifestly confirm the values of the annuities set down in the foregoing, or third, column; so that no doubt can be entertained of the truth of the said values.

And thus, by the help of Mr. Morgan's excellent methods, the computation of a table of the values of annuities for two joint lives, whose ages differ by the same number of years, is rendered as easily practicable, and as little liable to error, as that of a table of the values of annuities for single lives.

But it is evident that the number of the differences that may be supposed to subsist between the ages of two persons is so great that it is almost impossible to compute tables enough to exhibit the values of an annuity of one pound a year for two joint lives of all the different ages that may be taken. For this would require, at one rate of interest only, no fewer than 94 different tables, to wit, a table for two persons whose ages differ by 93 years, another for two persons whose ages differ by 92 years, a third for two persons whose ages differ by 91 years, a fourth for two persons whose ages differ by 90 years, and so on till we come to two persons of the same age. And

consequently, for the twelve different rates of interest above-mentioned, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. it would require twelve times 94, or near 1200, such tables: which it will, probably, never be thought worth while to cause to be computed. With respect therefore to the values of annuities for two joint lives, and still more with respect to those of annuities for three joint lives, (the varieties of which are still greater, and in a prodigious degree, than those of annuities for two joint lives,) we must be contented with a few tables accurately computed by the methods above-mentioned, for lives whose ages differ by only a moderate number of differences, far short of all the differences possible; and we must have recourse to methods of approximation for determining the values of such of these annuities as the tables we are possessed of do not exhibit. It is, however, desirable to have as many tables of the values of annuities for two joint lives accurately computed, as can be conveniently procured. And I think we have some reason to be surprized that so few tables of this kind have hitherto been published. To supply this defect in some degree I have procured ten different tables of the values of a life-annuity of one pound a year for two joint lives to be accurately computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. to wit, one table for two lives of the same age, another for two lives whose ages differ by five years, a third for two lives whose ages differ by 10 years, and a fourth, fifth, sixth, seventh, eighth, ninth, and tenth, for two lives whose ages differ by 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years. These tables are contained in pages 492, 493, 494, &c.—501; and in page 502 is given the value of an annuity of one pound a year for two joint lives of the ages of 93 years and 3 years; which is the only annuity that is possible, according to Monsieur de Parcieux's table of the probabilities of life, with a difference of 90 years between the ages.

After these tables, and a few remarks upon the difficulty of procuring complete sets of the values of all possible annuities for two joint lives, which make the subject of Art. ccccx in pages 502, 503, I have shewn how, by a certain method of *interpolation* between the values.

values of such annuities for two joint lives as are contained in the foregoing tables, we may derive very useful approximations to the values of other annuities for two joint lives that are not contained in them, at the same interest of $3\frac{1}{4}$ per cent. This method of interpolation is described and illustrated by examples in pages 503, 504, 505, 506, &c.—512. And the degree of exactness with which the numbers obtained by this method of interpolation, approach to, or exhibit, the true values of the annuities sought by them, is inquired into and shewn in pages 512, 513, 514, &c.—518. And by those means the reader is tolerably well enabled to find the value of one pound a year for the joint continuance of two lives of any ages, when the interest of money is $3\frac{1}{4}$ per cent.

I have then added the like number of tables of the values of an annuity of one pound a year for two joint lives, computed likewise from Monsieur de Parcieux's table of the probabilities of life, when the interest of money is $4\frac{1}{2}$ per cent. These tables are contained in pages 519, 520, 521, 522, &c.—528; and they exhibit the values of the said annuity of one pound a year for two joint lives, when the ages of the lives are equal, and when the differences of the said ages are (as before) 5 years, 10 years, 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years. And in Art. cccxxiii, page 529, is added the value of the said annuity of one pound a year for two joint lives of the ages of 93 years and 3 years, which is the only annuity that can be supposed, according to Monsieur de Parcieux's table of the probabilities of life, for two lives whose ages differ from each other by 90 years.

By the help of this second set of tables, and the method of interpolation before explained, the reader is tolerably well enabled to find the value of an annuity of one pound a year for the joint continuance of two lives of any ages whatsoever, when the interest of money is $4\frac{1}{2}$ per cent.

After this second set of tables of the values of annuities for two joint lives, which relates to the rate of $4\frac{1}{2}$ per cent. I have endeavoured to extend the use of these two sets of tables, which relate to the rates of $3\frac{1}{4}$ and $4\frac{1}{2}$ per cent. to the determination of the values of annuities for the same lives at the contiguous rates of interest of

3 per cent. 4 per cent. and 5 per cent. This is done by a method of proceeding not very different from that of the foregoing method of interpolation, and which may be called *The method of interpolation and continuation*. This method is described and explained in pages 530, 531, 532. And in pages 532, 533, 534, &c. — 539, an enquiry is made into the degree of exactness to which this method exhibits the values sought by it.

These methods of *interpolation* and *interpolation and continuation*, will only enable us to find near values of life-annuities for two joint lives at the same rates of interest as those which are computed in the two foregoing tables, to wit, the rates of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. and the three nearest rates of interest to them, and which differ from them by only $\frac{1}{4}$ per cent. to wit, 3 per cent. 4 per cent. and 5 per cent. In other rates of interest, that differ more considerably from those at which the tables are computed, it seems necessary to have recourse to some other method of approximating to the values of these annuities, that has no relation to those tables. This I have endeavoured to do in pages 540, 541, &c. — 546; in which I have given an expression of the value of an annuity of one pound a year for two joint lives, which I conceive to be, in most cases, a tolerable approximation to the true value of such annuity, and which may be applied with an equal probability of success in all the different rates of interest. This expression of the value of such an annuity for two joint lives is derived, by conjectural, but probable, reasonings, from the value of the like annuity for the older of the two single lives; which latter value will, if the interest of money is either 2 per cent. or $2\frac{1}{2}$ per cent. or 3 per cent. or $3\frac{1}{2}$ per cent. or 4 per cent. or $4\frac{1}{2}$ per cent. or 5 per cent. or 6 per cent. 7 per cent. 8 per cent. 9 per cent. or 10 per cent. be contained in one of the twelve tables of the values of annuities for single lives in pages 221, 222, 223, 224, &c. — 232. And by this means the said twelve tables of the values of annuities for single lives are rendered subservient to the purpose of finding the values of the like annuities for two joint lives: which seems to be a very convenient and useful principle to proceed upon in the valuation of these latter annuities, and of other annuities for more than two lives, which are still more numerous and complicated; provided the values of annuities for joint lives found by this method approach tolerably near to their

their true values. But, whether they do, or not, approach thus nearly to the said true values, can only be determined by trying them in a few cases, in which the true values have been actually computed, and comparing them with the said true values. This inquiry is made, with respect to the above-mentioned expression of the near value of an annuity of one pound a year for two joint lives (given in pages 540, 541, &c. — 546,) in pages 546, 547, 548, &c. — 550, by applying the said expression in a variety of examples to the investigation of near values of some of the annuities which are contained in the two foregoing sets of tables, and comparing the near values thereby obtained with the true values of the same annuities set down in the said tables: and the results of this comparison are set down in a table in pages 557 and 558. These results suggest a correction of the aforesaid expression (given in pages 540, 541, &c. — 546,) by multiplying it into the fraction $\frac{104}{100}$, which makes the said expression approach (in most cases) considerably nearer than before to the true values of the annuities sought by it. The results of the comparison of the near values, arising from this corrected expression, with the true values of the same annuities, are set down in another table in pages 558, 559, and 560. And in pages 560, 561, a remark is made on the said last table, in which the said corrected expression, to wit, $A \times \frac{104}{100} \times P \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_1}$, is finally recommended as a tolerably exact method of obtaining a near value of an annuity of one pound a year for two joint lives, when the interest of money is either less than 3 per cent. or greater than 5 per cent. and the before-mentioned methods of *interpolation*, and *interpolation and continuation* consequently cannot be applied.

The remaining part of the book relates to the values of annuities depending on three lives. In Art. CCCCLXXI, pages 561, 562, 563, a short and convenient, algebraick, expression is given of the value of an annuity of one pound a year for a given number of years, provided three lives of given ages shall all continue so long in being; which expression, whenever the said given number of years is equal to the greatest possible number of years during which the oldest of the three lives can be extended, will become the expression of the value of such an annuity during the whole joint continuance of the three lives.

lives. And in Art. cccclxxii, pages 564, 565, 566, an example is given of the computation of the value of such an annuity for the joint continuance of three lives of the ages of 75 years, 80 years, and 85 years, by means of the said expression. But, as the computation of these values of annuities for three joint lives occasionally, or as we want them, by means of the aforesaid expression, in the manner exemplified in Art. cccclxxiii, is in most cases, and especially when the lives are all young, extremely tedious;— and the immense variety of combinations with each other, which the ages of three different lives will admit of, makes it almost impossible to compute any competent numbers of these values;— it is still more necessary than in the case of annuities for two joint lives, to have recourse to a shorter, though less accurate, method of computing them. With this view I have endeavoured to find a method of deriving the value of an annuity of one pound a year for three joint lives of given ages from the value of the same annuity for the joint continuance of the two older of the said lives, which I have supposed to be either contained in some of the tables that have been computed for two joint lives, or to be derivable from those tables by the above-mentioned methods of *Interpolation* and *Interpolation and Continuation*, or to be obtainable by means of the corrected expression recommended for this purpose in pages 560, 561. And I have succeeded in some degree, (though not so well as I could wish,) in this endeavour, having found an expression for the value of such an annuity for three joint lives, which exhibits the said value, for the most part, within less than a 20th part of the truth. The method in which this expression is obtained, is explained in pages 568, 569, and 570. And the following twenty pages, to page 590, are employed in making a variety of trials of the degree of exactness with which this expression exhibits the values of annuities for three joint lives, by comparing the values of such annuities, obtained by means of the said expression, with the values of the same annuities accurately computed, as they are contained in certain tables of Mr. Morgan and Dr. Price. From all these trials, taken together, I have ventured to conclude in Art. cccxcv, page 590, that it seems reasonable to conjecture that the said expression given in pages 568, 569, and 570, to wit, $B \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_{ii}}{g \times P_i - b \times P_{ii}}$ will always give

us a tolerable approximation to the true value of an annuity of one pound a year for three joint lives of any ages whatsoever.

In pages 590, 591, 592, another method of approximating to the value of an annuity of one pound a year for the joint continuance of three lives of given ages, is delivered; which was published, and, I presume, invented, by the late very learned mathematician, Mr. Thomas Simpson, of Woolwich. And in pages 592, 593, an example is given of the computation of the value of such an annuity by this method in the case of three equal joint lives, all of the age of 60 years: and the near value of the said annuity, obtained by this computation, is found to be somewhat nearer than the near value of the same annuity resulting from the former expression, $B \times \frac{P_1}{P} \times$

$\frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$, to the true value of that annuity. And in the following five pages, to wit, 594, 595, 596, 597, and 598, further trials are made of the exactness of this method of Mr. Simpson; from all which it is concluded in page 599, that the said method of Mr. Simpson is, for the most part, more exact, as well as always much shorter and easier to practice, than the other approximation, by means of the expression $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$, and

upon the whole, deserves to be preferred to it. Yet, as there are now and then some instances in which the other method, by means of the expression $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$, comes nearer to the truth than this method of Mr. Simpson, I think it is convenient to be possessed of both methods, to the end that in doubtful cases we may resort to one of them as a kind of confirmation of the result obtained by the other, to a certain degree of exactness.

In Art. ccccx, page 599, it is observed, that, in all the trials of Mr. Simpson's approximation to the values of annuities for three joint lives, which have been made in the preceding pages, the differences of the near values thereby obtained, from the true values of the same annuities, respectively, seldom exceed an eighth part of a year's

year's annuity : which is a sufficient degree of exactness for all practical purposes, and consequently is a great recommendation of the said method.

The five remaining pages of the book, to wit, pages 600, 601, 602, 603, 604, relate to the values of annuities for the longest of two, or three, lives of given ages. As it has been shewn in the former part of this work, in the 4th and 11th corollaries to Prob. IV, that the values of all annuities of this kind may be derived from those of the same annuities for the same lives taken singly and jointly, it is shewn that, if there be three lives of different ages, and A be put for the value of an annuity of one pound a year for the youngest life, and B for the value of a like annuity for the middle life, and C for the value of a like annuity for the oldest life; and AB be put for the value of a like annuity for the joint continuance of the first, or youngest life, and the middle life, and AC for the value of the like annuity for the joint continuance of the first, or youngest life, and the oldest life, and BC for the value of a like annuity for the joint continuance of the middle life and the youngest life; and ABC be put for the value of a like annuity for the joint continuance of all the three lives; I say, it is shewn that, if this notation be used, the value of a like annuity of one pound a year for the longest of the first and second lives will be equal to $A + B - AB$, and the value of a like annuity for the longest of all the three lives will be equal to $A + B + C + ABC - AB - AC - BC$, or $A + B + C - AB - AC - BC + ABC$. Then, in Art. ccccxii, pages 601, 602, an example is given of the computation of the value of an annuity of one pound a year for the longest of two lives of the ages of 20 years and 30 years, by means of the expression $A + B - AB$; and in Art. dxiii, pages 602, 603, 604, an example is given of the computation of a like annuity of one pound a year for the longest of three lives of the ages of 20 years, 30 years, and 40 years, by means of the expression $A + B + C - AB - AC - BC + ABC$. After which example this treatise on life-annuities is concluded in page 604.

The Appendix to this treatise on Life-annuities consists of the abovementioned *Bill for establishing Life-annuities in Paris*, which the late Mr. Dowdeswell brought into the House of Commons in the year 1773, together with the two tables of the values of life-annuities, at the interest of 3 per cent: for the use of parishes in London and in the country, which Sir George Savile procured to be computed, under the inspection of Dr. Price, for the purposes of the said bill, and which were considered as a part of it. The reasons for reprinting these tables and the bill to which they refer, are stated in page 605.

A LIST of ERRATA, to be corrected in the following Work, and of some AMENDMENTS to be added to it.

In page 30, line 5th, instead of "£.2612" read "£.2696".

In page 31, line 5th, instead of "£3.7212, or £3. 14s. 5d." read "£3.7296, or £3. 14s. 7d."

In the same page, line 8th, instead of "£.2612" read "£.2696"; and in line 9th, instead of "£.7814" read "£.7898"; and in line 11th, instead of "£3.7212" read "£3.7296"; and in line 13th, instead of "£.7814" read "£.7898".

In page 32, dele the note at the end of Art. xxxiii, beginning thus, *N.B. This result is greater, and containing six lines.*

In page 34, line 4th, instead of "£.2612" read "£.2696"; and line 14th, instead of "£.8650" read "£.8734".

In page 48, Art. xlix, line 3^d, after the word "annuity" insert the words "of one pound a year."

In page 96, line 4th, dele the comma after the word "as".

xxxiv ERRATA AND AMENDMENTS.

In page 107, Art. xcvi, lines 8th and 9th, instead of the words "almost one half a year's payment, and consequently would be worth more than 21 years purchase," read these words, "about one half of one of those half-yearly payments, or a quarter of a year's annuity, and consequently would be worth very nearly 21 years purchase."

In page 123, the last line, instead of $\frac{1}{1.035} \times \frac{\text{£}}{16.448,528} = \text{£}15.892,$

297, or $\text{£}15.17s. 10d.$ read $\frac{1}{1.035} \times \frac{\text{£}}{16.402,375} = \text{£}15.847,$

705, or $\text{£}15.16s. 11\frac{1}{2}d.$

In page 136, line 2nd, after the words "one pound" insert the words "a year."

In page 137, line 7th, after the words "one pound" insert the words "a year."

— line 18th, after the word "pence" place a semicolon; and add these words, "which does not differ greatly from 2 pence, 3 pence, farthing, and 3 pence, half-penny, which are the differences of the four next preceding differences."

In page 159, line 5th, instead of $\frac{1}{r-1} \times \frac{1}{r^n \times r-1}$ read $\frac{1}{r-1}$

$$\frac{1}{r^n \times r - 1}$$

In page 161, Art. cxli, line 1st, instead of "n" read "a"; and in line 6th, or the line at the bottom of the page, instead of "equal to a" read "equal to 0".

In page 168 the operation of the algebraic division of the series $\frac{n-1}{n}$

$$\frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \&c. - \frac{1}{nr^n - 1} \text{ by } r - 1 \text{ might have been}$$

set forth more fully by inserting the last term of it, to wit, $-\frac{1}{nr^n - 1}$

after the &c. and at the same time inserting the corresponding terms in the quotient and in the series of successive dividends. If this had been done, (which, perhaps, it would have been better to do) the whole operation would have been as follows.

$$r-1)$$

ERRATA AND AMENDMENTS.

xxxv

$$(t-1) \frac{n-1}{n} - \frac{1}{n^2} - \frac{1}{n^3} - \frac{1}{n^4} - \text{&c.} - \frac{1}{n^{n-1}} \left(\frac{n-1}{n^2} + \frac{n-2}{n^3} + \frac{n-3}{n^4} + \frac{n-4}{n^5} + \text{&c.} + \frac{n-|n-1|}{n^{n-1}} \right),$$

$$\frac{n-1}{n} - \left| \frac{n-1}{n^2} \right| + \frac{n-2}{n^2} - \left| \frac{n-2}{n^3} \right| + \frac{n-3}{n^3} - \left| \frac{n-3}{n^4} \right| + \text{&c.} + \frac{1}{n^{n-1}}.$$

$$\frac{n-2}{n^2} - \left| \frac{n-2}{n^3} \right| + \frac{n-3}{n^3} - \left| \frac{n-3}{n^4} \right| + \text{&c.}$$

$$\frac{n-3}{n^3} - \left| \frac{n-3}{n^4} \right| + \frac{n-4}{n^4} - \left| \frac{n-4}{n^5} \right| + \text{&c.}$$

$$\frac{n-4}{n^4} - \left| \frac{n-4}{n^5} \right| + \frac{n-5}{n^5} - \left| \frac{n-5}{n^6} \right| + \text{&c.}$$

In page 216, in the fifth column of numbers, the 22nd number, 4,697,003, instead of a comma place a full stop “.” after the highest figure, 4.

In page 225, Table XVI, in the fourth column of numbers, in the first, the eighth, and the fifteenth numbers, which answer to the ages of 34 years, 41 years, and 48 years, place full stops, “.” “.” and “.” after the two highest figures, 16, 14, and 13, of those numbers, respectively.

In page 240, line 4th, after the &c. insert these words, “ and with $r\frac{1}{2} \times \frac{f_1}{2P} \times$.”

And in line 5th, instead of “ $\frac{f_1}{2P}$ ” read “ $\frac{f_1}{P}$ ”.

And in line 7th, instead of “ $\frac{PE-N+1}{rE-N+1}$ ” read “ $r\frac{1}{2} \times \frac{1}{2P} \times \frac{PE-N+1}{rE-N+1}$ ”.

And

xxxvi ERRATA AND AMENDMENTS.

And in lines 8th and 9th, after " $+ \frac{Piv}{r^5} +$ &c." read as follows,

$$+ \frac{r^{\frac{1}{2}}}{4} \times \frac{1}{P} + \frac{1}{2P} \times \text{the term } \frac{\pi E - N + 1}{rE - N + 1} = (\text{if we neg-}$$

lect this last quantity $\frac{1}{2P} \times \frac{\pi E - N + 1}{rE - N + 1}$ on account of its extreme smallness) &c." as in the book.

In page 300, line 5th from the bottom, instead of "this third method" read "this fourth method".

In page 302, line 20th, instead of " $\frac{1000,000}{554,338}$ " read " $\frac{1000,000}{554,338}$ ".

In page 368, the last line, in the number £4,711,195,1609, place a full stop before the four last figures, thus, 4711,195.1609.

In page 502, line 6th, after the words, "by 90 years," insert these words, "for the joint continuance of which an annuity will be of any value,"

— In line 9th, at the end of Art. ccccxix, add these words, "See below, page 529. Art. ccccxixxi.".

In page 560, line 2nd from the bottom, instead of "to the instances" read "in the instances".

In page 581, Art. cccclxxv, line 11th, instead of "lives in" read "lives in".

In page 592, Art. ccccxix, line 7th, after the word, "interest" insert the word "of".

In page 597, in the marginal abstract of Art. cccxcv, line 10th of the said abstract, after the words "than the near values" insert the words, "of the same annuities".

THE PRINCIPLES, &c.

ARTICLE I.

THE doctrine of life-annuities is by no means of so abstruse and difficult a nature as many people are apt to imagine. A moderate

Design of this tract.

share of common sense, or capacity to reason justly, and a knowledge of common arithmetick, are all the qualities that are necessary to a right understanding of the principles on which it is founded; even so far as to be able to compute the value of any proposed annuity for any given life or number of lives, if a person is disposed to undergo the labour of performing all the necessary arithmetical operations that arise in such a computation. To explain these principles in an easy and familiar manner, so as to make them intelligible to as many readers as possible, without having recourse to Algebra or the books written on the doctrine of chances, is the design of the following pages: which, as the subject of life-annuities is a matter of very general concern, will, I flatter myself, be considered by the publick as an useful and commendable undertaking.

II. A life-annuity is a set of equal sums of money to be paid at certain future times, at the distance of a year one after the other, during the whole course of the life of the person for whose life the annuity is granted. Definition of a life-annuity.

Thus, if a man grants an annuity of 100*l.* a year to a young man of twenty years of age, he thereby undertakes to pay him one sum of 100*l.* at the end of one year from the time of making him the grant, and another sum of 100*l.* at the end of two years from the same time, and a third sum of 100*l.* at the end of three years from the same time, and a fourth sum of 100*l.* at the end of four years from the same time; and so on, paying a new sum of 100*l.* at the end of every following year throughout the whole life of the grantee. If therefore we can find the values of these several single payments of 100*l.* each, which are to be made to the grantee of the life-annuity at the end of the several future years of his life, but of which no part is to be paid to his executors, or other representatives, when he is dead, we may, by adding all these values together, find the value of the sum of all these payments, or of the whole life-annuity. We must there-fore

Of the present value of a future contingent payment of a given sum of money.

fore endeavour to find some method of determining the present value of a single future payment of any given sum of money to be made to a person of any given age at the distance of any given number of years, or when he shall have attained to any greater given age. Now this value will depend upon two circumstances; which must therefore be previously agreed upon, before it can be known: namely, the rate of interest at which money may be improved, and the probability of the duration of human life. For, the higher the interest of money is, the less ought the grantor to take for his grant of a given sum of money, to be paid at a future time: because, when the interest of money is high, he may increase such lesser sum of money, in the interval of time between the grant and the future payment, so as to make it amount to as great a sum at that time as would have been produced by a greater sum of money received at the time of making the grant, and improved likewise at compound interest during the same interval, if the interest of money had been lower. And the greater is the frailty of human life, or the greater the probability that the grantee of the proposed future payment will die before it becomes due, the less also ought to be the sum of money paid to the grantor for his grant of it, because he will run a smaller risk of being obliged to pay it. We must therefore in every question of this sort determine beforehand at what rate of interest the money paid to the granters for such future payments may be improved, and with what degree of probability it may be expected that the persons to whom they are granted will, or will not, live till they become due.

Of the probability of the duration of human life at a given age.

III. Now, as to this latter circumstance of the degree of probability, that a person of a given age will, or will not, live to any other given age, or till the sum of money granted to him becomes due, it is obviously in many cases a matter of very great uncertainty, and will be often very different in different persons of the same age. The chance which a man of 30 years of age, who is in good health and leads a temperate and quiet life in the country, has to live 20 years, or till he is 50 years of age, is evidently much greater than that of another man of the same age of 30 years, and of the same degree of health and vigour of body, who is going into a hot and unhealthy climate, to which he has never been accustomed, as, for example, to Senegal in Africa: and it is likewise greater than that of another man of 30 years of age and of the same degree of health and vigour, but who lives in a capital city and in scenes of pleasure and debauchery; and still more evidently it is greater than that of another man of 30 who is of a weakly and unhealthy constitution of body, or who by his daily occupation is exposed to many dangers of his life, from which the generality of mankind are exempt, as is the case with soldiers and sailors in time of war. But these are circumstances out of the reach of calculation, and must be left to be considered by the persons who grant and purchase life-annuities according to their own judgment and discretion in

in the particular cases in which they occur. All that can be done by any general rules upon this subject, is to estimate the degree of probability with which it may reasonably be expected that a person of any given age will live to any other given age upon a supposition that he has neither a better nor a worse chance of doing so than the majority of other persons of the same age. And this medium, or average, chance of living is determined by tables that exhibit the numbers of persons which, out of a certain pretty large number of children of one, or two, or three years of age, (which is usually not less than 1000,) all living at the same time, are found (by methods of reasoning that are grounded on long serieses of observations,) to be living at the end of every subsequent year of human life to its extreme period, which some of the tables carry to 86, and others to more than 90, years. The instances of the prolongation of human life to more than 100 years are so unfrequent, that they are not thought to be worth attending to in forming any general rules upon this subject.

Of tables of the said probability.

IV. The most exact tables of this kind that have hitherto been published seem to be those of Mr. Kerseboom, and Monsieur de Parcieux; which are inserted in the Appendix to Mr. De Moivre's Treatise on the Valuation of Annuities. The former was published in an essay of the afore said Mr. Kerseboom on the number of people in the provinces of Holland and West-Friesland, written in the Dutch language, about the year 1738, (of which an account is given in the 9th volume of the Abridgement of the Philosophical Transactions, page 326,) and is said to have been formed from certain tables of assignable annuities for lives in Holland, which had been kept there for 125 years, and in which the ages of the several persons dying in that period had been truly entered. And Mr. de Parcieux's table was made, as Mr. De Moivre informs us, by a like use of the lists of the *French Tontines*, or *Long annuities*; and the numbers of it were verified by the *neurologies*, or mortuary registers, of several religious houses of both sexes. These seem to be the most solid and authentick grounds upon which it is possible to form any tables of this kind: whereas there are some circumstances of doubt and uncertainty in the methods of forming all the other tables of the probable duration of human life which prevent them from being intirely satisfactory. And therefore I conceive these two tables to be more exact and fit to be adopted in computing the values of life-annuities than any other tables I have seen; and particularly in computing the values of any annuities for lives which the government of this kingdom may at any time think fit to grant, if that method of raising money should hereafter be adopted, (as is the case at this time in Ireland,) or it should be thought expedient to discharge a part of the national debt in that way, by converting a part of the perpetual three per cent. annuities, payable at the Bank, into annuities for the lives of their respective proprietors, or for a term certain of 20 or 30 years and further for their lives. These two tables are as follows.

Of the tables of probability of Mr. Kerseboom and Monsieur de Parcieux.

These two tables are as follows.

B 2

T A B L E

The Principles of the Doctrine of

V. T A B L E I.

Representing the probabilities of the duration of human life at the several ages therein mentioned from the time of birth to the age of an hundred years: grounded on the registers of certain assignable annuities for lives granted by the government of Holland, which had been kept there for 125 years, and in which the ages of the several annuitants dying during that period had been truly entered.

By Mr. KERSSEBOOM.

Age.	Persons living.	Age.	Persons living.	Age.	Persons living.	Age.	Persons living.
0	1400	26	760	52	482	78	130
1	1125	27	747	53	470	79	115
2	1075	28	735	54	458	80	100
3	1030	29	723	55	446	81	87
4	993	30	711	56	434	82	75
5	964	31	699	57	421	83	64
6	947	32	687	58	408	84	55
7	930	33	675	59	395	85	45
8	913	34	665	60	382	86	36
9	904	35	655	61	369	87	28
10	895	36	645	62	356	88	21
11	886	37	635	63	343	89	15
12	878	38	625	64	329	90	10
13	870	39	615	65	315	91	7
14	863	40	605	66	301	92	5
15	856	41	596	67	287	93	3
16	849	42	587	68	273	94	2
17	842	43	578	69	259	95	1
18	835	44	569	70	245	96	0.6
19	826	45	560	71	231	97	0.5
20	817	46	550	72	217	98	0.4
21	808	47	540	73	203	99	0.2
22	800	48	530	74	189	100	0.0
23	792	49	518	75	175		
24	783	50	507	76	160		
25	772	51	495	77	145		

VI. The meaning of the foregoing table is this; That from the observations on which it is grounded there is reason to conclude that out of 1400 new-born children 1125 have lived to the end of one year, and 1075 to the end of two years, and 1030 to the end of three years; and so on throughout the table; the figures that are parallel and contiguous to every age expressing the number of persons out of the aforesaid original number of 1400 infants that are living at that age. As to the figures 0.6, 0.5, 0.4, 0.2, adjoining to the 96th, 97th, 98th, and 99th year of the hundred, which denote the decimal fractions $\frac{6}{10}$, $\frac{5}{10}$, $\frac{4}{10}$, and $\frac{2}{10}$, it is obvious that in a strict sense they are inapplicable to any human, or other living, creatures: and therefore they must be understood in the following manner. We must suppose the original number of new-born infants, who were the root, or basis, of the table, to have been increased tenfold, so as to become 14000 instead of 1400: in which case it is evident that all the following numbers in the table, representing the persons living at the several ages therein set down, will likewise be increased to ten times their former quantity, and therefore the number of persons living at the age of 95 years will be 10 instead of 1. Now the meaning of the figures 0.6, 0.5, 0.4, 0.2, and of the cyphers 0.0 adjoining to the 100th year, is that, out of the said 10 persons who would in such case have been found to be living at the age of 95 years, only 6 would have been found to be living at the age of 96, and 5 at the age of 97, and 4 at the age of 98, and 2 at the age of 99, and none at all at the age of an hundred years.

VII. The second table above-mentioned, given us by Mr. De Parcieux, and inserted by Mr. De Moivre in the appendix to his treatise on the valuation of annuities, is as follows.

The Principles of the Doctrine of

T A B L E II.

Representing the probabilities of the duration of human life at the several ages therein mentioned, from the age of 3 years to the age of 95: grounded on lists of the French Tontines, or Long Annuities, and verified by a comparison thereof with the necrologies, or mortuary registers, of several religious houses of both sexes.

By Monsieur DE PARCIEUX.

Age.	Persons living.	Age.	Persons living.	Age.	Persons living.	Age.	Persons living.	Age.	Persons living.
3 Yrs.	1000	23	790	43	636	63	423	83	71
4	970	24	782	44	629	64	409	84	59
5	948	25	774	45	622	65	395	85	48
6	930	26	766	46	615	66	380	86	38
7	915	27	758	47	607	67	364	87	29
8	902	28	750	48	599	68	347	88	22
9	890	29	742	49	590	69	329	89	16
10	880	30	734	50	581	70	310	90	11
11	872	31	726	51	571	71	291	91	7
12	866	32	718	52	560	72	271	92	4
13	860	33	710	53	549	73	251	93	2
14	854	34	702	54	538	74	231	94	1
15	848	35	694	55	526	75	211	95	0
16	842	36	686	56	514	76	192		
17	835	37	678	57	502	77	173		
18	828	38	671	58	489	78	154		
19	821	39	664	59	476	79	136		
20	814	40	657	60	463	80	118		
21	806	41	650	61	450	81	101		
22	798	42	643	62	437	82	85		

VIII. Both the foregoing tables represent human life as being more permanent, or the number of persons dying every year, out of a given number of persons alive at any given younger age, as being smaller, throughout both youth and middle age, and even till about the age of 70, than they appear to be by most other tables. And this is what we ought naturally to have expected, on account of the difference of the *data*, or grounds, on which these and those other tables have been constructed. For both these tables of Mr. Kerseboom and Mr. de Parcieux were formed, as we before observed, from the registers of the deaths of persons who had purchased life-annuities from the governments of Holland and France: whereas those other tables have been formed, for the most part, from the registers of the births and burials of all the inhabitants in general of the places for which they are computed. Now it is reasonable to believe that those persons who purchase annuities of a government are, for the most part, in easier circumstances, and less exposed to the dangers and hardships that destroy, or shorten, human life, than the rest of mankind, and, consequently, than the whole body of mankind (themselves included) upon an average: and therefore it is not likely that they should die off in the same average, or general, proportion, but at a slower rate.

Of the difference between the two foregoing tables of probabilities.

IX. But though both these tables of Mr. Kerseboom and Monsieur de Parcieux are formed from the registers of the deaths of government life-annuitants, and both of them represent the probability of the duration of human life as greater than it appears to be by any other tables, until towards the age of 70 years, yet they do not intirely agree with each other; but the table of Mr. de Parcieux represents that probability as still greater than that of Mr. Kerseboom, till towards the said advanced age of 70 years, and from that time somewhat less. The degree of difference between them in this respect may be collected from a comparison of them with each other at the several ages of 20, 30, 40, 50, 60, 70, and 80 years, which may be made in the following manner.

A comparison of them with each other with respect to persons of the age of 20 years.

X. Let us first take the persons living together at the age of 20 years in each of these tables, and see how many of those persons are represented in each of them to be living at the following ages of 30, 40, 50, 60, 70, 80, and 90, years, and then inquire in which of these two tables the proportions by which these numbers continually decrease are greatest.

Now

Now it appears by Mr. Kerseboom's table that out of 817 persons of 20 years of age, all living at the same time,

711	will have lived to the age of 30 years,
and 605	to the age of 40,
and 507	to the age of 50,
and 382	to the age of 60,
and 245	to the age of 70,
and 100	to the age of 80,
and 10	to the age of 90 years.

And by the table of Monsieur de Parcieux it appears that out of 814 persons of the same age of 20 years, all living at the same time,

734	will have lived to the age of 30 years,
and 657	to the age of 40,
and 581	to the age of 50,
and 463	to the age of 60,
and 310	to the age of 70,
and 118	to the age of 80,
and 11	to the age of 90 years.

It appears therefore that according to Monsieur de Parcieux's table there will be more persons found to have been living at the several ages of 30, 40, 50, 60, 70, 80, and 90, years, out of an original number of only 814, persons of the age of 20 years, all living at the same time, than are found to have been living at the same ages respectively, according to Mr. Kerseboom's table, out of the greater original number of 817 persons of the same age of 20 years all living at the same time; and that in the very considerable proportions of 734 to 711, 657 to 605, 581 to 507, 463 to 382, 310 to 245, 118 to 100, and 11 to 10. It is evident therefore that a life-annuity for the life of a person of 20 years of age is worth considerably more money according to Mr. de Parcieux's table than according to that of Mr. Kerseboom.

The like comparison of them with respect to the age of 30 years.

XI. In the next place we will compare these two tables together with respect to the age of 30 years.

Now

L I F E - A N N U I T I E S .

9

Now by Mr. Kerseboom's table it appears that out of 711 persons of the age of 30 years, all living at the same time,

605	will have lived to the age of 40 years,
and 507	to the age of 50,
and 382	to the age of 60,
and 245	to the age of 70,
and 100	to the age of 80,
and 10	to the age of 90 years.

And by the table of Monsieur de Parcieux it appears that out of 734 persons of 30 years of age, all living at the same time,

657	will have lived to the age of 40 years,
and 581	to the age of 50,
and 463	to the age of 60,
and 310	to the age of 70,
and 118	to the age of 80,
and 11	to the age of 90 years :

and consequently out of 711 persons of 30 years of age, all living at the same time, (which is the number of persons living at that age in Mr. Kerseboom's table,)

$(\frac{657 \times 711}{734}, \text{ or })$	636	will have lived to the age of 40 years,
and $(\frac{581 \times 711}{734}, \text{ or })$	563	to the age of 50,
and $(\frac{463 \times 711}{734}, \text{ or })$	448	to the age of 60,
and $(\frac{310 \times 711}{734}, \text{ or })$	300	to the age of 70,
and $(\frac{118 \times 711}{734}, \text{ or })$	114	to the age of 80,
and $(\frac{11 \times 711}{734}, \text{ or })$	10	to the age of 90 years.

It appears therefore that out of the same original number of 711 persons of 30 years of age, all living at the same time, there will be more persons found to be living at the several subsequent ages of 40, 50, 60, 70, and 80, years, according to Monsieur de Parcieux's table than according to that

that of Mr. Kerseboom, in the several very considerable proportions of 636 to 605, 563 to 507, 448 to 382, 300 to 245, and 114 to 100, and the same number, to wit, 10 persons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a person of 30 years of age would be worth a considerably greater sum of money according to Monsieur de Parcieux's table than according to that of Mr. Kerseboom.

The like comparison of them with respect to the age of 40 years.

XII. We will next compare these two tables together with respect to the age of 40 years.

Now by Mr. Kerseboom's table it appears that out of 605 persons of 40 years of age, all living at the same time,

507	will have lived	to the age of 50,
and 382		to the age of 60,
and 245		to the age of 70,
and 100		to the age of 80,
and 10		to the age of 90 years.

And by the table of Mr. de Parcieux it appears that out of 657 persons of 40 years of age, all living at the same time,

581	will have lived	to the age of 50 years,
and 463		to the age of 60,
and 310		to the age of 70,
and 118		to the age of 80,
and 11		to the age of 90 years:

and consequently out of 605 persons of 40 years of age, all living at the same time,

$(\frac{581 \times 605}{657}, \text{ or })$	540	will have lived	to the age of 50 years,
and $(\frac{463 \times 605}{657}, \text{ or })$	426		to the age of 60,
and $(\frac{310 \times 605}{657}, \text{ or })$	285		to the age of 70,
and $(\frac{118 \times 605}{657}, \text{ or })$	108		to the age of 80,
and $(\frac{11 \times 605}{657}, \text{ or })$	10		to the age of 90 years.

It appears therefore that out of the same original number of 605 persons of 40 years of age, all living at the same time, there will be more persons found to have been living at the several subsequent ages of 50, 60, 70, and 80 years according to Monsieur de Parcieux's table than according to that of Mr. Kerseboom, in the several considerable proportions of 540 to 507, 426 to 382, 285 to 245, and 108 to 100; and the same number, to wit, 10 persons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a person of 40 years of age would be worth considerably more by Mr. de Parcieux's table than by that of Mr. Kerseboom.

The like comparison of them with respect to the age of 50 years.

XIII. We will next compare these two tables with each other with respect to the age of 50 years.

Now by Mr. Kerseboom's table it appears that out of 507 persons of 50 years of age, all living at the same time,

382	will have lived to the age of 60 years,
and 245	to the age of 70,
and 100	to the age of 80,
and 10	to the age of 90 years.

And by the table of Monsieur de Parcieux it appears that out of 581 persons of the age of 50 years, all living at the same time,

463	will have lived to the age of 60 years,
and 310	to the age of 70,
and 118	to the age of 80,
and 11	to the age of 90 years:

and consequently out of 507 persons of 50 years of age, all living at the same time, (which is the number of persons living at that age in Mr. Kerseboom's table)

$(463 \times \frac{507}{581}, \text{ or } 404)$	will have lived to the age of 60 years,
and $(310 \times \frac{507}{581}, \text{ or } 270)$	to the age of 70,
and $(118 \times \frac{507}{581}, \text{ or } 103)$	to the age of 80,
and $(11 \times \frac{507}{581}, \text{ or } 10)$	to the age of 90 years.

It appears therefore that out of the same original number of 507 persons of 50 years of age, all living at the same time, there will be more persons found to have been living at the several subsequent ages of 60, 70, and 80 years, according to Mr. de Parcieux's table than according to that of Mr. Kerseboom, in the proportions of 404 to 382, 270 to 245, and 103 to 100; and the same number, to wit, 10 persons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a person of 50 years of age, as well as a life-annuity for the life of a person of 20, or 30, or 40, years of age, will be worth more according to the table of Monsieur de Parcieux than according to that of Mr. Kerseboom. But the difference of the values of such an annuity according to these two different tables will not be so great at the age of 50 as at those younger ages.

The like comparison of them with respect to the age of 60 years.

XIV. We will next examine the two tables with respect to the age of 60 years.

Now by Mr. Kerseboom's table it appears that out of 382 persons of 60 years of age, all living at the same time,

245	will have lived to the age of 70 years,
and 100	to the age of 80,
and 10	to the age of 90 years.

And by the table of Monsieur de Parcieux it appears that out of 463 persons of the age of 60 years, all living at the same time,

310	will have lived to the age of 70 years,
and 118	to the age of 80,
and 11	to the age of 90 years:

and consequently that out of 382 persons of 60 years of age, all living at the same time, (which is the number of persons living at that age in Mr. Kerseboom's table,)

$(\frac{310 \times 382}{463}, \text{ or } 256)$	will have lived to the age of 70 years,
and $(\frac{118 \times 382}{463}, \text{ or } 97)$	to the age of 80,
and $(\frac{11 \times 382}{463}, \text{ or } 9)$	to the age of 90 years.

It appears therefore that out of the same original number of 382 persons of 60 years of age, all living at the same time, more persons will be found to have been living at the age of 70 years by Mr. de Parcieux's table than by Mr. Kerseboom's in the proportion of 256 to 245; but that at the subsequent ages of 80 and 90 years fewer will be found to have been living by Mr. de Parcieux's table than by that of Mr. Kerseboom, in the proportions of 97 to 100, and 9 to 10. Now it is evident that the value of a life-annuity (which is, as we before observed, a set of equal future payments to be made at the end of every year during the continuance of the life for which it is granted,) depends more on the value of the payments that are to be made during the first years of its continuance than on those which are to be made at more remote periods, both because those first payments are nearer at hand, and because the probability of their becoming due by the continuance of the life for which they are granted is greater than the probability of the same event with respect to the latter payments. It seems reasonable therefore to conclude, (though this cannot be affirmed with certainty without actually making the calculation,) that a life-annuity for the life of a person of 60 years of age will be worth something more according to Mr. de Parcieux's table than according to that of Mr. Kerseboom, but not in so great a degree as at the younger ages before-mentioned.

The like comparison with respect to the age of 70 years.

XV. We will next examine the two tables with respect to the age of 70 years.

Now by Mr. Kerseboom's table it appears that out of 245 persons of 70 years of age, all living at the same time,

231	will have lived to the age of 71 years,
and 175	to the age of 75,
and 100	to the age of 80,
and 45	to the age of 85,
and 10	to the age of 90 years.

And by the table of Monsieur de Parcieux it appears that out of 310 persons of 70 years of age, all living at the same time,

291	will have lived to the age of 71 years,
and 211	to the age of 75,
and 118	to the age of 80,
and 48	to the age of 85,
and 11	to the age of 90 years:

and

and consequently that out of 245 persons of 70 years of age, all living at the same time (which is the number of persons living at that age in Mr. Kerseboom's table,)

$(\frac{291 \times 245}{310}, \text{ or } 230)$	will have lived to the age of 71 years,
and $(\frac{211 \times 245}{310}, \text{ or } 167)$	to the age of 75,
and $(\frac{118 \times 245}{310}, \text{ or } 93)$	to the age of 80,
and $(\frac{48 \times 245}{310}, \text{ or } 38)$	to the age of 85,
and $(\frac{11 \times 245}{310}, \text{ or } 9)$	to the age of 90 years.

It appears therefore that out of an original number of 245 persons of 70 years of age, all living at the same time, there will be fewer persons found to have been living at the several subsequent ages of 71, 75, 80, 85, and 90, years by Monsieur de Parcieux's table than by that of Mr. Kerseboom, in the several proportions of 230 to 231, 167 to 175, 93 to 100, 38 to 45, and 9 to 10. And consequently the value of a life-annuity for the life of a person of 70 years of age will be less according to Mr. de Parcieux's table than according to that of Mr. Kerseboom, though it has been seen that in all younger ages from 20 years upwards the value of a life-annuity is greater by the former table than by the latter.

The like comparison with respect to the age of 80 years.

XVI. In the last place we will examine the two tables with respect to the age of 80 years.

Now it appears by Mr. Kerseboom's tables that out of 100 persons of the age of 80 years, all living at the same time,

87	will have lived to the age of 81 years,
and 45	to the age of 85,
and 10	to the age of 90 years.

And

And by the table of Monsieur de Parcieux it appears that out of 118 persons of the age of 80 years, all living at the same time,

101 will have lived to the age of 81 years,
and 48 to the age of 85,
and 11 to the age of 90 years:

and consequently that out of 100 persons of the age of 80 years, all living at the same time, (which is the number of persons living at that age in Mr. Kerseboom's table,)

$\left(101 \times \frac{100}{118}, \text{ or } \right) 85$ will have lived to the age of 81 years,

and $\left(48 \times \frac{100}{118}, \text{ or } \right) 40$ to the age of 85,

and $\left(11 \times \frac{100}{118}, \text{ or } \right) 9$ to the age of 90 years.

It appears therefore that out of an original number of 100 persons of the age of 80 years, all living at the same time, there will be fewer persons found to have been living at the several subsequent ages of 81, 85, and 90 years according to Mr. de Parcieux's table than according to that of Mr. Kerseboom, in the several proportions of 85 to 87, 40 to 45, and 9 to 10. And consequently the value of a life-annuity for the life of a person of 80 years of age, as well as that of an annuity for the life of a person of 70 years of age, will be less according to Mr. de Parcieux's table than according to that of Mr. Kerseboom, though in the younger ages of 20, 30, and 40 years, the value of a life-annuity was considerably greater by the former table than by the latter.

Of the preference due to Monsieur de Parcieux's table of probabilities above that of Mr. Kerseboom, with respect to tables of life-annuities to be calculated for the use of Englishmen.

XVII. Thus much may be sufficient to shew the difference between these two tables of Mr. Kerseboom and Mr. de Parcieux. But which of them upon the whole deserves to be considered as the more exact, I will not pretend to determine. Only thus much I will venture to observe concerning them; That, as the soil and temperature of the air in England bear a greater resemblance, as I conceive, to the soil and temperature of the air in the northern parts of France than to those of Holland, which is so full of moist vapours arising from the waters amongst which it is situated; and the Dutch are in general reckoned to be shorter-lived than either

either the French or the English; it seems reasonable to suppose that Mr. de Parcieux's table, which is formed from observations made in France, is more likely to afford a just measure of the duration of the lives of Englishmen in the like situation and circumstances of life, that is, proprietors of government life-annuities, than the table of Mr. Kerseboom, which is formed from the like observations made in Holland. And therefore I conceive that, with respect to the valuation of annuities on the lives of persons living in England, and more especially of annuities to be granted at any time by the government, Mr. de Parcieux's table deserves to be preferred to the other. And accordingly I shall have recourse to it in the ensuing pages for the solution of the few questions, or examples, upon this subject which I shall have occasion to consider.

Of the Breslaw and London tables of the probabilities of the duration of human life.

XVIII. Mr. De Moivre has given us, in his appendix above-mentioned to his treatise on the valuation of annuities, two other tables of the probable duration of human life, which he considers as very useful for the purpose of computing the values of life-annuities. These are, 1st, Dr. Halley's table, which he formed from observations on the births and burials of the inhabitants of the city of Breslaw, (which is the capital of the duchy of Silesia in Germany,) during a series of five years in the latter part of the last century, to wit, the years 1687, 1688, 1689, 1690, and 1691; and 2^{dly}, that of Mr. Smart, which he formed from similar observations on the births and burials in London. This last table has been adopted by the late acute and very learned Mr. Thomas Simpson, of Woolwich, in his treatise called "*The doctrine of annuities and reversions*," and made the foundation of all his calculations of the values of annuities for lives, which he designed principally for the use of the inhabitants of London. And Dr. Price has likewise given us these two tables of Dr. Halley and Mr. Smart, together with two other tables of the same kind founded on observations of the births and burials at Norwich and Northampton, in his very useful treatise called "*Observations on reversionary payments*," which contains the greatest variety of tables for the valuation of annuities, both for lives and terms of years, of any book that I have met with. And to this last book, (which is deservedly in every body's hands,) I must refer my readers for such of these tables as they may have occasion to consult, as I think it would be needless to insert them in this tract in which I propose to make use only of Mr. de Parcieux's table for the solution of the few questions and examples concerning future payments depending upon lives, that will occur in it.

Of the manner of forming a table of the probabilities of the duration of human life.

XIX. The curious reader will probably be glad to be informed, in what manner these tables of the probable duration of human life are formed from observations of the births and burials of mankind at the several different ages of human life in any given district. For satisfaction in this particular I must refer him to the two following discourses; to wit, first, to Dr. Halley's celebrated tract in the Philosophical Transactions, intitled, "*An Estimate of the degrees of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslaw; with an attempt to ascertain the price of annuities upon lives.*" By Edmund Halley, R. S. S.; which is likewise published in the first volume of the *Miscellanea Curiosa*, page 280; and, secondly, to the before-mentioned treatise of Dr. Price upon reverſionary payments, in which he will find, from page 235 to page 276, an ample and curious discourse upon this subject, intitled, "*Observations on the proper method of constructing tables for determining the rate of human mortality, the number of inhabitants, and the values of lives, in any town or district, from bills of mortality in which are given the numbers dying annually at all ages.*" The part of Dr. Halley's discourse above-mentioned which relates to this subject is contained in the first five pages of it, as it is printed in the *Miscellanea Curiosa*, which is the part preceding the table. The subsequent part of the discourse sets forth the several uses to which the table may be applied; of which the four first uses are explained in a very clear and easy manner, so as to be intelligible to a reader not acquainted with the mathematicks, but the following parts are much more difficult, and require much more knowledge and attention to understand them rightly. This I mention in order to save the unlearned reader, who shall look into that discourse, unnecessary trouble.

XX. As we have mentioned in the 18th article several different tables of the probability of the duration of human life, which different authors have formed from different sets of observations, and particularly those of Dr. Halley and Messieurs Smart and Simpson, it will, I doubt not, be a satisfaction to the reader to be informed of what so great a judge of these matters as Mr. De Moivre has delivered as his opinion concerning their respective merits and uses. I shall therefore now insert in this place the remarks of that able writer upon the four tables of Dr. Halley, Mr. Kerſeboom, Mr. de Parcieux, and Messieurs Smart and Simpson, which he has inserted in the appendix to his learned treatise on the valuation of annuities. See his Doctrine of Chances, third edition, pages 345, 346, 347, 348. These remarks are as follows.

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Mr. De Moivre's remarks on the four tables of the probability of the duration of human life published by Dr. Halley, Mr. Kerseboom, Mr. de Parcieux, and Messieurs Smart and Simpson, inserted in his treatise on the valuation of annuities.

“ The first table is that of Dr. Halley, composed from the bills of mortality of the city of Breslaw; the best, perhaps, as well as the first of its kind; and which will always do honour to the judgement and sagacity of its excellent author.

“ Next follows a table of the ingenious Mr. Kerseboom, founded chiefly upon registers of the Dutch annuitants, carefully examined and compared for more than a century backward. And Monsieur de Parcieux by a like use of the lists of the *French Tomines*, or *Long Annuities*, has furnished us with Table III; whose numbers were likewise verified upon the *neurologies*, or mortuary registers of several religious houses of both sexes.

“ To these is added the table of Messieurs Smart and Simpson, adapted particularly to the city of London; whose inhabitants, for reasons too well known, are shorter-lived than the rest of mankind.

“ Each of these tables may have its particular use: The *Second* or *Third* in valuing the *better* sort of lives, upon which one would chuse to hold an annuity: The *Fourth* may serve for London, or for lives such as those of its inhabitants are supposed to be: while Dr. Halley's numbers, falling between the two extremes, seem to approach nearer to the general course of nature. And in cases of combined lives, two or more of the tables may perhaps be usefully employed.

“ Besides these, the celebrated Monsieur de Buffon has lately given us a new table, from the actual observations of Monsieur du Pré de Saint Maur of the French Academy. This gentleman, in order to strike a just mean, takes three populous parishes in the city of Paris, and so many country parishes as furnish him nearly an equal number of lives: and his care and accuracy in that performance have been such as to merit the high approbation of the learned Editor. It was therefore proposed to add this table to the rest; after having purged its numbers of the inequalities that necessarily happen in fortuitous things, as well as of those arising from the careless manner in which ages are given in to the parish-clerks; by which the years that are multiples of 10 are generally over-loaded.

“ But

“ But this having been done with all due care, and the whole reduced to Dr. Halley’s denomination of 1000 infants of a year old; there
 “ resulted only a mutual confirmation of the two tables; Mr. Du Pré’s
 “ table making the lives somewhat better as far as 39 years, and thence
 “ a small matter worse than they are by Dr. Halley’s.

“ We may therefore retain this last as no bad standard for mankind in
 “ general; till a better police, in this and other nations, shall furnish the
 “ proper *data* for correcting it, and for expressing the decrements of life
 “ more accurately and in larger numbers.

“ For which purpose the parish registers ought to be kept in a better
 “ manner, according to one or other of the forms that have been proposed
 “ by authors. Or, if we suppose the numbers annually born to have
 “ been nearly the same for an age past, the thing may be done at once,
 “ by taking the numbers of the living, with their ages, throughout every
 “ parish in the kingdom: as was in part ordered some time ago by the
 “ right reverend the bishops: but their order was not universally obeyed;
 “ for what reason we pretend not to guess. Certain it is that a *census* of
 “ this kind once established, and repeated at proper intervals, would fur-
 “ nish to our governours and to ourselves much important instruction, of
 “ which we are now in a great measure destitute: Especially if the whole
 “ was distributed into the proper classes of *married* and *unmarried*, in-
 “ *dustrious* and *chargeable* poor, *Artificers* of every kind, *Manufacturers*, &c.
 “ and if this was done in each county, city, and borough, separately;
 “ that particular useful conclusions might thence be readily deduced, as
 “ well as the general state of the nation discovered, and the rate according
 “ to which human life is wasting from year to year. See, on this subject,
 “ the judicious Observations of Mr. Corbyn Morris, addressed to Thomas
 “ Potter, Esq; in the year 1751.”

Thus far Mr. De Moivre. Since that time Dr. Price has published
 two more tables of the probability of the duration of human life formed
 from observations on the births and burials at Northampton and Norwich,
 which may be seen in his book above-mentioned, intitled, “ *Observations
 on reversionary payments*,” Tables IV and V, pages 317 and 318. And
 the grounds upon which they are constructed, as well as the method of
 forming them from those grounds, are set forth in the same book in the
 discourse intitled, “ *Observations on the proper method of constructing tables,
 &c.*” from page 240 to page 281, third edition.

XXI. I now proceed to the main design of this tract, which is to shew
 how, by the help of any one such table of the probabilities of the duration
 of human life as those above-mentioned, the value of any future contin-

gent payment, depending upon the continuance of one or more lives, may be computed. Now the great fundamental maxim upon which all computations of this kind are grounded, may be explained as follows.

The fundamental maxim of the doctrine of life-annuities.

In every bargain between two persons concerning a grant of a sum of money to be paid by the one to the other at a given future time, in case the grantee, or purchaser, shall be then alive, or in case the grantee and one or more other persons of given ages shall be then alive, the fair price of such future sum of money, according to a given rate of the interest of money and a given table of the probabilities of the duration of human life, is to be ascertained in the following manner. We must suppose, in the first place, that the grantor of the future sum of money makes several hundred grants of the same kind, and upon exactly the same conditions, to as many different grantees, or purchasers, all of the same age with the first grantee; and, in the second place, that these several purchasers and their companions, (or the persons upon the continuance of whose lives, as well as their own, their right to the said future sums depends,) die off, in the interval between the time of making the grants and the time of payment, in the same proportions as persons of the same ages respectively are represented to do in the table of the probabilities of the duration of human life by which the calculation is to be governed: and, in the third place, we must suppose that the several sums of money paid by the several grantees of these future payments to the grantor of them as the price thereof, are improved by the said grantor at compound interest, at the rate supposed in the question, during the whole interval of time between the time of making the grants and the time at which the payments become due. And then we must inquire what sum each of the said grantees ought to pay to the grantor to the end that, upon these three suppositions, he may, at the end of the said interval, or when the said payments become due, be neither a gainer nor a loser by the sum total of all his bargains, but be possessed of just enough money, arising from the sums formerly paid him by the said grantees, to satisfy all the demands which will then be made upon him. And the sum which ought thus to be paid him by each of the said grantees, when he makes a great number of the said grants to different persons, is the fair price which a single grantee ought to pay him for a grant of the same future sum of money, subject to the same conditions and contingencies, when he makes only one such grant.

This is a maxim which, I presume, will be admitted as self-evident; it being hardly possible to doubt of its truth. But if the reader should not admit it upon its own evidence, I confess I am unable to demonstrate it by means of any other proposition more evident than itself. And therefore
in

in this case I must desire him to consider it as a definition of what is meant in the following pages by the expressions of the *fair price*, or *true value* of such a future contingent payment; since it is only in that sense that the fair price, or true value, of such a future contingent payment can be collected from the tables of the probabilities of the duration of human life above-described.

Of the present values of future certain payments of money.

XXII. It appears from the foregoing article that, in order to find the true value of a future contingent payment, depending upon the continuance of one or more given lives, it is necessary that we should be able to determine, what sum of money, paid down at present to the grantor of the several future payments above-mentioned, will, if improved by him at compound interest at the rate of interest supposed in the question, during the whole interval of time before the said future payments become due, amount at that time to the whole sum of money which he will then be obliged to pay to his several grantees. This present sum of money which, being so improved at compound interest at a certain given rate of interest, will in a certain number of years amount to such larger given sum, is what the writers upon this subject usually call the *present value* of such larger sum at the said rate of interest. And, as this is a short and convenient expression to denote the said lesser sum, I shall frequently make use of it in the course of the following pages.

XXIII. Now the method of finding the present value of any given sum of money that is to be received at the end of a given number of years, according to any given rate of interest, is as follows.

P R O B L E M I.

To find the present value of any given sum of money which is payable at the end of any given number of years, according to any given rate of interest.

S O L U T I O N.

Let S be put for the given sum of money whose present value is to be determined; and n for the number of years at the end of which it is to be received. And let r be to 1 as the sum that arises by adding together any sum of money and its interest for one year according to the rate supposed in the question, is to the said principal sum of money alone, or without its interest. And let x be put for the present value of the given sum S , which is to be found. Then it is evident that x will be to x with its interest for one

one year as 1 to r , and consequently that x with its interest for one year will be equal to $\frac{rx}{1}$; or rx ; that is, the amount of x at the end of the first year will be rx . In like manner the amount of x at the end of the first year will be to the same amount together with the interest of it for one year, that is, to the amount of x at the end of the second year, as 1 to r ; and consequently the amount of x at the end of the second year will be equal to r multiplied into the amount of x at the end of the first year, or to r multiplied into rx , or to rrx . And in like manner it will appear that the amount of x at the end of the third year will be r^3x , and at the end of the fourth year r^4x , and at the end of the fifth year r^5x , and at the end of the n th, or last, year r^nx . Therefore r^nx is equal to the given sum S whose present value x is to be found. Therefore x is equal to $\frac{S}{r^n}$; that is, the present value of the given sum S is equal to the quotient that arises by dividing that given sum by that power of r whose index is the number of years at the end of which the said sum becomes payable. QEI.

An example of the foregoing solution.

Thus, for example, if the sum of 265*l.* is to be received at the end of three years, and the rate of interest is three per cent. we must proceed as follows.

Since the interest of money is 3 per cent. we shall have 100 : 103 :: 1 : r ; and consequently r will be $= \frac{103 \times 1}{100} = \frac{103}{100} = 1.03$.

Therefore rr is $= 1.03 \times 1.03 = 1.0609$, and $r^3 = rr \times r = 1.0609 \times 1.03 = 1.092,727$. Therefore x , or the present value of S , or 265*l.* is $= \frac{£.265}{1.092,727} = £.242.51$, or 242*l.* 10*s.* 2*d.* $\frac{1}{2}$. QEI.

COROLL. It follows from the foregoing solution that the present value of one pound at the end of n years is $\frac{1}{r^n}$ of a pound. For in this case S is equal to £1; and consequently x , or the present value of S , which is in all cases equal to $\frac{S}{r^n}$, is in this case equal to $\frac{1}{r^n}$, or $\frac{1}{r^n}$ of a pound. QED.

XXIV. If the interest of money is

2 per cent.	= 1.02,
2½ per cent.	= 1.025,
3 per cent.	= 1.03,
3½ per cent.	= 1.035,
4 per cent.	= 1.04,
4½ per cent.	= 1.045,
5 per cent.	= 1.05,
6 per cent.	= 1.06,
7 per cent.	= 1.07,
8 per cent.	= 1.08,
9 per cent.	= 1.09,
10 per cent.	= 1.10.

Of Mr. Smart's tables of interest.

XXV. If one pound sterling be denoted by 1, it is evident that r will be the amount of one pound sterling together with its interest for one year, or the amount of one pound sterling at the end of a year; and r^2 will be its amount at the end of two years; and r^3 its amount at the end of three years; and r^4 its amount at the end of four years; and, in general, r^n its amount at the end of n years. Now all these powers of r , from the first power to the hundredth, according to the twelve different values of it above-mentioned, to wit, 1.02, 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, and 1.10, (which correspond to the several rates of interest above-mentioned, of 2 per cent. 2½ per cent. 3 per cent. 3½ per cent. 4 per cent. 4½ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent.) or all these amounts of one pound sterling, at the end of one, two, three, and four, years, and of every following year as far as the hundredth, inclusively, according to those several rates of interest; together with the several amounts of it at the end of all the intermediate half years from the beginning of the first year to the end of the hundredth year; are ready calculated to our hands, to no less than eight places of decimal fractions, by Mr. John Smart in his most useful tables of interest published in quarto in the year 1726. These amounts, or powers of the different values of r , constitute what he calls his *first table of compound interest*, which takes up eight pages of his book, to wit, the 52d, 53d, 54th, 55th, 56th, 57th, 58th, and 59th pages. And in his *second table of compound interest*, contained in pages 60, 61, 62, 63, 64, 65, 66, 67, he has given us the reciprocals of these amounts, or powers of r , or the *present values* of one pound payable at the end of any number of years or half years, in the space of an hundred years, according to the said rates of interest, computed to the same degree of exactness. By these most useful tables the greater part of the labour of computing.

computing the value of a future contingent payment depending upon the continuance of a life or lives, or that of a life-annuity, (which is only the sum total of several such payments,) is taken away; as will-be evident from the method of computing the values of such payments, which we now proceed to explain.

P R O B L E M II.

XXVI. To find the sum of money which the purchaser of a future payment of one pound sterling, to be received at the end of any given number of years, provided the said purchaser shall be then living, ought to pay for it: the age of the said purchaser, and the rate of interest of money, and the probabilities of the duration of human life, being all given.

A solution of this problem in the case of a particular example.

Let the rate of interest of money be supposed to be 3 per cent. and the probabilities of the duration of human life such as they are represented to be in Monsieur de Parcieux's table above-mentioned. And let the number of years at the end of which the said sum of one pound is to be paid to the grantee, or purchaser of it, if he be then alive, be 30; and the age of the said grantee, or purchaser, 25 years.

Then, in the first place, we must look into Mr. de Parcieux's table to see how many persons of 25 years of age are there supposed to be all living at the same time. This number we shall find to be 774. We must therefore suppose that the grantor of the one pound to the purchaser proposed in the question does not confine himself to that single grant, but makes 773 more such grants, of one pound each, to as many different persons of the same age of 25 years, to be paid to them at the end of 30 years, or when they shall be 55 years old, if they shall then be living, but not to be paid to their executors, or other representatives, if they shall then be dead; that is, we must suppose that he makes 774 such grants in all, including that to the purchaser proposed in the question. And we must likewise suppose that all these 774 purchasers have the same chance, one with the other, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. This done, we must inquire how many of these 774 purchasers of one pound each will be alive at the end of 30 years, supposing them to die off in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 persons of the age of 25 years, all living at the same time, 526 will be alive at the age of 55 years, or at the distance of 30 years. Therefore
out

out of the said 774 purchasers of these future payments of one pound, to be received at the end of 30 years, 526 will live to be intitled to them. Therefore at the end of the said 30 years the grantor of these future payments will have 526 sums of one pound each to pay to the said surviving purchasers. And consequently, to the end that the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive at the time of making the said grants, 526 times the present value of one pound payable at the end of 30 years, when the interest of money is 3 per cent. or 526 times the sum which, being improved continually at compound interest during the said term of 30 years at the said rate of interest, will at the end of that time amount to one pound; because in that case, if he improves the said sum (of 526 times the present value of one pound) so received at compound interest at the said rate of 3 per cent. during the whole 30 years, it will in that time increase to just 526 pounds, which is the sum he will then be obliged to pay to the surviving purchasers. Now it appears by Mr. Smart's second table of compound interest, page 61, that the present value of one pound payable at the end of 30 years, without being liable to any contingency, when the interest of money is 3 per cent. is $.411,986,76$ of a pound. Therefore 526 times $.411,986,76$ of a pound, or $\pounds 216,705,035,76$, is the sum which the said grantor ought to receive at the time of making the said grants from all the 774 purchasers of them. Therefore the sum which each of them ought then to pay him is the 774th part of $\pounds 216,705,035,76$, or $.279,980,66$ of a pound, or, nearly, $.28$ of a pound, or $5s. 7d. \frac{1}{4}$. And consequently, by Art. 21, when he makes only one such grant to a purchaser of 25 years of age, he ought to receive for it the same sum of $.279,980,66$ of a pound, or $.28$ of a pound, or $5s. 7d. \frac{1}{4}$. Q.E.I.

XXVII. I have solved the foregoing problem in the case of a particular example for the sake of making the method of solution as clear and familiar as possible. But it is easy to see that the reasonings used in it extend to all other cases whatsoever, and consequently that the solution is really general. But now, that no doubt may be left on this head in the reader's mind, I shall repeat the solution, at the same length as before, in general terms and with a notation suited to them; by which means we shall obtain a short and general expression of the value of any such future and contingent payment as is hereby sought, which it will be easy to apply to any other instances.

A general solution of the foregoing problem.

XXVIII. Let i be put for one pound sterling, and r for the sum of one pound and its interest for one year according to the rate of interest given in the question, or the amount of one pound, improved at that rate of interest, at the end of a year. And let n denote the number of years
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at the end of which the sum of one pound is to be paid to the purchaser, if he is then alive, but not to his executor, or other representative, if he shall then be dead. And let N denote the number of years in the age of the purchaser at the time of his making the purchase of this one pound; and consequently $N + n$ the number of years contained in his age at the time he becomes intitled to receive it, if he lives so long.

Then, in the first place, we must look into the table of the probabilities of the duration of human life by which the calculation is to be governed, and which I shall suppose to be that of Mr. de Parcieux, to find how many persons of N years of age are there represented as living at the same time. This number we will call P . We must then suppose that the grantor of the future payment of one pound to the purchaser mentioned in the question does not confine himself to that single grant, but makes at the same time $P - 1$ more such grants to as many different purchasers of them, all of the age of N years; so that, including the grant to the purchaser mentioned in the question, he makes in all P such grants to P different purchasers, all of the age of N years. And we must further suppose that all these P purchasers have the same chance, one with the other, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. And we must likewise suppose them to die off, in the course of the n years which are to elapse before the payments become due, in the proportion in which persons of the same age are represented to die off in the same time in the table of the probabilities of the duration of human life adopted for the calculation. We must therefore look into that table to find how many persons out of P persons living at the age of N years are represented as living to the age of $N + n$ years. And this number we will call p . Then it is evident that out of all the P purchasers of the future sums of one pound each, payable at the end of n years, only p persons will be alive at the end of the said n years to claim their respective payments. Therefore at the end of the said n years the grantor of these future payments will have p sums of one pound each, to pay to the said surviving purchasers. Therefore, to the end that the said grantor, when these payments become due, may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive just p times the present value of one pound payable at the end of n years, when the interest of money is that which is denoted by the proportion of r to 1, or p times the sum which, being improved continually at compound interest during the space of n years at the said rate of interest, will at the end of the said term amount to one pound: because in that case, if he so improves the money he then receives, it will amount at the end of the said term to exactly p times one pound, so as to enable him to satisfy the just demands of the p surviving purchasers. Now the present value

value of one pound to be received at the end of n years, when the interest of money is that which is expressed by the proportion of r to 1, is just as much less than one pound as one pound is less than the amount of one pound improved at compound interest during n years at the same rate of interest. But the amount of one pound improved at compound interest for n years, at the rate of interest expressed by the proportion of r to 1, is r^n , or the n th power of r . Therefore the present value of one pound to be received at the end of n years, at the said rate of interest, is to 1*l.* as 1*l.* is to r^n . And consequently the said present value of one pound is equal to $\frac{1}{r^n}$ of a pound. Therefore p times $\frac{1}{r^n}$ of a pound, or $\frac{p}{r^n}$ £ is the sum which the said grantor ought to receive at the time of making the grants aforesaid from all the P purchasers of them. Therefore the sum which he ought to receive at that time from each of the said purchasers is the P th part of $\frac{p}{r^n}$ £ , or is equal to $\frac{p}{P \times r^n}$ £ . And consequently, by Art. 21, when the said grantor makes only one such grant to a purchaser of N years of age, he ought likewise to receive for it the same sum of $\frac{p}{P \times r^n}$ £ . QEI.

XXIX. This conclusion may be expressed in words in the manner following.

Find the present value of one pound certain, to be received at the end of the given number of years, according to the given rate of interest, by the help of Mr. Smart's second table of compound interest, or otherwise. Then find in the given table of the probabilities of the duration of human life at the several different ages of it, the number of persons living at the age of the purchaser. Then add to the age of the purchaser the number of years at the end of which the sum of one pound is made payable to him, and find in the said table the number of persons living at the said greater age. Then multiply the aforesaid present value of one pound certain by this latter number of persons living; and divide the product thence arising by the number of persons living at the age of the purchaser. And the quotient will be the value of this contingent sum of one pound, which is payable to the purchaser at the end of the given number of years, if he is then alive, expressed in decimal parts of a pound sterling.

XXX. COROLL. 1. If the sum of money to be received by the purchaser at the end of the given number of years is greater or less than one pound, it is evident that the price he ought to pay for it to the grantor will be greater or less than the price of the future payment of one pound in

the same proportion. Therefore if the sum to be received at the end of n years is denoted by the letter S , the price he ought to pay for it will be $\frac{P \times S}{P \times r^n}$. For as $\text{£}1$ is to S , so is $\frac{P}{P \times r^n}$, or the price of a future payment of $\text{£}1$, to $\frac{P \times S}{P \times r^n}$, or $\frac{P \times S}{P \times r^n}$.

XXXI. COROLL. 2. By finding in the method described in the solution of the foregoing problem the several values of one pound sterling to be received by a purchaser of a given age at the end of every future year of the whole space of time through which it is possible that his life may be extended, provided he is living at the time when every such payment becomes due, and adding these values all together, we shall obtain the value of a set of equal payments of one pound each to be made to the purchaser at the end of every future year, or of an annuity of one pound a year for his life. And this is the manner in which tables of the values of life-annuities are, or ought to be, computed.

An example of the calculation of a life-annuity according to the method described in the foregoing articles.

XXXII. As an example of this method of computing the values of life-annuities, we will here compute at length the value of an annuity of one pound for the life of a purchaser of 80 years of age, upon a supposition that the interest of money is 3 per cent. and that the probabilities of the duration of human life are such as they are represented to be in Mr. de Parcieux's table.

Now it appears by Mr. de Parcieux's table that, out of 118 persons of the age of 80 years, all living at the same time, 101 will live to the age of 81 years. And by Mr. Smart's second table of compound interest, page 60, it appears that the present value of one pound certain, to be received at the end of one year, when the interest of money is 3 per cent. is .9708 of a pound. (For though he carries this value to eight places of figures, I omit the four last figures, as unnecessary in the present computation.) Therefore by ART. 28, or 29, $\frac{101 \times .9708}{118}$, or $\frac{98.0508}{118}$, or

.8309 $\frac{1}{2}$, is the present value of the first year's payment of the annuity of one pound, which is to be made to the purchaser when he shall be 81 years old.

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In like manner it appears by Mr. de Parcieux's table that of the original number of 118 persons of the age of 80 years, all living at the same time, there will be

85	persons living	at the age of 82 years,
and 71		at the age of 83,
and 59		at the age of 84,
and 48		at the age of 85,
and 38		at the age of 86,
and 29		at the age of 87,
and 22		at the age of 88,
and 16		at the age of 89,
and 11		at the age of 90,
and 7		at the age of 91,
and 4		at the age of 92,
and 2		at the age of 93,
and 1		at the age of 94,
and none at all		at the age of 95.

And by Mr. Smart's second table of compound interest, page 60, it appears that, when the interest of money is 3 per cent. the present value of one pound certain, payable at the end of

2 years,	is	£
and at the end of 3 years,		.9425;
and at the end of 4 years,		.9151;
and at the end of 5 years,		.8884;
and at the end of 6 years,		.8626;
and at the end of 7 years,		.8374;
and at the end of 8 years,		.8130;
and at the end of 9 years,		.7894;
and at the end of 10 years,		.7664;
and at the end of 11 years,		.7440;
and at the end of 12 years,		.7224;
and at the end of 13 years,		.7013;
and at the end of 14 years,		.6809;
		.6611.

Therefore, by Art. 28 or 29, the value of the second year's rent, or payment of one pound, which is to be received by the purchaser when he is 82 years old, is $\frac{£}{118} \times .9425$, or $\frac{£}{80.1125}$, or .6789.

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118

And

And the value of the third year's rent, which is to be paid him when he is 83 years old, is $\frac{71 \times .9151}{118}$, or $\frac{65.0721}{118}$, or $\frac{.5514}{118}$.

And that of the fourth year's rent is $\frac{59 \times .8884}{118} = \frac{52.4156}{118} = .4442$.

And that of the fifth year's rent is $\frac{48 \times .8626}{118} = \frac{41.4048}{118} = .3508$.

And that of the sixth year's rent is $\frac{38 \times .8374}{118} = \frac{31.8212}{118} = .2612$.

And that of the seventh year's rent is $\frac{29 \times .8130}{118} = \frac{23.5770}{118} = .1998$.

And that of the eighth year's rent is $\frac{22 \times .7894}{118} = \frac{17.3668}{118} = .1472$.

And that of the ninth year's rent is $\frac{16 \times .7664}{118} = \frac{12.2624}{118} = .1039$.

And that of the tenth year's rent is $\frac{11 \times .7440}{118} = \frac{8.1840}{118} = .0693$.

And that of the eleventh year's rent is $\frac{7 \times .7224}{118} = \frac{5.0568}{118} = .0428$.

And

$$\text{And that of the twelfth year's rent is } \frac{\text{£} 4 \times .7013}{118} = \frac{\text{£} 2.8052}{118} = .0237.$$

$$\text{And that of the thirteenth year's rent is } \frac{\text{£} 2 \times .6809}{118} = \frac{\text{£} 1.3618}{118} = .0115.$$

$$\text{And that of the fourteenth year's rent is } \frac{\text{£} 1 \times .6611}{118} = \frac{\text{£} .6611}{118} = .0056.$$

Add all these values of the several yearly rents together; and their sum total, £3.7212, or 3*l*. 14*s*. 5*d*. will be the value of an annuity of one pound for the whole life of the person of 80 years of age who was supposed to be desirous of purchasing it. Q E I.

£	£	£	£
.8309	.2612	.0428	2.8562
.6789	.1998	.0237	.7814
.5514	.1472	.0115	.0836
.4442	.1039	.0056	
.3508	.0693		3.7212
<u>2.8562</u>	<u>.7814</u>	<u>.0836</u>	

XXXIII. The foregoing computation might have been somewhat shortened by first adding together all the several products £98.0508, £30.1125, £65.0721, £52.4156, &c. that arise from the multiplication of the numbers of persons living at the several ages of 81, 82, 83, 84, &c. years into the corresponding present values of the sum of one pound, and then dividing their sum total by 118, or the number of persons living at the age of 80 years, which is the age of the purchaser of the annuity. For as all these products are to be divided by 118, it is evident that the conclusion will be the same whether we first divide each of them by 118, and then add the quotients together, as has been done in the foregoing computation, or whether we first add the products themselves together, and then divide their sum by 118. And by this latter method of proceeding we shall have only one operation of division instead of several, which will considerably diminish the labour of the calculation. The addition of these products is as follows.

98.0508

<i>£</i>		<i>£</i>	<i>£</i>
98.0508		31.8212	5.0568
80.1125		23.5770	2.8052
65.0721		17.3668	1.3618
52.4156		12.2624	.6611
41.4048		8.1840	
337.0558		93.2114	9.8849
93.2114			
9.8849			
440.1521			

Therefore £440.1521 is the sum of all those products; which being divided by 118 gives us £3.7301, or 3*l*. 14*s*. 7*d*. $\frac{1}{4}$, for the value of an annuity of one pound for the life of a person of 80 years of age, when the interest of money is 3 per cent. Q.E.I.

N. B. This result is greater than the former by the small difference of the 100th part of a pound, or about 2*d*. $\frac{1}{2}$; which is owing, I imagine, to our not losing any part of the aforesaid products by this addition of them, whereas in dividing each of them by 118, according to the former method, we obtained quotients which were for the most part somewhat less than the truth. But this difference is not worth attending to.

XXXIV. The reader will observe that in adding together the fourteen products in the last article, and the quotients of the division of those products by 118 in Art. 32, I have made use of a round-about method of proceeding to obtain those sums; to wit, by first separating the said products and quotients, which were to be added together, into three different parcels, two of which consist of five terms and the third of four, and finding the sums of these several parcels of terms, and then adding these three sums together to obtain the sum total of the whole fourteen terms. My reason for doing this was because I find it easier to obtain the sum total of a great number of quantities in this manner than by adding them all up together in one operation; and I think there is less danger of making an error in the computation. But this is a matter in which every person, engaged in arithmetical calculations, must be governed by his own experience.

XXXV. The foregoing example of the computation of the value of a life-annuity for the life of a purchaser of fourscore years of age will do as well to illustrate the method of computing such annuities delivered in Coroll. 2*d*, Art. 31, as if we had taken a much younger life. And as
this

this was the sole intent of inserting any example of this kind, an old life was pitched upon, in preference to a young one of only 20 or 30 years, for the sake of avoiding the much greater number of multiplications and divisions that would have been necessary in computing the value of an annuity for such younger life, which might have appeared tedious to the reader.

XXXVI. COROLL. 3. The value of a remote life-annuity for the life of a purchaser of a given age, that is, of an annuity that is to take place only at the end of a given number of years, and to continue from that time to the end of the purchaser's life, according to any given rate of interest and any given table of the probabilities of human life, may likewise be computed by means of the foregoing problem.

For such a remote life-annuity is a set of equal sums of money which are to be paid to the purchaser at the end of every year from the time when the annuity is to take place, and which are to continue from that time forwards to the end of the purchaser's life. And consequently, by finding the values of the said equal contingent future payments from the time when the annuity is to commence to the end of the whole space of time through which it is possible the annuitant's life may be prolonged, (that is, according to Mr. de Parcieux's table, to the end of that number of years which, being added to the purchaser's present age, will make 94 years,) in the method described in the foregoing problem, and adding these values together, we shall obtain the value of the said remote life-annuity which is to commence after a given number of years.

XXXVII. Thus, if a person of 80 years of age purchases a life-annuity of one pound, to commence when he shall be 85 years of age, or whereof he is to receive the first payment when he is 86 years old, the interest of money being 3 per cent. and the probabilities of the duration of human life such as they are represented to be in Mr. de Parcieux's table, and it is required to find the value of such annuity, we must find by the help of the foregoing problem the values of the several future payments of one pound each which are to be made to him at the end of his 86th, 87th, 88th, 89th, 90th, 91st, 92d, 93d, and 94th years, if he attains to those respective ages, according to the said rate of interest and table of probabilities. And the sum of these values will be the value of the proposed distant life-annuity, that was required to be found. These values we have already found to be as follows.

TABLE
The value of a remote life-annuity, when the annuity is to commence at the end of a given number of years, and to continue from that time to the end of the purchaser's life, according to any given rate of interest and any given table of the probabilities of human life.

TABLE

The value of one pound a year to be received by a person of 80 years of age, according to the rate of interest and table of probabilities above-mentioned, when he shall be 86 years old, or

at the end of 6 years,	is	.2612;
And the value of £1 to be received by		
him at the end of 7 years,	is	.1998;
And at the end of 8 years,	is	.1472;
And at the end of 9 years,	is	.1039;
And at the end of 10 years,	is	.0693;
And at the end of 11 years,	is	.0428;
And at the end of 12 years,	is	.0237;
And at the end of 13 years,	is	.0115;
And at the end of 14 years,	is	.0056.

And the sum of all these values is $\pounds$.8650.

Therefore when the interest of money is 3 per cent. $\pounds$.8650, or 17s. 3d. $\frac{1}{2}$, is the value of an annuity of one pound for the life of a man of four score years of age, to commence when he shall be 85 years old, or whereof the first payment is to be made him when he shall be completely 86 years old. Q.E.I.

S C H O L I U M.

Concerning the bill for establishing certain remote life-annuities in parishes, which passed the House of Commons in the Spring of the year 1773.

XXXVIII. In this manner may be computed all those tables of remote life-annuities which were prepared in the Spring of the year 1773, and annexed to a bill at that time in the House of Commons, for enabling parishes to grant such annuities to their poor and industrious inhabitants. This bill was brought into the House of Commons by Mr. Dowdeswell, and seconded by Mr. George Rice, the member for Caertharthenshire, and supported by Sir George Savile, Sir Richard Sutton, Mr. Edmund Burke, Mr. Cornwall, Mr. Jackson, counsel to the Board of Trade, Mr. Thomas Townshend, junior, and many other members of parliament of eminent abilities. And it passed in that house upon a division, after a debate, by a majority of about two votes to one of all the members present. But it was thrown out by the House of Lords. As great pains had been taken in the framing this bill by Mr. Dowdeswell, (who brought it into the house,) Mr. Rice, and Sir George Savile, and many other gentlemen; who had often met together, for several hours at a time, at Sir George Savile's house in Leicester-Square to consider the several clauses of it; it may not be amiss to give my readers the following general account of it.

XXXIX. The

XXXIX. The design of this bill was to encourage the lower ranks of people to industry and frugality, by laying before them a safe and easy method of employing some part of the money they could save out of their wages, or daily earnings, in a manner that would be most strikingly for their benefit.

The design of the said bill.

It was observed that their wanting opportunities of this kind was probably one very principal cause of their neglecting so obvious a piece of prudence. That they knew, for the most part, but little of the public funds; and that, when it happened that they were acquainted with them, the smallness of the sums they would be intitled to receive as the interest of the money they could afford to lay out in them, was no encouragement to them to dispose of it in that way. For what inducement, for instance, can it be to a poor man who has saved ten pounds out of his year's wages to invest it in the 3 per cent. bank-annuities, to consider that it will produce him about six or seven shillings a year? It is but the wages of three days labour. And, if they lend their money to tradesmen of their acquaintance, as they sometimes do, it happens not unfrequently that their creditor becomes a bankrupt, and the money they had trusted him with is lost for ever; which discourages others of them from saving their money at all, and makes them resolve to spend it in the enjoyment of present pleasure.—But that, if they saw an easy method of employing the money they could spare in such a manner as would procure them a considerable income in return for it in some future period of their lives, without any such hazard of losing it by another man's folly or misfortune, it was probable they would frequently embrace it: and thus a diminution of the poor's rate on the estates of the rich, an increase of present industry and sobriety in the poor, and a more independant and comfortable support of them in their old age than they can otherwise expect, would be the happy consequences of such an establishment. To effect these useful purposes the bill provided as follows.

The principal provisions of it.

1st, That in every parish in England or Wales, in which there were two churchwardens and two or more overseers of the poor, or one churchwarden and three or more overseers of the poor, that is, four, or more, parish-officers intrusted with the care of the poor, it should be lawful for the body of the rateable inhabitants of such parish, that is, of those inhabitants who contributed to the poor's rate, to grant life-annuities, payable every quarter of a year, to such of the inhabitants thereof as should be willing to purchase them, at the prices set down in the tables annexed to the bill, which were computed upon a supposition that the interest of money was only 3 per cent.

2^{dly}, That the money received from the purchasers of these annuities should be vested in the 3 per cent. bank-annuities in the name of the parish which had granted it: and the dividends duly received by them every

half-year, and employed in the purchase of new stock, so as to be improved at compound interest, to the end that it may be able to answer the annuities bought with it when they shall become due.

3dly, That for the aforesaid purposes of granting these life-annuities, and receiving the money paid for them, and holding the stock purchased with it in the bank-annuities, and the other purposes of this bill, the said rateable inhabitants of every such parish should be made a body politic and corporate, and have a common seal.

4thly, That, if the parish-fund in the 3 per cent. bank-annuities should, by the mismanagement of it, or from any other cause, prove insufficient to supply the life-annuities charged upon it, the poor's-rate should be made a collateral security to the poor purchasers of these annuities for the payment of them, and should be increased to such a degree as should be sufficient to make good the deficiencies.

5thly, That no such annuity should be granted to any one person of more than 20*l.* sterling a year.

6thly, That no such annuity should be granted to any of the inhabitants of a parish but such as were legally settled in it, or had a right to be relieved by it in case they became poor and helpless.

7thly, That no such annuity granted to any man should commence before he was completely 50 years of age; nor to any woman before she was completely 35 years of age.

8thly, That no sum less than five pounds should be received by the managers of these annuities as the price of any such annuity.

9thly, That the ministers, and church-wardens and overseers of the poor, should be the managers of these annuities for the whole body politic and corporate of the rateable inhabitants of the parish, and should receive the money from the purchasers of them and vest it in the 3 per cent. bank-annuities, and receive the dividends, and employ them in the purchase of fresh stock, and pay the annuities to the purchasers when they became due; and that for the transacting of the said business at the Bank they should give a power of attorney to some person residing in London.

10thly, That nevertheless the said managers should not have the power of granting any of these annuities without the consent of the rateable inhabitants of the parish, who should be assembled in vestry for that purpose after public notice of such intended meeting given in the parish-church

church on two Sundays immediately after divine service. And in these meetings of the parishioners it should be necessary not only that the majority of them in number should consent to the granting the annuity proposed, but that those who so consented should have paid more than half the last poor's-rate paid by all the rateable inhabitants so assembled.

This restraint was intended to prevent the renters of small tenements in the parish from involving the parish in the contingent burthen on the poor's-rate that might arise from these annuities, against the will of the more substantial inhabitants.

And it was further provided that no such annuity should be granted unless there were present at the meeting, in which it was granted, at least twelve of the said rateable inhabitants of the parish, except in parishes where the whole number of rateable inhabitants was less than nineteen; and that in that case it should be necessary that at least two third parts of the whole number of inhabitants should be present at it.

And, in the 11th and last place, it was provided that the purchasers of these annuities should not be permitted to alienate them without first making an offer of them to the parish at the price they were worth at the time of such offer according to the tables annexed to the bill, or at some lower price: and that such of them as should, at the time of purchasing them, consent to a clause that should declare them to be absolutely unalienable, should, in consequence of such consent, be incapable of alienating them at all.

The reason of this restraint upon the alienation of these annuities was to guard the poor owners of them against their own folly and weakness, by making it impossible for them to sell their annuities for a small part of their true value, over a pot of ale and without a proper degree of deliberation.

The reason of computing the values of these life-annuities upon the supposition of so low a rate of interest as 3 per cent. was to make the fund arising from the money paid for them be amply sufficient to answer them when they should become due; so that it should be almost impossible, without great negligence in the management of this fund, that there should ever be a necessity of having recourse to an augmentation of the poor's-rate to make good its deficiencies. Yet even at this low rate of interest the purchasers of these annuities would usually get nine or ten per cent. for their money, if they purchased them only five years before the time of their commencement; and 30 or 40 per cent. if they would be content to wait for them 25 or 30 years; which men under 30 years of age might do without any inconvenience. And the hope of this, it was presumed, might be

be a sufficient inducement to them to employ some part of their money in this way, and to be diligent in their callings, and frugal in their expences, with that view.

The principal objection that seemed likely to be made to the said project.

XL. The only objection that seemed likely to be made to this project was the difficulty of carrying it into execution, arising from the inability of the minister and church-wardens and overseers of the poor of the parish to manage the money received from the purchasers of these annuities without an agent in London for that purpose; who would probably be, for the most part, either some stock-broker, or banker, or banker's clerk, or other man of business that dealt in money transactions, whom it might be difficult to engage in an employment of this kind without paying him for his trouble in a manner that the parish-fund could hardly afford. But this objection is not so strong as it appears to be: because the business of this kind to be done in London would not be so much as might at first be apprehended; and the price of brokerage upon buying and selling the parish-stock in the bank-annuities and receiving the dividends of it when they became due, is no great matter. But how far this difficulty was likely to hinder the execution of the bill could not be known with any tolerable degree of certainty without giving it a trial. And therefore the House of Commons passed it. But that the experiment might be as little hazardous as possible, and parishes might not be involved by it against their wills in the danger of these remote incumbrances, the bill was made entirely optional, and the rateable inhabitants of every parish were left at liberty to grant or not grant any of these annuities, as they should think fit, and even, after they had granted some such, to desist from granting any more. And this full liberty of proceeding herein according to their own judgments and inclinations was thought by the gentlemen who supported the bill in the House of Commons to be a compleat answer to the objection above-mentioned arising from the supposed difficulty of carrying it into execution.

In order to remove the said objection, the bill provided that every parish might chuse whether it would, or would not, adopt the said project.

An ingenious observation made in support of the said objection.

XLI. There was however a very ingenious and subtle observation made upon this answer, and in support of the foregoing objection, by a noble lord, of distinguished abilities, and who formerly filled the highest station in the law with great reputation. This was, "that the option above-mentioned was not given to the right persons, or to those who were most likely to be affected by the burthens which the granting these annuities might hereafter bring upon the parishes. For that the option was given to the rateable inhabitants of the parish, who were, for the most part, only renters of the lands they occupied; whereas the burthen upon the poor's-rate arising from the supposed deficiency of the annuity-fund was not likely to be felt till many years after the granting of the annuities, when

when the leases of the renters who had voted for the granting them, would be at an end, or, if they were renewed, would have been renewed at a lower rent than before, in consideration of the approaching and probable increase of the poor's rate arising from the said supposed deficiency; which would be an injury to the freeholders of the land, who were possessed of the permanent property of it:—and that therefore the consent of the said freeholders ought to be obtained to every act by which the lands of the parish might be exposed to the danger of such a future burthen.”

This observation seems to be somewhat refined; but will admit, as I conceive, of the following answer. The rateable inhabitants of parishes are of the three following sorts; either owners of the houses and lands which they occupy; or renters of them under long leases for 21 years, or for three lives, and often with a right of renewal; or renters of them under short leases for one or two years, or merely at the will of the owners without any leases. If they are of the first sort, they are the very persons in whom the noble author of the observation thinks the option of granting, or refusing to grant, these life-annuities ought to have been vested. If they are of the second sort, that is, renters of the lands they occupy under long leases, they then are more likely to feel the burthen brought upon the parish by the supposed augmentation of the poor's rate than the freeholder or owner of the reversion, and therefore are fitter than he is, according to the principle of the observation, to be trusted with the power of bringing this contingent burthen upon the parish. And lastly, if they are renters of the lands they occupy under short leases or at will, (which is the case supposed in the objection,) they are, in consequence of the precariousness of such a tenure, so much under the influence of their landlord, that, if he should but signify his pleasure to them, by his steward or by a letter, that he does not chuse that any of these annuities should be granted in the parish, lest his lands should be exposed to such a future increase of the poor's rate, they will be sure to give their votes against them. So that in all these cases the interests of the persons who are most likely to be affected by the apprehended burthen on the poor's rate, are sufficiently protected by the provision that vests this option in the rateable inhabitants of the parish. And besides, experience shews that the inhabitants of parishes in general, as well those who rent lands and houses by the year, or at will, as those who have more permanent interests in them, are wonderfully averse to every thing that has even a remote tendency to increase the poor's rate. And consequently there is no reason to apprehend that they would consent to grant any of these parish-annuities whenever there was the smallest danger of their being ill managed, and producing, in consequence thereof, an augmentation of the poor's rate to make good the deficiencies of their proper fund.

Some

Some other objections have also been made to this bill, which have been answered in a pamphlet, intitled, "*Considerations on the Bill now depending in the House of Commons for enabling parishes to grant life-annuities to poor persons, upon purchase, in certain circumstances, and under certain restrictions; being an appendix to the pamphlet intitled, 'A proposal for establishing life-annuities in parishes for the benefit of the industrious poor.'*" Sold by B. White, in Fleet-Street, 1773; to which I refer the reader.

I have this further to say in favour of the foregoing proposal, (which I hope will one day or other be again brought into parliament, and with better success;) that it has been carefully examined, and fully approved, by the learned and publick-spirited Dr. Price, of Newington-Green near Illington, the author of the "*Observations on reverſionary payments*" above-mentioned; and by the very acute and judicious Dr. Benjamin Franklin, and by Mr. Wedderburn, who at the time of this bill's paſſing the Houſe of Commons, was his Maſteſty's ſollicitor-general, and now (in Auguſt, 1780,) is lord chief juſtice of the court of Common Pleas and baron Loughborough.

Of the value
of an immedi-
ate, but im-
perfect, life-
annuity.

XII. COROLL. 4. The value of an immediate, but imperfect, life-annuity for the life of a purchaſer of a given age; that is, of an annuity that is to commence immediately, (ſo that the firſt payment thereof ſhall be received at the diſtance of only one year from the time of purchaſing it,) and to continue only during a certain number of years, which is leſs than that during which it is poſſible the purchaſer's life may be prolonged, and then to ceaſe, notwithstanding the purchaſer ſhould live on to a later period; may likewiſe be computed by the help of the foregoing problem.

For we need only in this caſe compute the values of the ſeveral future payments that are to be made to the purchaſer at the ends of the ſeveral years during which the annuity is to continue, and add them together into one ſum. And this ſum will be the value of the propoſed annuity. Q.E.I.

An example
of the calcu-
lation of the
value of ſuch
an annuity.

Thus, if a man of fourſcore years of age wanted to purchaſe an annuity of one pound a year for the five following years of his life, and no longer, according to the rate of intereſt and table of probabilities mentioned in Coroll. 2, the price he ought to pay for it would be the ſum total of the ſeveral ſums following, to wit, £.8309, £.6789, £.5514, £.4442, and £.3508, which are the values of the ſeveral payments of one pound which would become due to him at the end of the 81ſt, 82d, 83d, 84th, and 85th years of his age; which ſum is £2.8562, or 2l. 17s. 1d. $\frac{1}{2}$.

Note.

Note. An annuity of this kind might be useful to a man who had a reversionary interest in a good estate that was to fall in to him at the end of a certain, not very long, term of years. Thus, for example, if a man was the ground-landlord of a number of houses that paid him only a trifling sum in ground-rents, but which were to fall into his possession by the expiration of the leases in seven or eight years time, and then would afford him an ample income, an annuity of this kind, which should continue only till the said leases should expire, would be of great convenience to him.

XLIII. COROLL. 5. And in like manner the value of a distant Of the value of a distant and imperfect life-annuity. and imperfect life-annuity for the life of a purchaser of a given age; or of an annuity, depending on the purchaser's life, but that is not to commence till the end of a given number of years after the time of purchasing it, and is to continue only during a certain period of time less than the whole space through which it is possible the purchaser's life may be prolonged, and then is to cease, although the purchaser should live beyond it; may be computed by the help of the foregoing problem.

For we need only compute the values of the several future payments that are to be made to the purchaser at the ends of the several years during which the annuity is to continue. And the sum total of these values will be the value of the proposed annuity. QEI.

Thus, if a man of fourscore years of age wants to purchase an annuity of one pound a year, that shall begin at the end of five years, and continue during the five following years, if he lives so long, and then cease; so that he shall be intitled to receive the first payment of it at the end of the 86th year of his age, and to receive the other four payments at the ends of his 87th, 88th, 89th, and 90th years respectively, but nothing afterwards though he should live to a greater age; according to the rate of interest and table of probabilities mentioned in Coroll. 2; the price he ought to pay for such a distant and imperfect life-annuity would be the sum total of the several following sums, to wit, £.2612, £.1998, £.1472, £.1039, and £.0693, which are the values of the several payments of one pound which would be due to him at the ends of the 86th, 87th, 88th, 89th, and 90th years of his life respectively: which sum is £.7814, or 15s. 6d. $\frac{1}{2}$.

Thus much may suffice for the computation of the values of annuities depending upon one life. We now proceed to consider those which depend upon the joint continuance of two lives.

G PROBLEM

P R O B L E M III.

XLIV. To find the sum of money which the purchaser of a future payment of one pound sterling, to be received at the end of any given number of years, provided the said purchaser and another person, (who may be called *his companion*;) shall be then living, ought to pay for such future sum; the ages of the said purchaser and his companion, and the rate of interest of money, and the probabilities of the duration of human life, being all given.

A solution of this problem in the case of a particular example.

Let the rate of interest of money be supposed to be 3 per cent. and the probabilities of the duration of human life to be such as they are represented to be in Monsieur de Parcieux's table above-mentioned. And let the number of years at the end of which the said sum of one pound is to be paid to the purchaser of it, in case not only the said purchaser himself, but likewise his companion aforesaid, shall be then alive, be 30; and the age of the said purchaser 25 years; and that of his said companion 20 years.

Then, in the first place, we must look into Mr. de Parcieux's table to see how many persons of 25 years of age are there represented as all living at the same time. This number is 774. We must therefore suppose that the grantor of the one pound to the purchaser proposed in the question makes at the same time 773 more such grants of one pound to as many different persons, all of the same age of 25 years, to be paid them at the end of 30 years, or when they shall be 55 years old, if, not only the grantees themselves shall then be living, but certain other persons, who may be called *their companions*, who are of the same age of 20 years with the companion of the purchaser mentioned in the question: that is, we must suppose that the grantor makes 774 such grants in all, including that of the purchaser proposed in the question. And we must likewise suppose that all these 774 purchasers of these future payments of one pound have the same chance, one with another, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. This done, we must inquire how many of these 774 purchasers of one pound each will be alive at the end of 30 years, supposing them to die off in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 persons of the age of 25 years, all living at the same time, 526 will be alive at the age of 55 years, or at the end of 30 years. Therefore out of the said 774 purchasers of these future payments

payments of one pound each, only 526 will live to the end of the 30 years. And of these 526 surviving purchasers only some part will be intitled to demand these payments, to wit, those whose companions, who were of the age of 20 years at the time of making the grants, are likewise living at the end of the said 30 years. For the other surviving purchasers, whose companions are then dead, will, by the conditions of this problem, have no right to them. We must therefore, in the next place, inquire how many of the companions of the said 526 surviving purchasers will also be alive at the end of the said 30 years. Now the companions of these 526 surviving purchasers were at the time of making the grants just as many as those purchasers themselves, that is, 526. We must therefore inquire by Mr. de Parcieux's table how many of these 526 companions of the said 526 surviving purchasers, who were all living and of the age of 20 years at the time of making the grants, will be alive at the end of the said 30 years. Now it appears by Mr. de Parcieux's table that out of 814 persons of the age of 20 years, all living at the same time, only 581 will be living at the age of 50 years; and consequently out of 526 persons of the age of 20 years, all living at the same time, only $526 \times \frac{581}{814}$, or 375 will be alive at the age of 50 years. Therefore of the 526 companions of the 526 surviving purchasers only 375 will be living at the end of the said 30 years. Therefore only 375 out of the said 526 surviving purchasers will be intitled to receive the said payments of one pound each. Therefore at the end of the said 30 years the grantor of the said future payments will have only 375 sums of one pound each to pay to the surviving purchasers; which will be due to those 375 of them whose companions will be then alive. And consequently, to the end that the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive at the time of making the said grants 375 times the present value of one pound payable at the end of 30 years, when the interest of money is 3 per cent. or 375 times the sum which, being continually improved at compound interest during the said term of 30 years at that rate of interest, will, at the end of that time, amount to one pound; because in that case, if he improves the said sum of 375 times the present value of one pound, so received, at compound interest at the said rate of 3 per cent. during the whole 30 years, it will in that time increase to just 375 pounds, which is the sum which he will then be obliged to pay to the 375 surviving purchasers who, by the continuance of the lives of their respective companions, will be intitled to their several payments of one pound a piece. Now, by Mr. Smart's second table of compound interest, the present value of one pound payable at the end of 30 years when the interest of money is 3 per cent. is .419 of a pound. Therefore 375 times £.4119, or £154.4625, is the sum which the said grantor of these future payments ought to receive, at the time of making the said grants, from all the 774 purchasers of them.

them. Therefore the sum which he ought then to receive from each of the said purchasers is the 774th part of £154.4625, that is, £.1995, or, nearly, £.2, or 4 shillings. And consequently, by Art. 21, when he makes only one such grant of one pound, payable at the end of 30 years, to a purchaser of 25 years of age, provided a companion of the purchaser, who is of the age of 20 years at the time of making the grant, shall also be living at the end of the said 30 years, he ought to receive for it the same sum of $\frac{1}{10}$ of a pound, or four shillings. Q.E.I.

XLV. I now proceed to give a solution of this problem in general terms, as I did before of problem 2d, in Art. 28.

A general solution of the foregoing problem.

Let 1 be to r as one pound sterling, or any other sum of money whatsoever supposed to be put out at interest at the rate supposed in the question, is to the amount of the same sum at the end of a year, or to the sum total of the said sum and its interest for one year. And let n denote the number of years at the end of which the sum of one pound is to be paid to the purchaser, if both he and his companion are then alive. And let N denote the number of years in the age of the purchaser at the time of his making the purchase of this future payment of one pound; and consequently $N+n$ the number of years contained in his age at the time he becomes intitled to receive it. And let M denote the number of years in the age of the companion of the purchaser at the time of making the purchase; and consequently $M+n$ the number of years contained in his age at the time when the payment becomes due.

Then, in the first place, we must look into the table of the probabilities of the duration of human life by which the calculation is to be governed, and which I shall suppose to be that of Mr. de Parcieux, to find how many persons of N years of age are there represented as all living at the same time. This number we will call P . We must then suppose that the grantor of the future payment of one pound to the purchaser mentioned in the question does not confine himself to that single grant, but makes at the same time $P-1$ more such grants of one pound each to as many different purchasers of them; so that, including that to the purchaser in the question, he makes in all P such grants to P different purchasers. And we must further suppose that all these P purchasers have the same chance, one with the other, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. And we must likewise suppose them to die off, in the course of the n years which are to elapse before

before the payments become due, in the same proportion in which persons of the same age are represented to have died off in the same number of years in the table of the probabilities of the duration of human life that is adopted for the calculation. We must therefore look into that table to find how many persons out of P persons living at the age of N years are represented as living to the age of $N+n$ years. And this number we will call p . Then it is evident that out of all the P purchasers of the future sums of one pound each, payable at the end of n years, only p persons will be alive at the end of the said n years. And of these p surviving purchasers only some part will be intitled to demand these payments, to wit, those whole companions, who were of the age of M years at the time of making the grants, are likewise living at the end of the said n years. For the other surviving purchasers, whose companions are then dead, will, by the conditions of this problem, have no right to them. We must therefore, in the next place, inquire how many of the companions of the said p surviving purchasers will be alive at the end of the said n years; without concerning ourselves about the fate of the companions of the deceased purchasers, because it is immaterial to the present question how many of them, or whether all, or any, of them, are still alive. Now the companions of these p surviving purchasers were, at the time of making the grants, just as many as those purchasers themselves, that is, just p in number. We must therefore inquire how many of these p companions of the said p surviving purchasers will be alive at the end of the said n years. These companions of the said purchasers were of the age of M years at the time of making the said grants. We must therefore look into the table of the probabilities of the duration of human life adopted in the calculation, to find how many persons are there represented as living at the said age of M years, and how many at the subsequent age of $M+n$ years. Let the former of these numbers be denoted by Q , and the latter by q . Then, since the p companions of the p surviving purchasers of these future payments of one pound are supposed to die off, in the course of these n years, in the same proportion as the Q persons of the same age of M years are represented to have done in the said table of probabilities, the number of persons out of the said p companions of the said surviving purchasers that will be alive at the end of the said n years, will be to the number of those companions who are alive at the beginning of the said time in the same proportion as q is to Q ; and consequently will be equal to $\frac{pq}{Q}$, or $\frac{pq}{Q}$. Therefore only $\frac{pq}{Q}$ out of the whole number p of the said surviving purchasers will be intitled to these payments of one pound each at the end of the said n years. Therefore at the end of the said n years the grantor of the said future payments will have $\frac{pq}{Q}$ sums, of one pound each.

to pay to these $\frac{Pq}{Q}$ more fortunate surviving purchasers, whose companions will have lived, as well as themselves, to the end of the said n years. Therefore, to the end that the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive, at the time of making the said grants, $\frac{Pq}{Q}$ times the present value of one pound payable at the end of n years, when the interest of money is that which is expressed by the proportion of r to 1, or $\frac{Pq}{Q}$ times the sum which, being continually improved at compound interest during the said term of n years at that rate of interest, will, at the end of that time, amount to one pound: because in that case, if he improves the said sum of $\frac{Pq}{Q}$ times the present value of one pound, so received, at compound interest at the said rate of interest during the whole term of n years, he will thereby have augmented it at the end of that time to just $\frac{Pq}{Q}$ times one pound, or $\frac{Pq}{Q}$ pounds, which is the sum which he will then be obliged to pay to the $\frac{Pq}{Q}$ fortunate surviving purchasers of these future payments, whose companions will be then alive. Now the present value of one pound payable at the end of n years is $\frac{1}{r^n}$ of a pound. Therefore $\frac{Pq}{Q}$ times $\frac{1}{r^n}$ of a pound, or $\frac{Pq}{Qr^n}$ £, is the sum of money which the said grantor of these future payments ought to receive, at the time of making the said grants, from all the P purchasers of them. Therefore the sum which he ought then to receive from each of the said purchasers is the P th part of $\frac{Pq}{Qr^n}$ £, or is equal to $\frac{Pq}{Qr^n}$ £, or $\frac{Pq}{PQ} \times \frac{1}{r^n}$ £. And consequently, by Art. 21, when he makes only one such grant of one pound, payable at the end of n years, to a purchaser of N years of age, provided a companion of the purchaser, who is of the age of M years at the time of making the grant, shall also be living at the end of the said n years, he ought to receive for it from the said single purchaser the same sum of $\frac{Pq}{Qr^n}$ £, or $\frac{Pq}{PQ} \times \frac{1}{r^n}$ of a pound. Q.E.I.

XLVI. This conclusion may be expressed in words in the manner following.

Find the present value of one pound certain, to be received at the end of the number of years, (n) given in the problem, according to the rate of interest therein also given, and expressed by the proportion of r to 1; The foregoing present value will be $\frac{1}{r^n}$ of a pound. Then find in the given table ^{ing conclusion} which present value will be $\frac{1}{r^n}$ of a pound. Then find in the given table ^{expressed in} words.

of the probabilities of the duration of human life the number of persons living at the age of the purchaser; and call it P . Then add to the age of the purchaser the number of years at the end of which the payment will become due; and find in the said table the number of persons living at the said greater age; and call it p . Then find in the said table of probabilities the number of persons living at the age of the purchaser's companion; and call it Q . Then add to the age of the said companion of the purchaser the number of years at the end of which the payment will become due; and find in the said table the number of persons that are there represented to be living at the said greater age; and call it q . Then say as the number Q is to the number q , so is the number p to a fourth number; which will therefore be equal to $\frac{pq}{Q}$. Then multiply this fourth number into

$\frac{1}{r^n}$ of a pound, or $\frac{1}{r^n} \text{ £}$, or the present value of one pound certain, payable at the end of the given number of years. And lastly, divide the product, $\frac{pq}{Q} \times \frac{1}{r^n} \text{ £}$; thence arising, by P , the number of persons represented in the given table of probabilities to be living at the age of the purchaser. And the quotient of this division, or the quantity $\frac{pq}{PQ} \times \frac{1}{r^n} \text{ £}$, will be the value of the future contingent payment of one pound, which is payable to the purchaser at the end of the given number of years, provided both he and his companion are then alive, expressed in decimal parts of a pound sterling.

XLVII. COROLL. 1. If the sum of money to be received by the purchaser at the end of the given number of years is greater or less than one pound, it is evident that the price he ought to pay for it to the grantor will be greater or less than the price of the future payment of one pound in the same proportion. Therefore, if the sum to be received at the end of n years is denoted by $S \text{ £}$, the price which the purchaser ought to pay for it, upon the conditions supposed in the foregoing problem, will be $\frac{pq}{PQ} \times \frac{1}{r^n} S \text{ £}$.

For as as $\text{£}1$ is to $S \text{ £}$, so is $\frac{pq}{PQ} \times \frac{1}{r^n} \text{ £}$, (which is the price of a future payment of one pound,) to $\frac{pq}{PQ} \times \frac{1}{r^n} S \text{ £}$; which is therefore the price of a future payment of S pounds. QED.

Of the value
of an annuity
for the joint
lives of two
persons.

XLVIII. COROLL. 2. By finding in the method described in the solution of the foregoing problem the several values of one pound sterling, to be received by a purchaser of a given age at the end of every future year of the whole space of time during which it is possible that his life and that of his companion may be prolonged, provided the said purchaser and his companion, whose age is likewise given, shall be both living at the time when every such payment becomes due; and then adding these values all together into one sum; we shall obtain the value of a set of equal payments of one pound each to be received by the purchaser at the end of every future year during the continuance of the joint lives of the said purchaser and his companion, or of an annuity for their joint lives. And this is the way in which tables of the values of annuities for the joint lives of two persons of given ages ought to be computed.

An example of the calculation of an annuity for two joint lives according to the method described in the foregoing articles.

XLIX. As an example of this method of computing the values of annuities for two joint lives, let it be required to compute the value of an annuity for the joint lives of a purchaser aged 80 years and a companion aged 75 years, upon a supposition that the interest of money is 3 per cent. and the probabilities of the duration of human life such as they are represented to be in Mr. de Parcieux's table.

Now it appears by Mr. de Parcieux's table that out of 118 persons of the age of 80 years, all living at the same time, 101 will live to the end of one year, or to the age of 81 years; and out of 211 persons of the age of 75 years, all living at the same time, 192 will live to the end of one year, or to the age of 76 years. And by Mr. Smart's second table of compound interest, page 60, it appears that, when the interest of money is 3 per cent. or r is $= 1.03$, the present value of one pound certain, to be received at the end of a year, is .9708 of a pound, or $\text{£}.9708$. Therefore, in order to find the value of the first payment of this annuity for the joint lives of these two persons, we have $P = 118$, $p = 101$, $Q = 211$, $q = 192$, and $\frac{1}{r^n} = \frac{1}{r^n} = \text{£}.9708$, and consequently $\frac{Pq}{r^n} \times \frac{1}{r}$, or $\frac{Pq}{r^n} \times \frac{1}{r} = \frac{101 \times 192}{118 \times 211} \times \text{£}.9708 = \frac{19392}{24898} \times \text{£}.9708 = \frac{\text{£}18825.7536}{24898} = \text{£}.7561$. Therefore $\text{£}.7561$ is the value of the first year's payment of the annuity of one pound, which is to be made to him at the end of one year from the time of granting it, provided both he and his companion are then alive; at which time the purchaser himself will be 81 years old, and his companion 76.

In

In like manner it appears by Monsieur de Parcieux's table that of the original number of 118 persons of the age of 80 years, all living at the same time, there will be

85 persons living at the age of 82 years,	
and 71	at the age of 83,
and 59	at the age of 84,
and 48	at the age of 85,
and 38	at the age of 86,
and 29	at the age of 87,
and 22	at the age of 88,
and 16	at the age of 89,
and 11	at the age of 90,
and 7	at the age of 91,
and 4	at the age of 92,
and 2	at the age of 93,
and 1	at the age of 94,
and none at all	at the age of 95.

And by the same table it appears that of the original number of 211 persons of the age of 75 years, all living at the same time, there will be

173 persons living at the age of 77 years,	
and 154	at the age of 78,
and 136	at the age of 79,
and 118	at the age of 80,
and 101	at the age of 81,
and 85	at the age of 82,
and 71	at the age of 83,
and 59	at the age of 84,
and 48	at the age of 85,
and 38	at the age of 86,
and 29	at the age of 87,
and 22	at the age of 88,
and 16	at the age of 89 years.

It is needless to inquire how many persons out of the aforesaid original number of 211 persons of the age of 75 years, all living at the same time, will live beyond the age of 89 years; because, when the companion of the purchaser is more than 89 years old, that is, when more than 14 years shall have elapsed after the time of granting the annuity, the purchaser himself, who was 80 years old at the time of making the grant, will, according to Mr. de Parcieux's table, be dead, and consequently the annuity will be at an end.

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And

And by Mr. Smart's second table of compound interest, page 60, it appears that, when the interest of money is 3 per cent. the present value of one pound certain, payable at the end of

	2 years,	is	£
and at the end of	3 years,		.9425;
and at the end of	4 years,		.9151;
and at the end of	5 years,		.8884;
and at the end of	6 years,		.8626;
and at the end of	7 years,		.8374;
and at the end of	8 years,		.8130;
and at the end of	9 years,		.7894;
and at the end of	10 years,		.7664;
and at the end of	11 years,		.7440;
and at the end of	12 years,		.7224;
and at the end of	13 years,		.7013;
and at the end of	14 years,		.6809;
			.6611.

Therefore, by Art. 46, the value of the second year's rent, or payment of one pound, which is to be received by the purchaser when he is 82 years old, provided his companion is then alive, is = $\frac{85 \times 173}{118 \times 211} \times £.9425 =$

$$\frac{£}{118 \times 211} \frac{14705 \times .9425}{24898} = \frac{£}{24898} \frac{13859.4625}{24898} = £.5566.$$

And the value of the third year's rent, which is to be paid him when he is 83 years old, if his companion is then alive, is $\frac{71 \times 154}{118 \times 211} \times £.9151 =$

$$\frac{£}{24898} \frac{10934}{24898} \times £.9151 = \frac{£}{24898} \frac{10,005.7034}{24898} = £.4018.$$

And the value of the fourth year's rent, which is to be paid him when he is 84 years old, if his companion is then alive, is $\frac{59 \times 136}{118 \times 211} \times £.8884 =$

$$\frac{£}{24898} \frac{8024}{24898} \times £.8884 = \frac{£}{24898} \frac{7288.4336}{24898} = £.2927.$$

And

LIFE-ANNUITIES.

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And the value of the fifth year's rent is $\frac{48 \times 118}{118 \times 211} \times \text{£.8626} = \frac{48}{211}$

$$\times \text{£.8626} = \text{£.1962.}$$

And that of the sixth year's rent is $\frac{38 \times 101}{118 \times 211} \times \text{£.8374} = \frac{3838}{24898}$

$$\times \text{£.8374} = \text{£.1291.}$$

And that of the seventh year's rent is $\frac{29 \times 85}{118 \times 211} \times \text{£.8130} = \frac{2465}{24898}$

$$\times \text{£.8130} = \frac{2004.0450}{24898} = \text{£.0805.}$$

And that of the eighth year's rent is $\frac{21 \times 71}{118 \times 211} \times \text{£.7894} = \frac{1564}{24898}$

$$\times \text{£.7894} = \frac{1233.0428}{24898} = \text{£.0495.}$$

And that of the ninth year's rent is $\frac{16 \times 59}{118 \times 211} \times \text{£.7664} = \frac{944}{24898}$

$$\times \text{£.7664} = \frac{723.4816}{24898} = \text{£.0290.}$$

And that of the tenth year's rent is $\frac{11 \times 48}{118 \times 211} \times \text{£.7440} = \frac{528}{24898}$

$$\times \text{£.7440} = \frac{392.8320}{24898} = \text{£.0157.}$$

And that of the eleventh year's rent is $\frac{7 \times 38}{118 \times 211} \times \text{£.7224} = \frac{266}{24898}$

$$\times \text{£.7224} = \frac{192.1584}{24894} = \text{£.0077.}$$

And that of the twelfth year's rent is $\frac{4 \times 29}{118 \times 211} \times \text{£.7013} = \frac{116 \times 7013}{24898}$

$$= \frac{81.3508}{24898} = \text{£.0032.}$$

H 2

And

And that of the thirteenth year's rent is $\frac{2 \times 22}{118 \times 211} \times £.6809 = \frac{44}{24898}$

$$\times £.6809 = \frac{£.29.9596}{24898} = £.0012.$$

And that of the fourteenth and last year's rent is $\frac{1 \times 16}{118 \times 211} \times £.66m$

$$= \frac{16 \times £.6611}{24898} = \frac{£.10.5776}{24898} = £.0004.$$

The sum total of these values is £2.5197, or, nearly, £2.52, or 2l. 10s. 5d. which is therefore the value of the annuity proposed for the joint lives of the purchaser aged 80 years and his companion aged 75. *Q.E.I.*

£	.7561
	.5566
	.4018
	.2927
	.1962
	.1291
	.0805
	.0495
	.0290
	.0157
	.0077
	.0032
	.0012
	.0004
	2.5197

Of the value of a remote life-annuity for two joint lives. **L. COROLL. 3.** The value of a remote life-annuity, that is to begin at the distance of a given number of years, and to continue during the joint lives of the purchaser and his companion, whose ages are given, may be easily computed by means of the foregoing problem.

For we need only compute the values of the several payments that are to be received at the end of every year after the said given number of years during the longest possible continuance of the joint lives of the said purchaser

purchaser and his companion, that is, during the longest possible continuance of the life of the older of them, according to the table of the probabilities of the duration of human life, that is, according to Mr. de Parcieux's table, till he is 94 years old. And the sum of these values will be the value of the proposed remote annuity. Q.E.I.

LI. COROLL. 4. And in like manner the value of an immediate, but imperfect, life-annuity, that is to begin immediately, (or whereof the first payment is to be received at the end of a year,) but is to continue only during a certain number of years less than the utmost possible extent of the older of the two lives, provided both the purchaser and his companion continue to live during the said term, may be computed by means of the foregoing problem, by computing the values of the several payments that are to be received, at the end of every year during the time that the said annuity is to continue, and adding up those values into one sum. Of the value of an immediate, but imperfect, life-annuity for two joint lives.

LI. COROLL. 5. And in like manner we may compute the value of a remote and imperfect life-annuity depending on the joint lives of the purchaser and his companion; that is, of an annuity depending on their joint lives, which is to commence at the end of a given number of years, and is to continue only during a certain space of time less than the utmost possible extent of the older of the two lives, and then to cease, though both the purchaser and his companion should be still alive. Of the value of a remote and imperfect life-annuity for two joint lives.

For this value will be the sum of the values of the several payments that are to be received at the end of every year during the time that the annuity is to continue. Q.E.I.

LIII. COROLL. 6. It is easy to see that the method of reasoning used in the solution of the foregoing problem may be extended to the valuation of a future payment, to be received at the end of a given number of years, depending upon the continuance of three lives, or any greater number of lives whatsoever. In the case of three lives the additions to the two solutions in Art 44 and 45 will be as follows.

An extension of the particular solution of the foregoing problem to the valuation of a future payment of one pound depending on three joint lives.

LIV. In the particular example solved in Art. 44 let the right of the purchaser of 25 years of age to the future payment of one pound, at the end of 30 years, depend upon the continuance of two lives besides his-

own, instead of one; to wit, upon the life of a companion of the age of 20 years, as is supposed in that example, and upon the life of a second companion of the age of 10 years. And let 774 such grants of one pound each, to be paid at the end of 30 years, be made to as many different purchasers; as is likewise supposed in the said example: and let each of these 774 purchasers have two companions; the one of 20, the other of 10, years of age, upon the continuance of whose lives at the end of the said 30 years his right to the said future payment of one pound shall depend.

Then it is evident that the number of the surviving purchasers that will be intitled, at the end of the said 30 years, to these payments of one pound, will be less than it was before; because some of those whose first companions (who were aged 20 years at the time of making the grants) are still alive, will have lost their second companions, (who were 10 years of age at the time of making the grants,) and with them their right to this future payment. We must therefore inquire how many of the said surviving purchasers there will be whose first and second companions will be both alive at the end of the said 30 years. Now we have seen that the whole number of purchasers who will themselves be living at the end of the said 30 years is 526; and that of these 526 surviving purchasers there will be 375, whose first companions (who were 20 years of age at the time of making the grants,) will be also living at the end of the said term. But there will be only a part of these 375 purchasers whose second, as well as first, companions will be living at that time. We must therefore now inquire how many such there will be, that is, how many of the second companions of these 375 purchasers will be alive at the end of the said 30 years. Now these second companions of these 375 purchasers were also 375 in number at the time of making the grants; each of the said purchasers having one second, as well as one first, companion. We must therefore inquire how many out of these 375 second companions of the said 375 purchasers will be living at the end of the said 30 years. Now these second companions of the said purchasers were supposed to be 10 years of age at the time of making the said grants. And it appears that out of 880 persons of 10 years of age, all living at the same time, 657 will be living at the age of 40 years, or at the end of a term of 30 years. Therefore out of 375 persons of the same age of 10 years, all living at the same time, there will be $375 \times \frac{657}{880}$, or 280, who will live to the age of 40, or to the end of 30 years. Therefore only 280 of the second companions of the said 375 surviving purchasers will be alive at the end of the said 30 years. Therefore out of the whole number of 526 purchasers who will survive to the end of the said 30 years, there will be only 280 whose first and second companions will both be living at that time; and who will consequently

consequently be indebted to receive their respective payments of one pound each. Therefore 280 pounds is the sum which the grantor of these future payments will have then to pay to these surviving purchasers. And consequently 280 times the present value of one pound, payable at the end of 30 years, when the interest of money is 3 per cent. that is, 280 times 4.19 of a pound, or £115.3320, is the sum which he ought to receive, at the time of making the grants, from all the 774 purchasers of these future payments. Therefore the sum he ought to receive from each of the said purchasers at the time of making the grants, is the 774th part of £115.3320, or £,1490, or, nearly, £,150 or 3 shillings. And consequently, by Art. 21, the same sum of 3 shillings is the price which he ought to receive for such a future payment of one pound, payable at the end of 30 years, provided the purchaser and both his said companions are then alive, when he makes only one such grant. Q. E. I.

An extension of the general solution of the same problem to the valuation of a future payment of one pound depending on three joint lives.

L. V. In the general solution of the foregoing problem given in Art. 45, let the right of the purchaser to the future payment of one pound, to be received at the end of n years, depend upon the continuance of the lives of two companions, instead of one, besides his own life. And let the age of the purchaser himself be M years, and that of his first companion N years, as before, and that of his second companion L years. Further, let it be supposed, as before, that the grantor makes P such grants of future sums of one pound, to be received at the end of n years, to P different purchasers, each of whom has two companions, the one of the age of M years, and the other of the age of L years, upon the continuance of both whose lives to the end of the said n years his right to the said future payment will depend.

Then it is evident that the number of the surviving purchasers that will be intitled, at the end of the said n years, to these payments of one pound each, will be less than it was before: because some of those whose first companions will be still living at the end of that term, will have lost their second companions before that time, and with them their right to this future payment. We must therefore inquire how many of the said surviving purchasers there will be whose first and second companions will be both living at the end of the said n years. Now the whole number of purchasers who will themselves be living at the end of the said n years, is p . And of these p purchasers it has been shewn in Art. 45 that there will be $p \times q$, whose first companions (who were of the age of M years at the time

of making the grants,) will be living at the end of the said term. But of these only a part will have their second, as well as their first, companion living at the end of the said term. We must therefore now inquire how great a part this will be of the said $p \times q$ purchasers whose first companions will be then alive, that is, how many of the second companions of these $p \times q$ purchasers will be alive at the end of the said n years.

Now it is evident that these second companions of the said $p \times q$ purchasers were also $p \times q$ in number at the time of making the said grants; because each of the said purchasers had a second, as well as a first, companion. We must therefore inquire how many of these $p \times q$ second companions of the said $p \times q$ purchasers will be living at the end of the said

n years, upon a supposition that they will die off in the course of the said n years in the proportion represented in the table of the probabilities of the duration of human life by which the calculation is to be governed. Now these second companions of the said purchasers were supposed to be L years of age at the time of making the grants. We must therefore look into the table of probabilities to see how many persons are there represented to be living at the age of L years. Call this number T . We must then look again into the table to see how many persons are there represented to be living at the age of $L + n$ years. Call this number t . Then, since out of T persons of the age of L years, all living at the same time, only t persons will live to the age of $L + n$ years, or to the end of n years, it follows that out of the $p \times q$ second companions of the afore-

said $p \times q$ purchasers, who were all living at the time of making the said grants, and were then L years old, only $p \times q \times \frac{t}{T}$ will be alive at the end

of the said n years. Therefore only $p \times q \times \frac{t}{T}$ of the said surviving purchasers will be intitled at the end of the said n years to demand their respective payments of one pound. Therefore the sum which the grantor of the said payments will be obliged to pay to the said purchasers at the end of the said n years, will be $p \times q \times \frac{t}{T}$ pounds. Therefore the sum

which

which he ought to receive, at the time of making the said grants, from all the P purchasers of the said future payments, is $P \times q \times t$ times the present

value of one pound certain, payable at the end of n years, that is, $P \times q \times t \times \frac{Q}{r^n}$

times $\frac{1}{r^n}$ of a pound, or $P \times q \times t \times \frac{1}{r^n} \times \frac{Q}{r^n}$. Therefore the sum which he

ought to receive from each of the said purchasers, at the time of making the said grants, as the price of his future payment of one pound, is the

P th part of $P \times q \times t \times \frac{1}{r^n} \times \frac{Q}{r^n}$ or is equal to $\frac{P}{r^n} \times \frac{q}{r^n} \times \frac{t}{r^n} \times \frac{Q}{r^n}$. And

consequently, by Art. 21, the price which he ought to receive from a single purchaser of such a future payment, when he makes only one such

grant, is likewise equal to $\frac{P}{r^n} \times \frac{q}{r^n} \times \frac{t}{r^n} \times \frac{Q}{r^n}$ or $\frac{Pq}{r^{3n}} \times \frac{t}{r^n} \times \frac{Q}{r^n}$. Q.E.D.

LVI. COROLL. 7. By finding, in the method described in the last corollary, the several values of one pound sterling, to be received by a purchaser of a given age at the end of every future year of the whole space of time during which it is possible that his life and those of his two companions, whose ages are likewise given, may be prolonged, provided that the said purchaser and both his said companions shall be living at the time when every such payment becomes due; and then adding these values all together into one sum; we shall obtain the value of a set of equal payments of one pound each, to be received by the purchaser at the end of every year during the continuance of the joint lives of the said purchaser and his two companions, or of an annuity for their joint lives. And this is the way in which tables of the values of annuities for the joint lives of three persons of given ages ought to be computed.

Of the value of an annuity for three joint lives.

Of the value of future payments depending on the continuance of the longest of two or more given lives.

LVII. We come now in the last place to consider the value of a future sum of money depending upon the continuance of any one of two or more lives whose ages are given.

P R O B L E M IV.

Of the value of a future sum of one pound sterling, to be received at the end of any given number of years, if either the said purchaser or another certain person (who may be called his companion,) shall be then living: the ages of the said purchaser and his companion, and the rate of interest of money, and the probabilities of the duration of human life, being all given.

years in case of the continuance of either of two given lives.

A solution of this problem in the case of a particular example.

LVIII. Let the rate of interest of money be supposed to be 3 per cent and the probabilities of the duration of human life to be such as they are represented to be in Mr. de Parcieux's table above-mentioned. And let the space of time at the end of which the said sum of one pound is to be paid to the purchaser of it, if he is then living, or to his companion, if the purchaser himself is then deceased, and his said companion is still alive, be 30 years: and the age of the said purchaser 25 years; and that of his said companion 20 years.

Then in the first place we must look into Mr. de Parcieux's table to see how many persons of 25 years of age are there represented to be all living at the same time. This number is 774. We must therefore suppose that the grantor of the future payment of one pound to the purchaser proposed in the question, makes at the same time 773 more such grants of one pound to as many different purchasers, all of the same age of 25 years, to be paid to them at the end of 30 years, or when they shall be 55 years old, if they shall then be living, and if they shall then be dead, but certain other persons, (who may be called their companions,) who are of the same age of 20 years with the companion of the purchaser mentioned in the question, shall be then alive, to be paid to their said companions respectively: that is, we must suppose that the grantor makes 774 such grants in all, including that to the purchaser proposed in the question. And we must likewise suppose that all these 774 purchasers of these future payments of one pound have the same chance, one with another, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. This done, we must inquire how many of these 774 purchasers of these future payments will be alive at the end of 30 years, supposing them to die off in that interval of time in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 persons of the age of 25 years, all living at the same time, 526 will be alive at the age of 55 years, or at the end of 30 years. Therefore, out of the said 774 purchasers

purchasers of these future payments of one pound each, 526 will live to the end of the said 30 years; and consequently 248 will have died in the mean time. But by the conditions of this problem, (which differ widely from those of the last problem,) all these 526 surviving purchasers of these future payments will be intitled to receive them, and likewise all the surviving companions of the deceased 248 purchasers. We must therefore inquire, by the means of Mr. de Parcieux's table, how many of the companions of the said deceased 248 purchasers will be alive at the end of the said 30 years. Now the number of the companions of the said deceased 248 purchasers, at the time of making the grants, was 248, each of the said purchasers having had one companion. And their age, at the time of making the grants, was 20 years. Now it appears from Mr. de Parcieux's table that out of 814 persons of the age of 20 years, all living at the same time, 581 will be living at the age of 50 years, or at the end of 30 years. Therefore out of the 248 companions of the deceased purchasers, (who were all living at the time of making the said grants, and were then 20 years of age,) $248 \times \frac{581}{814}$, or 177, will be living at the end of the said 30 years. Therefore the grantor at the end of the said 30 years will have 526 pounds to pay to the 526 surviving purchasers, and 177 pounds to pay to the surviving 177 companions of the 248 deceased purchasers; that is, he will have, in all, 526 and 177 pounds, or 703 pounds, to pay to both. To the end therefore that the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive from all the purchasers of the said future payments, at the time of making the said grants, 703 times the present value of one pound, payable at the end of 30 years, when the interest of money is 3 per cent. that is, 703 times .4119 of a pound, or £289.5657. Therefore the sum which he ought to receive, at the time of making the said grants, from each of the said purchasers, who are 774 in number, is the 774th part of £289.5657, or .3741 of a pound. Therefore, by Art. 21, the sum which he ought to receive from a single purchaser, as the price of such a future payment of one pound, when he makes only one such grant, is likewise £.3741, or 7s. 5d. $\frac{1}{2}$. Q.E.I.

A general solution of the foregoing problem.

LIX. Let i be to r as the sum of one pound sterling, or any other sum of money whatsoever, that is put out to interest at the rate supposed in the question, is to the amount of the same sum at the end of a year, or to the sum total of the said sum and its interest for a year. And let n denote the number of years at the end of which the sum of one pound is to be paid to the purchaser, if he is then alive, or, if he is then dead but his companion is still alive, to his companion. And let N denote the number of years in the age of the purchaser, at the time of his purchasing this future payment

payment of one pound, and M the number of years in the age of his companion at the same time: whence it is evident that $N+n$ will be the number of years in the age of the purchaser, and $M+n$ the number of years in the age of his companion, at the end of the said n years, or when the payment becomes due.

Then, in the first place, we must look into the table of probabilities of the duration of human life by which the calculation is to be governed, to find how many persons of N years of age are there represented to be living at the same time. This number we will call P . We must then suppose that the grantor of the said future payment of one pound to the purchaser mentioned in the question does not confine himself to that single grant, but makes at the same time $P-1$ more such grants, of one pound each, to as many different purchasers of them, all of the same age of N years; so that, including the grant to the purchaser in the question, he makes, in all, P such grants to P different purchasers. And we must further suppose that all these P purchasers have the same chance, one with another, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. And we must likewise suppose them to die off, in the course of the said n years which are to elapse before the said payments become due, in the same proportion in which persons of the same age are represented to have died off in the same number of years in the table of the probabilities of the duration of human life that is adopted for the calculation. We must therefore look into that table to find how many persons out of P persons of the age of N years, all living at the same time, are represented as living to the age of $N+n$ years. And this number we will call p . Then it is evident that out of all the P purchasers of the future sums of one pound each, payable at the end of n years, only p persons will be alive at the end of the said n years. Therefore the number of these purchasers who will have died in the said n years is $P-p$. But by the conditions of this problem (which differ widely from those of the last problem,) all the said p surviving purchasers will be intitled to receive their respective payments of one pound, and likewise all the surviving companions of the $P-p$ deceased purchasers. We must therefore inquire, by means of the aforesaid table of probabilities, how many of the companions of the said deceased $P-p$ purchasers will be alive at the end of the said n years. Now the number of the companions of the said deceased $P-p$ purchasers, at the time of making the grants, was the same with that of the said deceased purchasers, that is, $P-p$; because each of these deceased purchasers was supposed to have one companion. And the age of these companions, at the time of making the grants, was M years. We must therefore look into the table of probabilities aforesaid, to find how many persons are there represented to be living at the age of M years, and how many at the subsequent age of $M+n$ years. Let the former of these numbers be denoted by Q , and the latter

latter by q . Then, since the $P-p$ companions of the $P-p$ deceased purchasers of these future payments are supposed to die off, in the course of these n years, in the same proportion as the Q persons of the same age of M years are represented to have done in the said table of probabilities, the number of persons out of the said $P-p$ companions of the said $P-p$ deceased purchasers, that will be alive at the end of the said n years, will be to the number of those companions who were alive at the beginning of the said time, in the same proportion as q is to Q ; and consequently will be equal to $\frac{Q}{P-p} \times q$. Therefore the grantor of the said future payments, will, at

the end of the said n years, be obliged to pay p sums of one pound each to the p surviving purchasers themselves, and likewise $\frac{P-p}{Q} \times q$ sums of one pound each to the $\frac{P-p}{Q} \times q$ surviving companions of the $P-p$ deceased purchasers; that is, he will in all have $p + \frac{P-p}{Q} \times q$ sums of one pound to pay to them both. Therefore, to the end that he may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive from all the purchasers of the said future payments, at the time of making the said grants, $p + \frac{P-p}{Q} \times q$ times the present value of one pound,

payable at the end of n years, when the interest of money is that which is expressed by the proportion of r to 1, that is, $p + \frac{P-p}{Q} \times q$ times

$\frac{1}{r^n}$ of a pound, or $\frac{p + \frac{P-p}{Q} \times q}{r^n}$ times $\frac{1}{r^n} £$, or $\frac{pQ + Pq - pq}{r^n} \times \frac{1}{r^n} £$. Therefore the sum which he ought to receive, at the time of making the said grants, from each of the said purchasers, whose number is P , is the P th part of $\frac{pQ + Pq - pq}{r^n} \times \frac{1}{r^n} £$, or is equal to $\frac{pQ + Pq - pq}{P r^n} \times \frac{1}{r^n} £$. And

consequently, by Art. 21, the sum which he ought to receive from a single purchaser, as the price of such a future payment of one pound, upon the conditions above-mentioned, when he makes only one such grant, is likewise equal to this same quantity, $\frac{pQ + Pq - pq}{P r^n} \times \frac{1}{r^n} £$, (or $\frac{pQ + Pq - pq}{P r^n} \times \frac{1}{r^n} £$)

$$\times \frac{1}{r^n} £, \text{ or } \frac{pQ + Pq - pq}{P r^n} \times \frac{1}{r^n} £. \quad Q.E.D.$$

Of the relation of the said value to the values of the same payment of one pound sterling to be made at the end of the same number of years in case of the continuance of the single lives of the same two persons and of the continuance of their joint lives.

For the value found by the foregoing problem is $\frac{p+q-pq}{p \cdot q \cdot p \cdot q} \times \frac{1}{r^n} £$,

or $\frac{p}{p} \times \frac{1}{r^n} £ + \frac{q}{q} \times \frac{1}{r^n} £ - \frac{pq}{p \cdot q} \times \frac{1}{r^n} £$. And it has been shewn in Problem 2d, Art. 28, that $\frac{p}{p} \times \frac{1}{r^n} £$ is the value of a future payment of one

pound, to be received at the end of n years in case a purchaser of the age of N years shall be then alive, and that $\frac{q}{q} \times \frac{1}{r^n} £$ is the value of a future payment of one pound, to be received at the end of n years in case a purchaser of the age of M years shall be then alive: and it has been shewn in Problem 3d, Art. 45, that $\frac{pq}{p \cdot q} \times \frac{1}{r^n} £$ is the value of a future payment of one pound, to be received at the end of n years in case a purchaser of the age of N years and a companion of the age of M years, shall be both living at that time. Therefore the value found in the foregoing problem is equal to the excess of the sum of the two former values above the third, or last, value; or the value of a future payment of one pound, to be received at the end of n years in case that either of two given lives shall be then in being, is equal to the excess of the sum of the two values of the same future payment, to be received at the same time, in case each of the same two lives shall be then in being, above the value of the same future payment, to be received at the same time, in case both the said lives shall be then in being. QED.

Of the value of a future payment of S pounds depending on the same contingencies.

LXI. COROLL. 2. If the sum of money to be received by the purchaser at the end of the given number of years is greater or less than one pound, it is evident that the price he ought to pay for it will be greater or less than the price of the future payment of one pound in the same proportion. Therefore for a future payment of S pounds, to be received at the end of n years in case either himself or his companion, whose ages are N and M years, shall be then alive, he ought to pay the sum of

$$\frac{p+q-pq}{p \cdot q \cdot p \cdot q} \times \frac{S}{r^n} £.$$

LXII. COROLL. 3. By finding in the method described in the solution of the foregoing problem the several values of one pound sterling, to be received by a purchaser of a given age at the end of every future year of the whole space of time during which it is possible that either his life or that of his companion, whose age is also supposed to be given, may be prolonged, in case that either the said purchaser or his said companion shall be living at the time when every such payment becomes due; and then adding these values all together into one sum; we shall obtain the value of a set of equal payments of one pound each, to be received by the purchaser at the end of every future year during the lives of the said purchaser and his companion and the life of the longer liver of them, that is, of an annuity of one pound for the lives of the said purchaser and his companion and the life of the longer liver of them.

LXIII. COROLL. 4. The value of an annuity of one pound for the lives of two persons of given ages and the life of the longer liver of them, is equal to the excess of the sum of the values of two separate annuities of one pound each, for the single lives of the same persons, above the value of an annuity of one pound for their joint lives.

In the foregoing articles P denoted the number of persons represented in the table of the probabilities of the duration of human life adopted for the calculation, to be living at the age of the purchaser, or at the age of N years; and Q denoted the number of persons therein represented to be living at the age of the purchaser's companion, or at the age of M years.

Now let P' denote the number of persons therein represented to be living at the age of $N+1$, and P'' those living at the age of $N+2$, and P''' those living at the age of $N+3$, and P^{iv} those living at the age of $N+4$; and so on for the following years; the Roman numeral figures placed over every new letter P denoting the number of years by which every new age, corresponding to it, exceeds the age of N years, or the age of the purchaser. And, in like manner, let Q' denote the number of persons represented in the said table to be living at the age of $M+1$ years; and Q'' the number of persons living at the age of $M+2$ years; and Q''' those living at the age of $M+3$ years; and Q^{iv} those living at the age of $M+4$ years; and so on for the following years; the Roman numeral figures placed over every new letter Q denoting the number of years by which every new age corresponding to it, exceeds the age of M years, or the age of the purchaser's companion.

Then it is evident from the foregoing problem that the value of a sum of one pound, to be received at the end of one year, in case either

either the purchaser or his companion shall be then alive, will be equal to $\frac{P^i + Q^i - P^i Q^i}{P^i Q^i} \times \frac{1}{r} £$; and the value of the same sum, to be received at the end of two years, subject to the same contingency, will be equal to $\frac{P^{i+1} + Q^{i+1} - P^{i+1} Q^{i+1}}{P^{i+1} Q^{i+1}} \times \frac{1}{r^2} £$; and the value of the same sum, to be received at the end of three years, subject to the same contingency, will be $\frac{P^{i+2} + Q^{i+2} - P^{i+2} Q^{i+2}}{P^{i+2} Q^{i+2}} \times \frac{1}{r^3} £$; and the value of the same sum, to be received at the end of four years, subject to the same contingency, will be $\frac{P^{i+3} + Q^{i+3} - P^{i+3} Q^{i+3}}{P^{i+3} Q^{i+3}} \times \frac{1}{r^4} £$; and so on for the following years.

Therefore the sum of these values continued to the utmost possible extent of the younger of these two lives, that is, the value of a life-annuity of one pound for the lives of the purchaser and his companion and the life of the longer liver of them, is equal to the following set of values; to wit,

$$\frac{P^i + Q^i - P^i Q^i}{P^i Q^i} \times \frac{1}{r} £$$

$$+ \frac{P^{i+1} + Q^{i+1} - P^{i+1} Q^{i+1}}{P^{i+1} Q^{i+1}} \times \frac{1}{r^2} £,$$

$$+ \frac{P^{i+2} + Q^{i+2} - P^{i+2} Q^{i+2}}{P^{i+2} Q^{i+2}} \times \frac{1}{r^3} £,$$

$$+ \frac{P^{i+3} + Q^{i+3} - P^{i+3} Q^{i+3}}{P^{i+3} Q^{i+3}} \times \frac{1}{r^4} £,$$

$$+ \frac{P^v + Q^v - P^v Q^v}{P^v Q^v} \times \frac{1}{r^5} £,$$

+ &c. continued to the utmost extent of human life;

And

And consequently to the following set of quantities; to wit,

$$\begin{aligned}
 & \frac{P^i}{P} \times \frac{1}{r} k + \frac{Q^i}{Q} \times \frac{1}{r} k - \frac{P^i Q^{i+1}}{P Q} \times \frac{1}{r} k \\
 & + \frac{P^{i+1}}{P} \times \frac{1}{r^2} k + \frac{Q^{i+1}}{Q} \times \frac{1}{r^2} k - \frac{P^{i+1} Q^{i+2}}{P Q} \times \frac{1}{r^2} k \\
 & + \frac{P^{i+2}}{P} \times \frac{1}{r^3} k + \frac{Q^{i+2}}{Q} \times \frac{1}{r^3} k - \frac{P^{i+2} Q^{i+3}}{P Q} \times \frac{1}{r^3} k \\
 & + \frac{P^{i+3}}{P} \times \frac{1}{r^4} k + \frac{Q^{i+3}}{Q} \times \frac{1}{r^4} k - \frac{P^{i+3} Q^{i+4}}{P Q} \times \frac{1}{r^4} k \\
 & + \frac{P^v}{P} \times \frac{1}{r^5} k + \frac{Q^v}{Q} + \frac{1}{r^5} k - \frac{P^v Q^v}{P Q} \times \frac{1}{r^5} k \\
 & + \&c. \text{ continued to the utmost extent of human life;}
 \end{aligned}$$

And consequently to the three following sets of quantities, which are the same with the quantities last mentioned, but only are placed in a different order, (those which before were placed in perpendicular columns, one under the other, being now placed in separate horizontal lines,) to wit,

$$\begin{aligned}
 \text{1st, } & \frac{P^i}{P} \times \frac{1}{r} k + \frac{P^{i+1}}{P} \times \frac{1}{r^2} k + \frac{P^{i+2}}{P} \times \frac{1}{r^3} k + \frac{P^{i+3}}{P} \times \frac{1}{r^4} k \\
 & + \&c. \text{ continued to the utmost extent of human life;}
 \end{aligned}$$

$$\begin{aligned}
 \text{And 2dly, } & \frac{Q^i}{Q} \times \frac{1}{r} k + \frac{Q^{i+1}}{Q} \times \frac{1}{r^2} k + \frac{Q^{i+2}}{Q} \times \frac{1}{r^3} k + \frac{Q^{i+3}}{Q} \times \frac{1}{r^4} k \\
 & + \frac{Q^v}{Q} \times \frac{1}{r^5} k + \&c. \text{ continued to the utmost extent of human life;}
 \end{aligned}$$

$$\begin{aligned}
 \text{And 3dly, } & -\frac{P^i Q^i}{P Q} \times \frac{1}{r} k - \frac{P^{i+1} Q^{i+1}}{P Q} \times \frac{1}{r^2} k - \frac{P^{i+2} Q^{i+2}}{P Q} \times \frac{1}{r^3} k \\
 & - \frac{P^{i+3} Q^{i+3}}{P Q} \times \frac{1}{r^4} k - \frac{P^v Q^v}{P Q} \times \frac{1}{r^5} k - \&c. \text{ continued to the utmost extent of} \\
 & \text{human life;}
 \end{aligned}$$

that is, the value of the life-annuity of one pound for the lives of the said purchaser and his companion and the life of the longer liver of them, is equal to the excess of the sum of the first and second of the three last sets of quantities above the third set.

K

Now

Now it appears from Problem 2d, Coroll. 2, Art. 31, that the first of these three sets of quantities, to wit, $\frac{P^i}{P} \times \frac{1}{r} \ell + \frac{P^{i+1}}{P} \times \frac{1}{r^2} \ell + \frac{P^{i+2}}{P} \times \frac{1}{r^3} \ell + \frac{P^{i+3}}{P} \times \frac{1}{r^4} \ell + \frac{P^{i+4}}{P} \times \frac{1}{r^5} \ell + \text{&c.}$ continued to the utmost extent of human life, is the value of an annuity of one pound for the life of the purchaser, who was supposed to be of the age of N years, which corresponds to the number P in the table.

And it appears, in like manner, that the 2d of these 3 sets of quantities, to wit, $\frac{Q^i}{Q} \times \frac{1}{r} \ell + \frac{Q^{i+1}}{Q} \times \frac{1}{r^2} \ell + \frac{Q^{i+2}}{Q} \times \frac{1}{r^3} \ell + \frac{Q^{i+3}}{Q} \times \frac{1}{r^4} \ell + \frac{Q^{i+4}}{Q} \times \frac{1}{r^5} \ell + \text{&c.}$ continued to the utmost extent of human life, is the value of an annuity of one pound for the life of the said purchaser's companion, who was supposed to be of the age of M years, which corresponds to the number Q in the table.

And it appears from Prob. 3, Coroll. 2, Art. 48, that the last of the said 3 sets of quantities, to wit, $\frac{P^i Q^i}{P Q} \times \frac{1}{r} \ell + \frac{P^{i+1} Q^{i+1}}{P Q} \times \frac{1}{r^2} \ell + \frac{P^{i+2} Q^{i+2}}{P Q} \times \frac{1}{r^3} \ell + \frac{P^{i+3} Q^{i+3}}{P Q} \times \frac{1}{r^4} \ell + \frac{P^{i+4} Q^{i+4}}{P Q} \times \frac{1}{r^5} \ell + \text{&c.}$ continued to the utmost extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser and his companion, whose ages N and M correspond to the numbers P and Q in the said table.

Therefore the value of a life-annuity of one pound for the lives of the said purchaser and his companion, and the life of the longer liver of them, is equal to the excess of the sum of the values of two separate life-annuities of one pound each, for the single lives of the said purchaser and his companion, above the value of an annuity of one pound for their joint lives. Q.E.D.

LXIV. Therefore when tables of the values of life-annuities for single lives and for two joint lives have been computed and prepared for use, it seems to be unnecessary, and hardly worth the labour of it, to compute a third table of the values of life-annuities for the lives of two persons of given ages, and the life of the longer liver: because the values of these latter annuities may be so easily derived, by means of the foregoing corollary, from the values of the corresponding annuities of the two former kinds, which are supposed to be set down in the tables.

LXV. COROLL. 5. The value of a remote life-annuity of one pound, that is to begin at the ^{Of the value} ~~Range~~ of a given number of years, and to continue during the lives of ^{of a remote} ~~the purchaser and his companion and the life~~ annuity of one pound for the longer liver of them, ^{annuity of one} ~~whose ages are given, is equal to the excess~~ pound for the life of the longer liver of the sum of two separate remote annuities of one pound each, for the single lives of the same two persons, that are to begin at the same future period, above the value of a third remote annuity of one pound, that is also to begin at the same future period and to continue during the joint lives of both the said persons.

For all the reasoning used in the foregoing corollary applies equally to these remote life-annuities and to those compleat and immediate life-annuities that are the subject of that corollary; of which compleat annuities the former remote annuities are only the latter parts. This I take to be so evident, that a repetition of that reasoning to apply it to this case would probably be tedious to the reader. And therefore I omit it.

LXVI. COROLL. 6. And in like manner the value of an immediate, but imperfect, life-annuity of one pound, that is to begin immediately (or whereof the first payment is to be received at the end of a year,) but is to continue only during a certain number of years, less than the utmost extent of human life, if either the purchaser or his companion, whose ages are given, continues to live through that period, is equal to the excess of the sum of the values of two separate, immediate, and imperfect life-annuities of one pound each, for the single lives of the purchaser and his companion, to continue during the same limited period, if each of them continues to live so long, above the value of a like immediate and imperfect annuity of one pound for the joint lives of the said purchaser and his companion during the same term of years, if they both continue to live so long.

This also may be easily collected from the reasoning used in Coroll. 4, these annuities being only the first parts of the compleat life-annuities therein mentioned.

LXVII. COROLL. 7. And in like manner the value of a remote and imperfect life-annuity of one pound, for the lives of the purchaser and his companion, whose ages are given, and the life of the longer liver of them; that is, of an annuity that is to begin at the end of a given number of years, and is to continue only during a certain space of time, less than the utmost possible extent of the life of the younger of them, if either of them lives to the end of the said period, and then to cease, though one, or both, of them should be still alive; is equal to the excess of the sum of the values ^{Of the value} ~~of a remote~~ of a remote and imperfect life-annuity of one pound for the life of the longer liver of two persons whose ages are given.

of two separate annuities, of one pound each, for the single lives of the said purchaser and his companion, that should commence at the end of the same given number of years as the former and continue only during the same time, in case each of them should continue to live so long, and then cease, though they should be still alive, above the value of a like remote and imperfect life-annuity for the joint lives of the same persons, that should commence at the end of the same number of years, and continue during the same space of time, if they both shall continue to live so long.

This also may be easily collected from the reasoning used in Coroll. 4; these remote and imperfect life-annuities for the lives of the purchaser and his companion and the life of the survivor of them, being only the middle parts of the compleat life-annuities of the same kind therein mentioned. And to this corollary I therefore refer the reader.

Of the value of a future payment depending on the continuance of any one of three given lives.

LXVIII. COROLL. 8. The reasoning used in the solution of the foregoing problem may be extended to the valuation of a future payment, to be received at the end of a given number of years in case any one of three persons, or more, whose ages are given, shall be then alive. In the case of three lives the additions necessary to be made to the two solutions in Art. 58 and 59, will be as follows.

An investigation of the said value in the case of a particular example.

LXIX. In the particular example solved in Art. 58 let the right of the purchaser, of 25 years of age, to the future payment of one pound, at the end of 30 years, be extended to two other persons besides himself, instead of one; so that, if either the purchaser himself or either of his said companions shall be then alive, the said future sum shall be payable by the grantor of it to one of the said three persons. And to make the case more clear and definite, let it be supposed that the older of these two persons is called his *first companion* and the younger his *second companion*; and that, if the purchaser himself is alive at the end of the said 30 years, the said sum of one pound shall be payable to him alone, though either or both his said companions should be also living at the same time; and that, if he is then dead, but his companions are both alive, it shall be paid to the elder of the two, or his first companion; and that, if only one of them is then alive, it shall be paid to the said only surviving companion. And let the age of the said purchaser's older, or first, companion, be 20 years,

as was supposed in the foregoing example; and that of his younger, or second, companion 10 years; at the time of making the grant. And further, let it be supposed that the grantor of the said future payment of one pound makes 774 such grants of one pound each, to be received at the end of 30 years, to as many different purchasers, all of the same age of 25 years as the purchaser proposed in the question; and that the grant to the purchaser proposed in the question is one of those 774 grants; and that each of these purchasers has two companions, to wit, an older, or first, companion of 20 years of age, and a younger, or second, companion of 10 years of age; and that each of the sums so granted is to be paid by the said grantor, at the end of the said space of 30 years, provided the purchaser himself, or either of his two companions, is then alive; namely, to the purchaser himself, if he is then alive; or otherwise to the older of his two companions, if they are then both alive; or, if only one of them is then alive, to the said only surviving companion.

These things being supposed, it is evident that the number of persons that will be intitled to receive these payments of one pound each at the end of the said 30 years, will be greater than in the case supposed in the solution of the foregoing problem, Art. 58: because, not only all the 526 surviving purchasers themselves, together with the 177 surviving first companions of the 248 deceased purchasers, making in all 703 persons, will be intitled to these payments, as they were in that solution, but there will also be some surviving second companions of the deceased purchasers who will also have a right to them. What the number of these will be, we must now proceed to inquire.

Now it is evident, in the first place, from the conditions of this question, that the surviving second companions of the 526 surviving purchasers can have no claim to these payments of one pound; because they are to be made to the said surviving purchasers themselves. And, for the like reason, it is evident, in the second place, that the second companions of those deceased purchasers whose first companions are alive at the end of the said 30 years, can have no claim to these payments; because it is provided that when both the companions of a deceased purchaser are alive at the time his payment becomes due, it shall be made to the said purchaser's first companion, and not to his second companion. Now it has been shewn in Art. 58 that out of the 248 purchasers, who will have died in the course of the said 30 years, there will be 177, whose first companions (who were 20 years old at the time of making the grants,) will be alive at the end of the said 30 years. Therefore the number of the said deceased purchasers, whose first companions will also be dead before the end of the said 30 years, is the excess of 248 above 177 persons, that is, 71 persons. There are therefore 71 deceased purchasers, whose second companions will have a right

to receive these payments of one pound each, if they live to the end of the said 30 years. We must therefore inquire how many of the second companions of these 71 deceased purchasers will live to the end of the said 30 years, supposing them to die off in the proportion set forth in Mr. de Parcieux's table of the probabilities of the duration of human life. Now these companions are evidently 71 in number, because each of the said 71 deceased purchasers had one second as well as one first companion. And the age of these second companions, at the time of making the grants, is supposed to be 10 years. Now it appears from Mr. de Parcieux's table that out of 880 persons of the age of 10 years, all living at the same time, 657 will live to the age of 40 years, or to the end of a term of 30 years. Therefore out of the said 71 second companions of the said deceased purchasers there will be $71 \times \frac{657}{880}$, or 53, who will live to the end of the said

30 years. Therefore the whole number of persons intitled to receive the said payments of one pound each, at the end of the said 30 years, will be, first, the 526 surviving purchasers, and, secondly, the 177 surviving first companions of the 248 deceased purchasers, and, thirdly, the said 53 surviving second companions of those 71 of the said 248 deceased purchasers whose first companions will have died before the end of the said 30 years; that is, in all, 756 persons. Therefore at the end of the said 30 years the aforesaid grantor will be obliged to pay to the said surviving purchasers and the said first and second companions of the purchasers that are deceased, the sum of 756 pounds. Therefore, to the end that, when the said payments become due, the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive, at the time of making the said grants, 756 times the present value of one pound certain, payable at the end of 30 years, when the interest of money is 3 per cent. that is, 756 times .4119 of a pound, or £311.3964, from all the 774 purchasers of these future payments. Therefore the sum which each of the said purchasers ought then to pay him, as the price of the said future payment of one pound to be received at the end of the said 30 years, is the 774th part of £311.3964, or £.4023, or 8s. $\frac{1}{4}d$. And consequently, by Art. 21, this sum of 8s. $\frac{1}{4}d$. is likewise the price which a single purchaser ought to pay for a grant of such a future payment of one pound, to be received at the end of 30 years, if either himself or either of his two companions aforesaid shall be then alive, when the grantor of it makes only one such grant. Q E I.

LXX. In the foregoing article it was supposed that, where both the companions of the deceased purchasers happened to live to the end of the said 30 years, the said sums of one pound were to be paid to their first, or older, companions, who were 20 years of age at the time of making the grants

grants of them, in preference to their second, or younger, companions, who were only 10 years old at that time. But the values of these future sums of one pound will be exactly the same, if we should suppose that they are to be paid to the said second, or younger, companions of the said deceased purchasers in preference to the first, or older. This is what we might reasonably expect and suppose to be true without a particular demonstration. But, to remove all doubt from the reader's mind, we shall prove it distinctly by applying the reasoning used in the foregoing article to this new supposition, in the manner following.

Upon this new supposition it is evident that the persons who will be entitled to receive the said payments of one pound at the end of the said 30 years, will be, in the first place, those of the 774 purchasers themselves who will then be alive, whose number has been shewn to be 526; and, in the second place, those of the second companions of the 248 deceased purchasers who will then be living; and, in the third place, after subtracting from the said 248 deceased purchasers those of them whose second companions will have lived to the end of the said 30 years, those of the first companions of the remaining deceased purchasers (that is, of those deceased purchasers whose second companions have died in the course of the said 30 years,) who will also then be living. We must therefore inquire by the means of Mr. de Parcieux's table, in the first place, how many of the second companions of the said 248 deceased purchasers will be alive at the end of the said 30 years: and then, subtracting this number from that of all the deceased purchasers, that is, from 248, in order to obtain the number of the said deceased purchasers whose second companions will have died in the course of the said 30 years, we must inquire, in the next place, by means of the said table, how many of the first companions of the said remaining number of the said deceased purchasers will be alive at the end of the said 30 years.

Now it is evident that the second companions of the 248 deceased purchasers are also 248 in number, each of the said purchasers having had one second as well as one first companion. And these second companions are supposed to have been 10 years old at the time of making the grants. Now it appears by Mr. de Parcieux's table above-mentioned that out of 880 persons of 10 years of age, all living at the same time, 657 will be alive at the age of 40 years, or at the end of 30 years. Therefore out of the aforesaid 248 second companions of the 248 deceased purchasers there will be $248 \times \frac{657}{880}$, or 185, persons alive at the end of the said 30 years.

Therefore the number of the said second companions of the said 248 deceased purchasers who will have died in the course of the said 30 years, will be the excess of 248 above 185, that is, 63 persons. Therefore there are

are 63 persons out of the said 248 deceased purchasers whose second companions will have died in the course of the said 30 years. But, though the second companions of these 63 deceased purchasers will be dead at the end of the said 30 years, yet some of their first companions, who were of the age of 20 years at the time of making the grants, will probably be still alive at that time; and, if they are so, will be intitled to these payments of one pound each. We must therefore inquire, by Mr. de Parcieux's table, how many of the said first companions of these remaining 63 deceased purchasers will be alive at the end of the said 30 years. Now these first companions of the said 63 deceased purchasers are likewise 63 in number at the time of making the said grants; because each of the said purchasers is supposed to have had at that time two companions. And the age of these first companions at the time of making the said grants, was supposed to be 20 years. Now it appears by Mr. de Parcieux's table that out of 814 persons of the age of 20 years, all living at the same time, 581 will be alive at the age of 50 years, or at the end of 30 years. Therefore of the said 63 first companions of the said 63 deceased purchasers, there will be $63 \times \frac{581}{814}$, or 45, persons living at the end of the said

30 years, and will be intitled to receive their respective payments of one pound each. Therefore the whole number of persons intitled to receive the said payments, at the end of the said 30 years, will be, first, the aforeaid 526 surviving purchasers, and, secondly, the 185 surviving second companions of the 248 deceased purchasers, and thirdly, the 45 surviving first companions of the 63 deceased purchasers whose second companions will have died in the course of the said 30 years; that is, in all, 756 persons, as before. Therefore at the end of the said 30 years the aforeaid grantor will be obliged to pay to the said surviving purchasers, and the said first and second companions of the purchasers that are deceased, the sum of 756 pounds. Therefore, to the end that, at the expiration of the said 30 years, the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive, at the time of making the said grants, 756 times the value of one pound certain, payable at the end of 30 years, when the interest of money is 3 per cent. that is, 756 times .4119 of a pound, or £311.3964, from all the 774 purchasers of the said future payments. Therefore the sum which he ought then to receive from each of the said purchasers as the price of his said future sum of one pound, is the 774th part of £311.3964, or 4023 $\frac{1}{2}$, or 8s. $\frac{1}{4}$. And consequently, by Art. 21, this sum of £.4023, or 8s. $\frac{1}{4}$, is likewise the price which a purchaser ought to pay the grantor for such a future payment, when the latter makes only one such grant; which is the same price which was to be paid for it on the former supposition of the first, or older, companion's being intitled to receive the said payment in preference to the younger. QED.

LXXI. In the two foregoing articles it is shewn that the price to be paid for the future sum of one pound, to be received at the end of 30 years, if the purchaser or either of his two companions shall be then alive, is exactly the same, whether, in case of the purchaser's death and the survivorship of both his companions at the end of the said time, the purchaser's first, or older, companion be intitled to receive it in preference to his second, or younger, companion, or his second, or younger, companion, be intitled to receive it in preference to the first: for that in both cases, if we suppose 774 such grants of future sums of one pound to be made to as many different purchasers upon the conditions above-mentioned, the number of persons living at the end of the said 30 years who will be intitled to receive the said sums, will be the same, to wit, 756 persons. Now that this is not accidental, or peculiar to the ages and numbers that occur in this particular example, but would have been true, if any other ages than those of 25, 20, and 10 years, had been pitched upon, and any other number of years than 30 had been supposed to intervene before the said future payments became due, will appear by setting down the several factors and divisors by whose multiplication and division of each other the said resulting number of 756 claimants of these sums of one pound is obtained in both cases: which may be done in the manner following.

In Art. 69 the number of the persons intitled to receive the said payments of one pound each at the end of the said 30 years was composed of the whole number of purchasers who were then alive, which was 526, and of the 177 first, or older, companions of the deceased 248 purchasers, and of the 53 surviving second companions of those (248—177. or) 71 deceased purchasers whose first companions had died in the course of the said 30 years.

Now the said number, 177, of the surviving first companions of the deceased 248 purchasers, is = $248 \times 581 = 774 - 526 \times 581 = \frac{774 \times 581}{526 \times 581}$.

And 53, the number of the surviving second companions of the said 71 deceased purchasers, whose first companions will have died in the course of the said 30 years, is = $71 \times 657 = 248 - 177 \times 657 = \frac{248 \times 657}{177 \times 657} - \frac{177 \times 657}{177 \times 657}$

$$\begin{aligned}
 &= \frac{774 - 526 \times 657}{880} - \frac{177 \times 657}{880} = \frac{774 \times 657}{880} - \frac{526 \times 657}{880} - \frac{177 \times 657}{880} \\
 &= \frac{774 \times 657}{880} - \frac{526 \times 657}{880} - \frac{774 \times 581 \times 657}{814} + \frac{526 \times 581 \times 657}{814} = \frac{774 \times 657}{880} \\
 &\quad - \frac{526 \times 657}{880} - \frac{774 \times 581 \times 657}{814} + \frac{526 \times 581 \times 657}{814 \times 880}.
 \end{aligned}$$

L

Therefore

Therefore $526 + 177 + 53$, or the whole number of persons intitled to receive these payments at the end of the said 30 years, is $= 526 + 774 \times 581$

$$\begin{array}{r} \text{—} 526 \times 581 + 774 \times 657 \text{—} 526 \times 657 \text{—} 774 \times 581 \times 657 + 526 \times 581 \times 657, \\ 814 \qquad 880 \qquad 880 \qquad 814 \times 880 \qquad 814 \times 880 \end{array}$$

And in Art. 70 the number of persons intitled to receive the said payments of one pound each at the end of the said 30 years, is composed of the said 526 surviving purchasers, and of the 185 surviving second, or younger, companions of the 248 deceased purchasers, and of the 45 surviving first, or older, companions of those (248—185, or) 63 deceased purchasers whose second companions had died in the course of the said 30 years.

Now the said number, 185, of the second companions of the 248 deceased purchasers, is $= 248 \times 657 = 774 - 526 \times 657 = 774 \times 657$

$$\text{—} 526 \times 657. \\ 880.$$

And 45, the number of the surviving first companions of those 63 purchasers whose second companions have died in the course of the said 30 years, is $= 63 \times 581 = 248 - 185 \times 581 = 248 \times 581 - 185 \times 581$

$$\begin{array}{r} = 774 - 526 \times 581 - 185 \times 581 = 774 \times 581 - 526 \times 581 - 185 \times 581 \\ 814 \qquad 814 \qquad 814 \qquad 814 \qquad 814 \qquad 814 \\ = 774 \times 581 - 526 \times 581 - 774 \times 657 \times 581 + 526 \times 657 \times 581 = 774 \times 581 \\ 814 \qquad 814 \qquad 880 \qquad 814 \qquad 880 \qquad 814 \qquad 814 \\ \text{—} 526 \times 581 - 774 \times 657 \times 581 + 526 \times 657 \times 581. \\ 814 \qquad 880 \times 814 \qquad 880 \times 814 \end{array}$$

Therefore $526 + 185 + 45$, or the whole number of persons intitled to receive these payments at the end of the said 30 years is $= 526 + 774 \times 657$

$$\begin{array}{r} \text{—} 526 \times 657 + 774 \times 581 - 526 \times 581 - 774 \times 657 \times 581 + 526 \times 657 \times 581, \\ 880 \qquad 814 \qquad 814 \qquad 880 \times 814 \qquad 880 \times 814 \end{array}$$

Now

Now it is easy to observe that this expression consists of the same number of terms, to wit, seven, as the expression found above for the number of claimants of these sums of one pound upon the former supposition of preferring the surviving first companions of the purchasers to their surviving second companions; and that the terms of both the expressions are the same, being composed of the same factors and divisors, though placed in a somewhat different order. For in both expressions the first term is 526; and the second term of the first expression is $\frac{774 \times 581}{814}$, which makes the

fourth term in the second expression; and the third term of the first expression is $\frac{526 \times 581}{814}$, with the mark of subtraction prefixed to it, which

makes the fifth term in the second expression, where it has the same mark of subtraction; and the fourth term of the first expression is $\frac{774 \times 657}{880}$,

which makes the second term of the second expression; and the fifth term of the first expression is $\frac{526 \times 657}{880}$, with the mark of subtraction prefixed to

it, which makes the third term in the second expression, where it has the same mark of subtraction prefixed to it; and the sixth term of the first expression is $\frac{774 \times 581 \times 657}{814 \times 880}$, with the mark of subtraction prefixed to it; and

$\frac{774 \times 657 \times 581}{880 \times 814}$ with the mark of subtraction prefixed to it, is the sixth

term of the second expression, which is equal to the sixth term of the former expression, because it is composed of the same factors both in the numerator and denominator, though placed in a different order; and the seventh term in the first expression is $\frac{526 \times 581 \times 657}{814 \times 880}$, which is composed

of the same factors with, and therefore is equal to, the seventh term of the second expression, or $\frac{526 \times 657 \times 581}{880 \times 814}$. Therefore the whole first expression

is equal to the whole second expression, or the number of persons intitled to receive these payments at the end of the said 30 years is the same upon both the said suppositions. QED.

A general investigation of the value of a future payment of one pound depending on the continuance of any one of three given lives.

LXXII. We proceed now to extend the general solution of the foregoing problem given in Art. 59 to the case of a grant of a future payment of one pound made to a purchaser, to be received at the end of a given number of years, in case either the purchaser himself, or either of two other persons named by him, and who may be called his *companions*, shall be then alive; the ages of the purchaser and his two companions, and the rate of interest of money, and the table of the probabilities of the duration of human life, being all given.

Let the rate of interest of money be expressed, as before, by the proportion of r to 1. And let n denote the number of years at the end of which the said sum of one pound is to be received; and N the number of years in the age of the purchaser himself; and M the number of years in the age of the purchaser's first, or older, companion; and L the number of years in the age of his second, or younger, companion. And let the said sum of one pound be payable, at the end of the said n years, to the said purchaser himself, if he is then alive; and, if he is then dead, but both his companions are alive, to his first, or older, companion; and, if he also is then dead, but his second, or younger, companion is then alive, to his said second, or younger, companion. And let P denote the number of persons represented in the given table of probabilities to be living at the age of N years, or the age of the purchaser; and Q the number of persons therein represented to be living at the age of M years, or the age of the purchaser's first, or older, companion; and r the number of persons therein represented to be living at the age of L years, or the age of the purchaser's second, or younger, companion; and p , q , and s the numbers of persons therein represented to be living at the ages of $N+n$, $M+n$, and $L+n$ respectively. And let it be supposed that the grantor of the said future sum of one pound, to be received at the end of n years, makes not only one such grant, but P such grants to P different purchasers, each of whom has two companions, an older, or first, companion of the age of M years, and a younger, or second, companion of the age of L years; of which P grants that to the purchaser in the question is one; and that each of these grants is subject to the same conditions as the grant to the purchaser in the question.

Then it is evident that the number of persons that will be intitled to receive these payments of one pound each at the end of the said n years, will be greater than in the case supposed in the solution of the foregoing problem, Art. 59; because not only all the P purchasers who will be living at the end of the said 30 years, together with the $\overline{P-p} \times q$ surviving first
companions

companions of the $P-p$ deceased purchasers, making in all $p + \frac{P-p \times q}{q}$

persons, will be intitled to these payments, as they were in that solution, but there will also be some surviving second companions of the said deceased purchasers who will have a right to them. What the number of these will be, we must now proceed to inquire.

Now it is evident, in the first place, from the conditions of this question that the surviving second companions of the p purchasers, who will be alive at the end of the said n years, can have no claim to the said sums of one pound; because they are to be paid to the said surviving purchasers themselves. And, for the like reason, it is evident, in the second place, that the second companions of those deceased purchasers whose first companions are alive at the end of the said n years, can have no claim to these payments; because it is provided that, when both the companions of a deceased purchaser are alive at the time the payment becomes due, it shall be received by the said purchaser's first, or older, companion in preference to the second. Now it has been shewn in Art. 58 that out of the $P-p$ purchasers who will have died in the course of the said n years, there will be $\frac{P-p \times q}{q}$ persons whose first, or older, companions, who were M years of

age at the time of making the grants, will be alive at the end of the said n years. Therefore the number of the said deceased purchasers whose first, or older, companions will also be dead at the end of the said n years, is the excess of $P-p$ above $\frac{P-p \times q}{q}$, or $\frac{Pq-pq}{q}$, persons, or is equal to $P-p$

$\frac{Pq-pq}{q} = \frac{Pq-pq-Pq+pq}{q}$. There are therefore $\frac{Pq-pq-Pq+pq}{q}$

deceased purchasers whose second companions will have a right to receive these payments of one pound each, if they live to the end of the said n years. We must therefore inquire how many of the second companions of these $\frac{Pq-pq-Pq+pq}{q}$ deceased purchasers will live to the end of the said

n years, supposing them to die off in the proportion set forth in the given table of the probabilities of the duration of human life. Now these second companions of the said deceased purchasers are evidently just as many as the said deceased purchasers themselves, that is, $\frac{Pq-pq-Pq+pq}{q}$; because

each of the said deceased purchasers had one second as well as one first companion. And the age of these second companions at the time of making the grants, is supposed to be L years. Now the numbers of persons represented in the given table of probabilities to be living at the ages of L years and $L+n$ years are supposed to be T and t . Therefore, since out of T persons of L years of age, all living at the same time, t persons

persons will be alive at the end of n years, it is evident that of the aforesaid $P\mathcal{Q}-p\mathcal{Q}-Pq+pq$ second companions of the said deceased purchaser,

who are all living, and of the age of L years, at the time of making the grants, there will be $t \times \frac{P\mathcal{Q}-p\mathcal{Q}-Pq+pq}{T}$, or $\frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$,

alive at the end of the said n years. Therefore the whole number of persons intitled to receive the said payments of one pound each at the end of the said n years, will be, first, the p surviving purchasers themselves, and, secondly, the $\frac{P-p \times q}{T}$, or $\frac{Pq-pq}{T}$, surviving first companions of the

$P-p$ deceased purchasers, and, thirdly, the said $\frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$ surviving second companions of the said $P\mathcal{Q}-p\mathcal{Q}-Pq+pq$ deceased pur-

chasers, whose first companions will have died in the course of the said n years: that is, the number of the persons intitled to receive these payments will be, in all, $p + \frac{Pq-pq}{T} + \frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$, or

$p + \frac{Pq-pq}{T} + \frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$. Therefore at the end of the said

n years the aforesaid grantor will be obliged to pay to the said surviving purchasers and the said first and second companions of the purchasers that are deceased, the sum of $p + \frac{Pq-pq}{T} + \frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$ pounds.

Therefore, to the end that the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive, at the time of making the said grants, $p + \frac{Pq-pq}{T} + \frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$

times the present value of one pound certain, payable at the end of n years, when the interest of money is expressed by the proportion of r to 1; that is, $p + \frac{Pq-pq}{T} + \frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$ times $\frac{1}{r^n}$ of a pound, or $\frac{1}{r^n} \times$

from all the P purchasers of these future payments. Therefore the sum which he ought then to receive from each of the said purchasers is the P th part of $p + \frac{Pq-pq}{T} + \frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pq t}{T}$ times $\frac{1}{r^n}$ £, or is equal to

$\frac{p}{P} + \frac{q}{P} - \frac{pq}{P} + \frac{t}{P} - \frac{pt}{P} - \frac{qt}{P} + \frac{pqt}{P}$ times $\frac{1}{r^n}$ £. And consequently, by

Art. 21, the price which a purchaser ought to pay for such a grant of a future

future sum of one pound, to be received at the end of n years, upon the conditions above-mentioned, when the grantor of it makes only one such grant, is likewise equal to $\frac{p}{P} + \frac{q}{Q} - \frac{pq}{PQ} + i - \frac{pi}{P} - \frac{qi}{Q} + \frac{pqi}{PQ}$ times

$\frac{1}{P}$. Q E I.

LXXIII. In the foregoing article it was supposed that when both the companions of the deceased purchasers were alive at the end of the said n years, the said sums of one pound were to be paid to their first, or older, companions, who were of the age of M years at the time of making the grants, in preference of their second, or younger, companions, who were at that time of the age of L years. But the values of these future sums of one pound will be exactly the same, if we should suppose that they are to be paid to the said second, or younger, companions of the said deceased purchasers in preference to the first or older. This may be shewn in the manner following.

Upon this new supposition it is evident that the persons who will be intitled to receive the said sums of one pound at the end of the said n years, will be composed, in the first place, of the p surviving purchasers themselves, and, in the second place, of the surviving second, or younger, companions of the $P-p$ deceased purchasers, and, in the third place, of the surviving first, or older, companions of those of the said $P-p$ deceased purchasers whose second companions will have died in the course of the said n years. Now, since of T persons of the age of L years, or of the age of the second companions of the said purchasers at the time of making the grants, t persons are found in the given table of probabilities to be living at the end of n years, it follows, that of the $P-p$ second companions of the $P-p$ deceased purchasers, who were living at the time of making the grants, there will be $\frac{P-p}{T} \times t$, or $\frac{P-p}{T} \cdot t$, persons alive at the end of the

said n years. Therefore the number of deceased purchasers whose second companions will die in the course of the said n years is $P-p - \frac{P-p}{T} \cdot t$, or

$\frac{PT - pT - Pt + pt}{T}$; and consequently the number of the first companions

of these deceased purchasers is likewise equal to $\frac{PT - pT - Pt + pt}{T}$. But

the age of these first companions is that of M years. And it appears by the table that of Q persons of the age of M years, all living at the same time,

time, there will be q persons living at the end of n years. Therefore of the said $P_T - p_T - P_i + p_i$ first companions of the said deceased purchaser,

who were living at the time of making the grants, there will be living at the end of the said n years only $\frac{P_T - p_T - P_i + p_i}{q} \times q$, or $P_T q - p_T q - P_i q + p_i q$,

persons. Therefore the whole number of persons intitled to receive the said payments of one pound each at the end of the said n years is $p + P_i - p_i$

+ $\frac{P_T q - p_T q - P_i q + p_i q}{q}$, or $p + P_i - p_i + \frac{P_T q}{q} - \frac{p_T q}{q} - \frac{P_i q}{q} + \frac{p_i q}{q}$.

Therefore this will be the number of pounds which the grantor of the said future payment will have to pay at the end of the said n years. Therefore at the time of making the grants he ought to receive from all the P purchasers of these future payments a sum equal to $p + \frac{P_i}{q} - \frac{p_i}{q} + \frac{P_T q}{q} - \frac{p_T q}{q}$

— $P_i q + p_i q$ times the present value of one pound payable at the end of $\frac{P_T q}{q}$ years, or $p + \frac{P_i}{q} - \frac{p_i}{q} + \frac{P_T q}{q} - \frac{p_T q}{q} - P_i q + p_i q$ times $\frac{1}{q}$ of a pound; and

consequently from each of the said purchasers the P th part of the former sum, or $\frac{p + \frac{P_i}{q} - \frac{p_i}{q} + \frac{P_T q}{q} - \frac{p_T q}{q} - P_i q + p_i q}{P} \times \frac{P}{q}$ times $\frac{1}{q}$ of a pound.

Therefore, by Art. 21, this is likewise the sum which a purchaser ought to pay to the grantor for a grant of such a future sum of one pound, when he makes only one such grant. Now this sum of $\frac{p + \frac{P_i}{q} - \frac{p_i}{q} + \frac{P_T q}{q} - \frac{p_T q}{q} - P_i q + p_i q}{P}$

— $\frac{1}{q} + p_i q$ times $\frac{1}{q}$ of a pound, is evidently equal to the sum which was

found in the last article to be the price of such a grant of a future payment of one pound to be received at the end of n years, in case the purchaser himself who was aged N years, or either of his two companions, who were aged M years and L years at the time of making the grant, should be then alive, upon a supposition that his first, or older, companion should be intitled to the said payment in preference to his second, or younger, companion, namely, the sum of $\frac{p + \frac{P_i}{q} - \frac{p_i}{q} + \frac{P_T q}{q} - \frac{p_T q}{q} - P_i q + p_i q}{P}$ times

$\frac{1}{q}$ of a pound; because these two expressions consist of the same terms connected together by addition and subtraction in the same manner, and differing from each other only in the order in which they are placed, which cannot

of one pound, to be received at the same time in case a person of the age of L years, which corresponds to the number T in the said table, or of the age of the aforesaid purchaser's second, or younger, companion, shall be then alive. And, by Prob. 3, Coroll. 6, Art. 55, $\frac{p'q'}{pqT} \times \frac{1}{r^n} £$ is the value of a future payment of one pound, to be received at the end of n years in case a purchaser of the age of N years, which corresponds to the number P in the table, and two companions of the said purchaser, one of the age of M years, which corresponds to the number Q in the table, and the other of the age of L years, which corresponds to the number T in the table, shall all three be living at the end of the said time. And, by Prob. 3,

Art. 45, $\frac{p'q}{pQ} \times \frac{1}{r^n} £$ is the value of a future payment of one pound, to be received at the end of n years if both a purchaser of the age of N years, which corresponds to the number P in the table, and a companion of the age of M years, which corresponds to the number Q in the table, shall be living at that time: and $\frac{p'}{pT} \times \frac{1}{r^n} £$ is the value of one pound, to be re-

ceived at the end of n years if two persons of the ages of N and L years, which correspond to the numbers P and T in the table, shall be then alive:

and $\frac{q'}{qT} \times \frac{1}{r^n} £$ is the value of a future payment of one pound, to be received at the end of n years, if two persons of the ages of M and L years, which correspond to the numbers Q and T in the table, shall be then alive. Therefore the value of a future payment of one pound, to be received at the end of n years if either the purchaser aforesaid, aged N years, or either of his two companions, aged M years and L years, shall be then alive, is equal to the excess of the sum of the four following values, to wit, the values of the same payment, to be received at the same time, depending upon the single lives of the said purchaser and his two companions, and the value of it, to be received at the same time in case of the joint continuance of the lives of all the three, above the sum of the three following values, to wit, the values of the same payment, to be received at the same time, in case of the joint continuance of the two lives of the purchaser and his first, or older, companion, and in case of the joint continuance of the lives of the purchaser and his second, or younger, companion, and in case of the joint continuance of the lives of his two companions. Q.E.D.

LXXV. COROLL. 10. By finding in the method described in the two last corollaries the several values of one pound sterling, to be received at the end of every year of the whole space of time during which it is possible that either a purchaser of a given age, or either of his two companions, whose ages are likewise given, may be prolonged, in case that either the said purchaser or either of his said companions shall be living at the time when every such payment becomes due; and then adding these values all together into one sum; we shall obtain the value of a set of equal payments of one pound each, to be received by the purchaser or his companions at the end of every future year during the lives of either of them, that is, the value of an annuity of one pound for the lives of the said purchaser and his two companions, and the life of the longest liver of them.

Of the value of an annuity of one pound sterling for the life of the longest liver of three persons of given ages.

LXXVI. COROLL. 11. The value of an annuity of one pound sterling for the lives of three persons of given ages and the life of the longest liver of them, is equal to the excess of the sum of the four following values, to wit, the value of an annuity of one pound for the life of the first of the said three persons, the value of a like annuity for the life of the second of them, and the value of a like annuity for the life of the third of them, and the value of a like annuity for the joint lives of all the three, above the sum of the three following values, to wit, the value of a like annuity of one pound during the joint lives of the first and second persons, and the value of a like annuity during the joint lives of the first and third persons, and the value of a like annuity during the joint lives of the second and third persons.

Of the relation of the said value to the values of the like annuities of one pound for the single lives of the same three persons, and for the joint lives of every two of them, and for the joint lives of all the three.

Let the first of these three persons be supposed, as before, to be the purchaser of the said annuity of one pound, and the other two be called his companions. And let N be the number of years in the age of the said purchaser; and M the number of years in the age of the said purchaser's first, or older, companion; and L the number of years in the age of his second, or younger, companion. And let P denote, as before, the number of persons represented in the table of the probabilities of the duration of human life adopted for the calculation, to be living at the age of the purchaser, or at the age of N years; and Q denote the number of persons therein represented to be living at the age of the purchaser's first, or older, companion, or the age of M years; and T the number of persons therein represented to be living at the age of the purchaser's second, or younger, companion, or the age of L years. And further, let P' denote the number of persons represented in the said table to be living at the age of $N+1$ years; and P'' the number of those living at the age of $N+2$ years; and P''' the number of those living at the age of $N+3$ years; and P_{iv} the number of those living at the age of $N+4$ years; and so on for the following years;

M_2

years; the Roman numeral figures placed over every new letter P denoting the number of years by which every new age, corresponding to it, exceeds the age of N years, or the age of the purchaser. And, in like manner, let Q denote the number of persons represented in the said table to be living at the age of $M+1$ years; and Q' the number of persons living at the age of $M+2$ years; and Q'' the number of persons living at the end of $M+3$ years; and Q''' the number of persons living at the end of $M+4$ years; and so on, for the following years; the Roman numeral figures placed over every new letter Q denoting the number of years by which every new age corresponding to it, exceeds the age of M years, or of the purchaser's first, or older, companion. And, lastly, let T denote the number of persons represented in the said table to be living at the age of $L+1$ years; and T' the number of persons living at the age of $L+2$ years; and T'' the number of persons living at the age of $L+3$ years; and T''' the number of persons living at the age of $L+4$ years; and so on for the following years; the Roman numeral figures placed over every new letter T denoting the number of years by which every new age, corresponding to it, exceeds the age of L years, or of the purchaser's second, or younger, companion.

Then it is evident from Coroll. 8. that the value of a sum of one pound to be received at the end of one year, in case either the purchaser or one of his two companions shall be then alive, will be equal to

$$\frac{P' + Q' + T' - P'Q' - P'T' - Q'T' + P'Q'T'}{P \quad Q \quad T \quad P \quad Q \quad T \quad P \quad Q \quad T} \times \frac{1}{r} \text{ of a pound; because in this case } n = 1, \text{ and } p, q, t \text{ are respectively equal to } P', Q', \text{ and } T'.$$

And in like manner it is evident that the value of the same sum of one pound, to be received at the end of two years, subject to the same contingency, is equal to

$$\frac{P'' + Q'' + T'' - P''Q'' - P''T'' - Q''T'' + P''Q''T''}{P \quad Q \quad T \quad P \quad Q \quad T \quad P \quad Q \quad T} \times \frac{1}{r^2} \text{ of a pound; because in this case } n \text{ is equal to } 2, \text{ and } p, q, t \text{ are respectively equal to } P'', Q'', \text{ and } T''.$$

And in like manner the value of the same sum, to be received at the end of three years, subject to the same contingency, is equal to

$$\frac{P''' + Q''' + T''' - P'''Q''' - P'''T''' - Q'''T''' + P'''Q'''T'''}{P \quad Q \quad T \quad P \quad Q \quad T \quad P \quad Q \quad T} \times \frac{1}{r^3} \text{ of a pound; because in this case } n \text{ is } = 3, \text{ and } p, q, t, \text{ are equal to } P''', Q''', T''', \text{ respectively}$$

And

And in like manner the value of the same sum, to be received at the end of four years, subject to the same contingency, is equal to

$$\frac{P_{IV} + \frac{Q_{IV}}{2} + \frac{T_{IV}}{7} - \frac{P_{IV} Q_{IV}}{P} - \frac{P_{IV} T_{IV}}{P} - \frac{Q_{IV} T_{IV}}{2} + \frac{P_{IV} Q_{IV} T_{IV}}{P}}{\frac{1}{r^4}} \times \frac{1}{r^4}$$

of a pound; because in this case n is = 4, and p, q, t , are equal to P_{IV}, Q_{IV}, T_{IV} , respectively.

And in the same manner we may find the values of one pound, to be received at the end of the fifth, sixth, seventh, and every following year of the whole space of time during which it is possible that the lives of the aforesaid purchaser, or either of his two companions, may be prolonged.

Therefore the sum of these values, continued to the utmost possible extent of the youngest of those three lives, that is, the value of a life-annuity of one pound for the lives of the purchaser and his two companions, and the life of the longest liver of them, is equal to the following set of values, to wit,

$$\frac{P^I + \frac{Q^I}{2} + \frac{T^I}{7} - \frac{P^I Q^I}{P} - \frac{P^I T^I}{P} - \frac{Q^I T^I}{2} + \frac{P^I Q^I T^I}{P}}{\frac{1}{r}} \times \frac{1}{r}$$

$$+ \frac{P^{II} + \frac{Q^{II}}{2} + \frac{T^{II}}{7} - \frac{P^{II} Q^{II}}{P} - \frac{P^{II} T^{II}}{P} - \frac{Q^{II} T^{II}}{2} + \frac{P^{II} Q^{II} T^{II}}{P}}{\frac{1}{r^2}} \times \frac{1}{r^2}$$

$$+ \frac{P^{III} + \frac{Q^{III}}{2} + \frac{T^{III}}{7} - \frac{P^{III} Q^{III}}{P} - \frac{P^{III} T^{III}}{P} - \frac{Q^{III} T^{III}}{2} + \frac{P^{III} Q^{III} T^{III}}{P}}{\frac{1}{r^3}} \times \frac{1}{r^3}$$

$$\times \frac{1}{r^3} f_3,$$

$$+ \frac{P^{IV} + \frac{Q^{IV}}{2} + \frac{T^{IV}}{7} - \frac{P^{IV} Q^{IV}}{P} - \frac{P^{IV} T^{IV}}{P} - \frac{Q^{IV} T^{IV}}{2} + \frac{P^{IV} Q^{IV} T^{IV}}{P}}{\frac{1}{r^4}} \times \frac{1}{r^4} f_4,$$

$$+ \frac{P^V + \frac{Q^V}{2} + \frac{T^V}{7} - \frac{P^V Q^V}{P} - \frac{P^V T^V}{P} - \frac{Q^V T^V}{2} + \frac{P^V Q^V T^V}{P}}{\frac{1}{r^5}} \times \frac{1}{r^5} f_5,$$

+ &c. continued to the utmost extent of human life;

And

And consequently to the following set of quantities, to wit,

$$\begin{aligned} \frac{P^I}{P} \times \frac{1}{r^I} k &+ \frac{Q^I}{Q} \times \frac{1}{r^I} k &+ \frac{T^I}{T} \times \frac{1}{r^I} k &- \frac{P^I Q^I}{P Q} \times \frac{1}{r^I} k &- \frac{P^I T^I}{P T} \times \frac{1}{r^I} k \\ &- \frac{Q^I T^I}{Q T} \times \frac{1}{r^I} k &+ \frac{P^I Q^I T^I}{P Q T} \times \frac{1}{r^I} k, \end{aligned}$$

$$\begin{aligned} + \frac{P^{II}}{P} \times \frac{1}{r^I} k &+ \frac{Q^{II}}{Q} \times \frac{1}{r^I} k &+ \frac{T^{II}}{T} \times \frac{1}{r^I} k &- \frac{P^{II} Q^{II}}{P Q} \times \frac{1}{r^I} k \\ &- \frac{P^{II} T^{II}}{P T} \times \frac{1}{r^I} k &- \frac{Q^{II} T^{II}}{Q T} \times \frac{1}{r^I} k &+ \frac{P^{II} Q^{II} T^{II}}{P Q T} \times \frac{1}{r^I} k, \end{aligned}$$

$$\begin{aligned} + \frac{P^{III}}{P} \times \frac{1}{r^I} k &+ \frac{Q^{III}}{Q} \times \frac{1}{r^I} k &+ \frac{T^{III}}{T} \times \frac{1}{r^I} k &- \frac{P^{III} Q^{III}}{P Q} \times \frac{1}{r^I} k \\ &- \frac{P^{III} T^{III}}{P T} \times \frac{1}{r^I} k &- \frac{Q^{III} T^{III}}{Q T} \times \frac{1}{r^I} k &+ \frac{P^{III} Q^{III} T^{III}}{P Q T} \times \frac{1}{r^I} k, \end{aligned}$$

$$\begin{aligned} + \frac{P^{IV}}{P} \times \frac{1}{r^I} k &+ \frac{Q^{IV}}{Q} \times \frac{1}{r^I} k &+ \frac{T^{IV}}{T} \times \frac{1}{r^I} k &- \frac{P^{IV} Q^{IV}}{P Q} \times \frac{1}{r^I} k \\ &- \frac{P^{IV} T^{IV}}{P T} \times \frac{1}{r^I} k &- \frac{Q^{IV} T^{IV}}{Q T} \times \frac{1}{r^I} k &+ \frac{P^{IV} Q^{IV} T^{IV}}{P Q T} \times \frac{1}{r^I} k, \end{aligned}$$

$$\begin{aligned} + \frac{P^V}{P} \times \frac{1}{r^I} k &+ \frac{Q^V}{Q} \times \frac{1}{r^I} k &+ \frac{T^V}{T} \times \frac{1}{r^I} k &- \frac{P^V Q^V}{P Q} \times \frac{1}{r^I} k \\ &- \frac{P^V T^V}{P T} \times \frac{1}{r^I} k &- \frac{Q^V T^V}{Q T} \times \frac{1}{r^I} k &+ \frac{P^V Q^V T^V}{P Q T} \times \frac{1}{r^I} k, \end{aligned}$$

+&c. continued to the utmost extent of human life.

And consequently to the seven following sets of quantities, which are the same with the quantities last-mentioned, but only are placed in a different order, (those which before were placed in perpendicular columns, one under the other, being now placed in separate horizontal lines;) to wit,

$$\text{1st, } \frac{P^I}{P} \times \frac{1}{r^I} k \quad + \frac{P^{II}}{P} \times \frac{1}{r^I} k \quad + \frac{P^{III}}{P} \times \frac{1}{r^I} k \quad + \frac{P^{IV}}{P} \times \frac{1}{r^I} k$$

$$+ \frac{P^V}{P} \times \frac{1}{r^I} k \quad + \&c. \text{ continued to the utmost extent of human life:}$$

And

And 2dly, $\frac{Q}{2} \times \frac{1}{r} f + \frac{Q^2}{2} \times \frac{1}{r^2} f + \frac{Q^3}{2} \times \frac{1}{r^3} f + \frac{Q^4}{2} \times \frac{1}{r^4} f + \&c.$ continued to the utmost extent of human life;

And 3dly, $\frac{T}{7} \times \frac{1}{r} f + \frac{T^2}{7} \times \frac{1}{r^2} f + \frac{T^3}{7} \times \frac{1}{r^3} f + \frac{T^4}{7} \times \frac{1}{r^4} f + \frac{T^5}{7} \times \frac{1}{r^5} f + \&c.$ continued to the utmost extent of human life;

And 4thly, $\frac{P}{p} \times \frac{1}{r} f + \frac{P^2}{p^2} \times \frac{1}{r^2} f + \frac{P^3}{p^3} \times \frac{1}{r^3} f + \frac{P^4}{p^4} \times \frac{1}{r^4} f + \frac{P^5}{p^5} \times \frac{1}{r^5} f + \&c.$ continued to the utmost extent of human life;

And 5thly, $\frac{P^1 T^1}{p^1 q^1} \times \frac{1}{r} f + \frac{P^2 T^2}{p^2 q^2} \times \frac{1}{r^2} f + \frac{P^3 T^3}{p^3 q^3} \times \frac{1}{r^3} f + \frac{P^4 T^4}{p^4 q^4} \times \frac{1}{r^4} f + \frac{P^5 T^5}{p^5 q^5} \times \frac{1}{r^5} f + \&c.$ continued to the utmost extent of human life;

And 6thly, $\frac{Q^1 T^1}{2^1 7^1} \times \frac{1}{r} f + \frac{Q^2 T^2}{2^2 7^2} \times \frac{1}{r^2} f + \frac{Q^3 T^3}{2^3 7^3} \times \frac{1}{r^3} f + \frac{Q^4 T^4}{2^4 7^4} \times \frac{1}{r^4} f + \frac{Q^5 T^5}{2^5 7^5} \times \frac{1}{r^5} f + \&c.$ continued to the utmost extent of human life;

And 7thly, $\frac{P^1 Q^1 T^1}{p^1 q^1 2^1} \times \frac{1}{r} f + \frac{P^2 Q^2 T^2}{p^2 q^2 2^2} \times \frac{1}{r^2} f + \frac{P^3 Q^3 T^3}{p^3 q^3 2^3} \times \frac{1}{r^3} f + \frac{P^4 Q^4 T^4}{p^4 q^4 2^4} \times \frac{1}{r^4} f + \frac{P^5 Q^5 T^5}{p^5 q^5 2^5} \times \frac{1}{r^5} f + \&c.$ continued to the utmost extent of human life;

that

that is, the value of a life-annuity of one pound for the lives of the said purchaser and his two companions, and the life of the longest liver of them, is equal to the excess of the sum of the first, second, third, and seventh, of the seven last sets of quantities above the sum of the fourth, fifth, and sixth sets.

Now it appears from Prob. 2, Coroll. 2, Art. 31, that the first of these seven sets of quantities, to wit, $\frac{P^i}{P} \times \frac{1}{r^i} L$ $+\frac{P^{ii}}{P} \times \frac{1}{r^i} L$ $+\frac{P^{iii}}{P} \times \frac{1}{r^i} L$

$+\frac{P^{iv}}{P} \times \frac{1}{r^i} L$ $+\frac{P^v}{P} \times \frac{1}{r^i} L$ $+\&c.$ continued to the utmost extent of hu-

man life, is the value of an annuity of one pound for the life of the purchaser, who was supposed to be of the age of N years, which corresponds to the number P in the table.

And, in like manner, the second of these seven sets of quantities, to wit,

$\frac{2^i}{2} \times \frac{1}{r^i} L$ $+\frac{2^{ii}}{2} \times \frac{1}{r^i} L$ $+\frac{2^{iii}}{2} \times \frac{1}{r^i} L$ $+\frac{2^{iv}}{2} \times \frac{1}{r^i} L$ $+\frac{2^v}{2} \times \frac{1}{r^i} L$

$+\&c.$ continued to the utmost extent of human life, is the value of an annuity of one pound for the life of the said purchaser's first, or older, companion, who was supposed to be of the age of M years, which corresponds to the number 2 in the table.

And, in like manner, the third of these seven sets of quantities, to wit,

$\frac{3^i}{3} \times \frac{1}{r^i} L$ $+\frac{3^{ii}}{3} \times \frac{1}{r^i} L$ $+\frac{3^{iii}}{3} \times \frac{1}{r^i} L$ $+\frac{3^{iv}}{3} \times \frac{1}{r^i} L$ $+\frac{3^v}{3} \times \frac{1}{r^i} L$

$+\&c.$ continued to the utmost extent of human life, is the value of an annuity of one pound for the life of the said purchaser's second, or younger, companion, who was supposed to be of the age of L years, which corresponds to the number 3 in the table.

And

And it appears from Prob. 3, Coroll. 2, Art. 48, that the fourth of the said seven sets of quantities, to wit,

$$\frac{P^i Q^i}{P^i Q^i} \times \frac{1}{r} \ell + \frac{P^{i'} Q^{i'}}{P^i Q^i} \times \frac{1}{r^2} \ell$$

$$+ \frac{P^{i''} Q^{i''}}{P^i Q^i} \times \frac{1}{r^3} \ell + \frac{P^{i'''} Q^{i'''}}{P^i Q^i} \times \frac{1}{r^4} \ell + \&c. \text{ continued}$$

to the utmost extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser and his said first, or older, companion, whose ages N and M correspond to the letters, or numbers, P and Q in the said table.

And in like manner the fifth of the said seven sets of quantities, to wit,

$$\frac{P^i Q^i}{P^i Q^i} \times \frac{1}{r} \ell + \frac{P^{i'} Q^{i'}}{P^i Q^i} \times \frac{1}{r^2} \ell + \frac{P^{i''} Q^{i''}}{P^i Q^i} \times \frac{1}{r^3} \ell + \frac{P^{i'''} Q^{i'''}}{P^i Q^i} \times \frac{1}{r^4} \ell$$

$$+ \frac{P^{i''''} Q^{i''''}}{P^i Q^i} \times \frac{1}{r^5} \ell + \&c. \text{ continued to the utmost extent of human life, is}$$

the value of an annuity of one pound for the joint lives of the said purchaser and his said second, or younger, companion, whose ages N and L correspond to the numbers P and Q in the said table.

And in like manner the sixth of the said seven sets of quantities, to wit,

$$\frac{P^i Q^i}{P^i Q^i} \times \frac{1}{r} \ell + \frac{P^{i'} Q^{i'}}{P^i Q^i} \times \frac{1}{r^2} \ell + \frac{P^{i''} Q^{i''}}{P^i Q^i} \times \frac{1}{r^3} \ell + \frac{P^{i'''} Q^{i'''}}{P^i Q^i} \times \frac{1}{r^4} \ell$$

$$+ \frac{P^{i''''} Q^{i''''}}{P^i Q^i} \times \frac{1}{r^5} \ell + \&c. \text{ continued to the utmost extent of human life, is}$$

the value of an annuity of one pound for the joint lives of the said purchaser's two companions, whose ages M and L correspond to the numbers Q and Q in the said table.

And, lastly, it appears from Prob. 3d, Coroll. 7, Art. 56, that the seventh, or last, of the said seven sets of quantities, to wit,

$$\frac{P^i Q^i}{P^i Q^i} \times \frac{1}{r} \ell + \frac{P^{i'} Q^{i'}}{P^i Q^i} \times \frac{1}{r^2} \ell + \frac{P^{i''} Q^{i''}}{P^i Q^i} \times \frac{1}{r^3} \ell + \frac{P^{i'''} Q^{i'''}}{P^i Q^i} \times \frac{1}{r^4} \ell$$

$$+ \frac{P^{i''''} Q^{i''''}}{P^i Q^i} \times \frac{1}{r^5} \ell + \&c. \text{ continued to the utmost}$$

extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser and his two companions, whose ages N , M , and L correspond to the numbers P , Q , and Q .

N

Therefore

Therefore the value of an annuity of one pound for the three lives of the said purchaser and his two companions, and the life of the longest living of them, is equal to the excess of the sum of the values of three separate life-annuities of one pound each for the single lives of the said purchaser and his two companions, together with the value of an annuity of the same sum for the joint lives of all the three, above the sum of the three values of a like annuity for the joint lives of the said purchaser and his first, or older, companion, and for the joint lives of the said purchaser and his second, or younger, companion, and for the joint lives of both his said companions. QED.

Short and general Expressions for the Values of the
Annuities that are the Subject of the foregoing
Problems.

LXXVII. Having now explained, in as full and clear a manner as I was able, the grounds and reasons of the methods above delivered for the calculation of the values of annuities, so far as relates to annuities for given numbers of years, not depending upon the continuance of any life, or upon any other contingent event, and to annuities for given numbers of years depending upon the continuance of a single life of a given age, or upon the joint continuance of two, or of three, lives, of given ages, or upon the continuance of any one of two, or of three, lives of given ages, the interest of money, in all these cases, being likewise given; it will now be convenient to recapitulate the substance of the conclusions that have been obtained in the foregoing articles, by exhibiting short and general expressions of the values of these several kinds of annuities, to which the reader, whenever he shall have occasion to compute any of these annuities, may readily have recourse. This may be done in the manner following.

A short expression of the value of an annuity of one pound per annum for a given number of years, not depending on the continuance of any life or on any other uncertain event, the interest of money being also given.

LXXVIII. Let r denote the value of one pound sterling together with its interest for one year at any given rate of interest: and let n be the number of years during which an annuity of one pound per annum is to continue.

Then by Art. 23, pages 21 and 22, it is evident that the present value of this annuity will be $= £1 \times$ the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ continued to the term $\frac{1}{r^n}$, or to n terms.

A method of obtaining another and still shorter expression of the same value.

LXXIX. As this series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ is a geometrical progression, the sum of its terms may be found in a single short expression without taking the trouble of computing them all separately, and then adding them into one sum total. This may be done by means of the following Lemma.

A L E M M A.

LXXX. The sum of the terms of every decreasing geometrical progression is equal to the quotient that arises by dividing the excess of the square of its first, or greatest, term above the product, or rectangle under its second term and its least term, by the excess of its first, or greatest, term above its second term.

For, if $A, B, C, D, E,$ are a series of terms in geometrical proportion,

we shall have
 $A : B :: B : C,$
 and
 $A : B :: C : D,$
 and
 $A : B :: D : E.$

Therefore, by El. 5, 12, the sum of all the antecedents $A, B, C, D,$ will be to the sum of all the consequents $B, C, D, E,$ as the first antecedent A is to the first consequent B . But the sum of all the antecedents $A, B, C, D,$ is the sum of all the terms of the series except the last term E ; and

N^2

and the sum of all the consequents B, C, D, E , is the sum of all the terms of the series except the first term A . Therefore, if the sum of all the terms of the series be called S , we shall have $S - E : S - A :: A : B$. Therefore $B \times S - E$ will be $= A \times S - A$, or $BS - BE$ will be $= AS - AA$. Therefore, adding AA to both sides, we shall have $AA - BE + BS = AS$; and, subtracting BS (which is less than AS , because B is less than A , and which therefore is also less than the other side of the equation, to wit, $AA - BE + BS$) from both sides, we shall have $AA - BE = AS - BS = S \times A - B$. Therefore S , or the sum of all the terms, A, B, C, D, E , will be equal to $\frac{AA - BE}{A - B}$, that is, to the quotient that arises by dividing the excess of AA , the square of the first, or greatest, term A , above BE , the product, or rectangle under the second term B and the least term E , by the excess of the first, or greatest, term A , above the second term B . And it is easy to see that the same reasonings will take place, and consequently that the same conclusion will follow from them, if the series A, B, C, D , &c. should consist of any other number of terms whatever, as well as when it consists of five terms. QED.

LXXXI. Now in the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \frac{1}{r^8}$, &c. $+\frac{1}{r^n}$, $\frac{1}{r}$ is $= A$, and $\frac{1}{r^2}$ is $= B$, and $\frac{1}{r^n}$ is $= E$. Therefore

AA is $= \frac{1}{r^2}$, and BE is $= \frac{1}{r^2} \times \frac{1}{r^n} = \frac{1}{r^{n+2}}$, and consequently $AA - BE$ is $= \frac{1}{r^2} - \frac{1}{r^{n+2}} = \frac{r^{n+2} - r^2}{r^{n+4}} = \frac{r^n - 1}{r^{n+2}}$. And $A - B$ is $= \frac{1}{r} - \frac{1}{r^2} = \frac{r - 1}{r^2}$. And consequently $\frac{AA - BE}{A - B}$ is $= \frac{\frac{r^n - 1}{r^{n+2}}}{\frac{r - 1}{r^2}} = \frac{r^n - 1}{r^{n+2}} \times \frac{r^2}{r - 1} = \frac{r^n - 1}{r^{n+1}}$. Therefore

$\frac{r^n - 1}{r^{n+1}} = \frac{r^n}{r^{n+1}} - \frac{1}{r^{n+1}} = \frac{1}{r - 1} - \frac{1}{r^{n+1}}$. Therefore

A second, and very short, expression of the value of an annuity of one pound a year for a given number of years certain. QEI.

LXXXII. I shall

LXXXII. I shall now proceed to give an example of the computation of the value of an annuity of one pound for a given number of years by means of both these expressions, that is, the expression $1 \times$ the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$ and the expression $\frac{1}{1} \times \left[\frac{1}{r-1} - \frac{1}{r^n \times r-1} \right]$.

An example of the computation of the value of an annuity of one pound for a given number of years by means of the expression $1 \times$ the series

$$\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}.$$

LXXXIII. Let n , or the given number of years during which the annuity is to continue, be 30 years; and let the rate of the interest of money be $3\frac{1}{4}$ per cent.

Then will r be = 1.035, and $\frac{1}{r}$ will be = $\frac{1}{1.035}$ = .966,184. And

$$\frac{1}{r^2} \text{ will be } = .933,511, \text{ and } \frac{1}{r^3} = .901,943,$$

$$\text{and } \frac{1}{r^4} = .871,442, \text{ and } \frac{1}{r^5} = .841,973,$$

$$\text{and } \frac{1}{r^6} = .813,501, \text{ and } \frac{1}{r^7} = .785,991,$$

$$\text{and } \frac{1}{r^8} = .759,412, \text{ and } \frac{1}{r^9} = .733,731,$$

$$\text{and } \frac{1}{r^{10}} = .708,919, \text{ and } \frac{1}{r^{11}} = .684,946,$$

$$\text{and } \frac{1}{r^{12}} = .661,783, \text{ and } \frac{1}{r^{13}} = .639,404,$$

$$\text{and } \frac{1}{r^{14}} = .617,782, \text{ and } \frac{1}{r^{15}} = .596,891,$$

and

A computation of the same annuity for 30 years by means of the

$$\text{expression } £1 \times \frac{\frac{1}{r-1} - \frac{1}{r^n \times r-1}}{r-1}.$$

LXXXIV. Here r is $= 1.035$; and consequently $\frac{1}{r}$ is $= \frac{1}{1.035} = .966,184$; and $\frac{1}{r^n}$ is $= \frac{1}{r^n} = .356,278$; and $r-1$ is $= .035 = \frac{35}{1000}$; and $\frac{1}{r-1}$ is $= \frac{1}{.035} = 1 \times \frac{1000}{35} = 28.571,428$; and $\frac{1}{r^n \times r-1}$, or $\frac{1}{\frac{1}{r^n} \times \frac{1}{r-1}}$ is $= .356,278 \times \frac{1000}{35} = \frac{356.278}{35} = 10.179,37$. Therefore $\frac{1}{r-1} - \frac{1}{r^n \times r-1}$ is $= 28.571,42 - 10.179,37 = 18.392,05$. Therefore the value of the proposed annuity of one pound per annum for 30 years, when the interest of money is $3\frac{1}{4}$ per cent. is $£1 \times 18.392,05$, or $£18.392,05$, or $18\text{ l. } 7\text{ s. } 10\text{ d. } \frac{1}{2}$, as before. Q.E.I.

LXXXV. Note. A table of the values of annuities of this kind, shewing the present value of an annuity of one pound a year for any number of years certain, not exceeding 90 years, when the interest of money is at 5, 4, and 3, per cent. has been given us by the late learned Mr. Thomas Simpson, of Woolwich, in the 114th and 115th pages of his comprehensive little treatise, called, "*The doctrine of annuities and reversions, &c.*" in small octavo, which was published in the year 1742. And another table of the same kind had been published before by Mr. John Smart in the year 1726 in his very useful work above-mentioned in Art. 25, intitled "*Tables of Interest, &c.*" This latter table is computed to eight places of decimal fractions; and for the twelve different rates of interest mentioned above in Art. 24, to wit, 2 per cent. $2\frac{1}{4}$ per cent. 3 per cent. $3\frac{1}{4}$ per cent. 4 per cent. $4\frac{1}{4}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. and for every number of years, and even half-years, in the space of an hundred years. So that nothing can be expected, or need be desired, more completely convenient for the purpose of estimating the values of annuities for terms of years certain, than this table. It is Mr. Smart's fourth table of compound interest, pages 76, 77, 78, 79, 80, 81, 82, 83. of his valuable book above-mentioned.

A Short

A short expression of the value of an annuity of one pound per annum for a given number of years, depending on the continuance of one life of a given age, the rate of the interest of money being also given.

LXXXVI. Let r be, as, before, the sum of one pound together with its interest for one year according to the given rate of interest. And let N be the number of years in the age of the person on whose life the annuity is to depend, and E be the greatest number of years through which it is supposed to be possible for human life to be extended, according to the table of the probabilities of the duration of human life adopted for the calculation; which number is in Monsieur de Parcieux's table 94 years. Then will $E - N$ be the greatest number of years through which, according to such supposition, it is possible that the life of the annuitant of the given age of N years can be extended. Let n be any number of years not greater than $E - N$ years. And let the annuity of one pound per annum be granted for the term of n years, provided the said annuitant of the age of N years shall so long live, but otherwise to cease at his death. Let P be the number of persons of the said age of N years represented to be all living at the same time in Monsieur de Parcieux's table of the probabilities of the duration of human life, or in such other table of these probabilities as is thought proper by the calculator to be adopted as the ground of his calculation. And let P' be the number of persons represented in the said table to be living at the age of $N + 1$ years; and P'' the number living at the age of $N + 2$ years; and P''' the number living at the age of $N + 3$ years; and P^{IV} , P^V , P^{VI} , P^{VII} , P^{VIII} , P^{IX} , P^X , &c. the numbers living at the several following ages of $N + 4$ years, $N + 5$ years, $N + 6$ years, $N + 7$ years, $N + 8$ years, $N + 9$ years, $N + 10$ years, &c. respectively.

Then will the present value of an annuity of one pound a year for the space of n years, in case the said life of N years shall so long continue, be equal to the following expression, to wit, $1 \times$ the series $\frac{P'}{P_r} + \frac{P''}{P_r^2} + \frac{P'''}{P_r^3}$

$+ \frac{P^{IV}}{P_r^4} + \frac{P^V}{P_r^5} + \frac{P^{VI}}{P_r^6} + \frac{P^{VII}}{P_r^7}$ + &c. continued to n terms, or to the term

$\frac{P^n}{P_r^n}$, or to the expression $\frac{1}{P} \times$ the series $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{IV}}{r^4}$
 $+ \frac{P^V}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ + &c. continued to n terms, or to the term $\frac{P^n}{r^n}$.

This is evident from Problem 2, and its second corollary, Art. 26, 27, 28, 29, 31, pages 24, 25, 26, 27, 28.

LXXXVII. If

LXXXVII. If n years is the greatest number of years through which it is possible (according to the table of probabilities of the duration of human life adopted in the calculation,) for the given life of N years to be

extended, the said expression $\frac{1}{P} \times$ the series $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{iv}}{r^4}$

$+ \frac{P^v}{r^5} + \frac{P^{vi}}{r^6} + \frac{P^{vii}}{r^7} + \&c.$ continued to n terms, or to the term $\frac{P^n}{r^n}$, will be the value of an annuity of one pound *per annum* for the whole life of a person of N years of age: but, if n is less than the said complement of N years to the utmost duration of human life, the said expression will be less than the value of an annuity of one pound *per annum* for the whole life of a person of the age of N years, and will be the value of an immediate, but imperfect, life-annuity during n years of the life of a person of that age. This is evident from Art. 31, 32, 33, pages 28, 29, 30, 31, 32.

An example of the computation of the value of an immediate and complete life-annuity of one pound per annum for the whole life of a person of a given age, by means of the foregoing expression.

LXXXVIII. Let it be required to find the value of an annuity of one pound *per annum* for the life of a man of fourscore years of age, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and upon a supposition that the interest of money is 3 per cent.

Here n , or the number of years through which the annuity is to continue, in case the life of fourscore years of age shall last so long, is the greatest possible number of years through which, according to Monsieur de Parcieux's table, a life of fourscore years of age can be extended, that is, 94—80 years, or 14 years. Therefore the series $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{iv}}{r^4} + \frac{P^v}{r^5} + \frac{P^{vi}}{r^6} + \frac{P^{vii}}{r^7} + \&c.$ in the foregoing expression must be continued to 14 terms; which may be computed as follows.

P is = 118, $P' = 101$, $P'' = 85$, $P''' = 71$, $P^{iv} = 59$, $P^v = 48$, $P^{vi} = 38$, $P^{vii} = 29$, $P^{viii} = 22$, $P^{ix} = 16$, $P^{x} = 11$, $P^{xi} = 7$, $P^{xii} = 4$, $P^{xiii} = 2$, and $P^{xiv} = 1$. And r is = 1.03, and $\frac{1}{r} = \frac{1}{1.03} = .9708$,

O

and

and $\frac{1}{r^2} = .9425$, $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$,

$\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, $\frac{1}{r^9} = .7664$, $\frac{1}{r^{10}} = .7440$, $\frac{1}{r^{11}} = .7224$,

$\frac{1}{r^{12}} = .7013$, $\frac{1}{r^{13}} = .6809$, and $\frac{1}{r^{14}} = .6611$. Therefore the expression

$\frac{1}{P} \times \text{the series } \frac{P^i}{r} + \frac{P^{i+1}}{r^2} + \frac{P^{i+2}}{r^3} + \frac{P^{i+3}}{r^4} + \frac{P^{i+4}}{r^5} + \frac{P^{i+5}}{r^6} + \frac{P^{i+6}}{r^7} + \frac{P^{i+7}}{r^8} + \frac{P^{i+8}}{r^9} + \frac{P^{i+9}}{r^{10}} + \frac{P^{i+10}}{r^{11}} + \frac{P^{i+11}}{r^{12}} + \frac{P^{i+12}}{r^{13}} + \frac{P^{i+13}}{r^{14}}$ will be equal to

$\frac{1}{118} \times \text{the series } 101 \times .9708 + 85 \times .9425 + 71 \times .9151 + 59 \times .8884 + 48 \times .8626 + 38 \times .8374 + 29 \times .8130 + 22 \times .7894 + 16 \times .7664 + 11 \times .7440 + 7 \times .7224 + 4 \times .7013 + 2 \times .6809 + 1 \times .6611$
 $= \frac{1}{118} \times \text{the series } 98.0508 + 80.1125 + 65.0721 + 52.4156 + 41.4048 + 31.8212 + 23.5770 + 17.3668 + 12.2624 + 8.1840 + 5.0568 + 2.8052 + 1.3618 + .6611$
 $= \frac{1}{118} \times 440.1521 = 1 \times \frac{440.1521}{118} = 1 \times 3.7301 = 3.7301 = 3, 14, 7\frac{1}{2}$

Consequently the value of an annuity of one pound per annum for the whole life of a man of fourscore years of age, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and when the interest of money is 3 per cent. is 3*l*. 14*s*. 7*d*. $\frac{1}{2}$. Q.E.I.

An example of the computation of the value of an immediate, but imperfect, life-annuity, depending on the life of a person of a given age, by means of the same expression.

LXXXIX. Let it be required to find the value of an annuity of one pound *per annum* for the first five years of the life of a man of fourscore years of age, according to the same table of the probabilities of the duration of human life and the same rate of interest as before.

For

For this purpose we need only take the first five terms of the foregoing

series $\frac{P^1}{r} + \frac{P^{1'}}{r^2} + \frac{P^{1''}}{r^3} + \frac{P^{1'''}}{r^4} + 8c.$ and multiply their sum into

the fraction $\frac{1}{p}$; and the product will be the value of the proposed annuity.

These terms are $101 \times .9708 + 85 \times .9425 + 71 \times .9151 + 59 \times .8884 + 48 \times .8626$, which are equal to $98.0508 + 80.1125 + 65.0721 + 52.4156 + 41.4048 = 337.0558$; which, being multiplied into the fraction $\frac{1}{p}$, or $\frac{1}{118}$, is $= \text{£}1 \times \frac{337.0558}{118} = \text{£}1 \times 2.8564 = \text{£}2.8564$

$= 2l. 17s. 1d. \frac{1}{2}$. Therefore the value of an annuity of one pound *per annum* for the first five years of the life of a man of fourscore years of age, according to the foregoing suppositions of the duration of human life and the interest of money, is $2l. 17s. 1d. \frac{1}{2}$. QEI.

XC. And, if the annuity to be purchased is to be a remote one, or to be paid at the distance of more than one year, a short expression of its value, similar to the foregoing expressions, may be found as follows.

A short and general expression of the value of a remote annuity of one pound per annum for a given number of years, depending on a life of a given age, when the interest of money is also given.

Let r be, as before, the value of one pound together with its interest for a year at the given rate of the interest of money. And let m be the number of years at the end of which the annuity is to commence, so that the first payment of it shall be made at the end of $m+1$ years. And let N be the number of years in the age of the purchaser of the annuity at the time of purchasing it; and let E , as before, be the whole number of years through which it is possible, according to the table of probabilities adopted in the calculation, for human life to be extended; which in Monsieur de Parcieux's table is 94 years. Then will $N+m$ be the number of years in the age of the said purchaser at the time of the commencement of the said annuity, and $N+m+1$ the number of years in his age at the time when the first payment of the said annuity will become due; and $E-N+m$, or $E-N-m$, will be the greatest possible number of years, through which the life of the said purchaser can be extended after he shall have attained the age of $N+m$ years, and the annuity shall have commenced. Let n be any number of years not greater than $E-N-m$; and let P^{m+1}

denote the number of persons of the age of $N+m+1$ years represented, in the table of probabilities of life adopted in the calculation, to be all living at the same time; and P_{m+1} the number of persons represented in the said table to be living at the age of $N+m+2$ years; and P_{m+11} the number of persons represented to be living at the age of $N+m+3$ years; and P_{m+1v} the number of persons living at the age of $N+m+4$ years; and P_{m+v} the number of persons living at the age of $N+m+5$ years; and so on for all the following ages in the table.

Then will the value of an annuity of one pound per annum to commence at the distance of m years, (so that the first payment of it shall be made at the end of $m+1$ years,) and to continue during n years, provided a person aged N years at the time of the purchase shall so long live, but to cease as soon as such person shall be dead, be equal to the following expression, to wit,

$$\frac{1}{P} \times \text{the series } \frac{P_{m+1}}{r_{m+1}} + \frac{P_{m+11}}{r_{m+2}} + \frac{P_{m+111}}{r_{m+3}} + \frac{P_{m+1v}}{r_{m+4}} + \frac{P_{m+v}}{r_{m+5}} + \frac{P_{m+v1}}{r_{m+6}} + \frac{P_{m+v11}}{r_{m+7}} + \&c. \text{ continued to } n \text{ terms, or to the term } \frac{P_{m+n}}{r_{m+n}}. \text{ This is}$$

evident from Art. 36 and 37, pages 33, 34.

An example of the computation of the value of a remote annuity, depending upon a life of a given age, by means of the foregoing expression.

XCI. Let it be required to find the value of an annuity of one pound *per annum* for the life of a person of fourscore years of age, but which shall not commence till five years after the purchase of it, so that the first payment of it shall be made to the said person of fourscore years of age at the end of six years, or when he shall be fourscore and six years of age, if he shall be then living, and which shall continue during the whole remainder of the life of the said purchaser; the interest of money being 3 per cent. (as in the last example,) and the probabilities of the duration of human life such as they are represented to be in Monsieur de Parcieux's table.

Here N , or the number of years in the age of the purchaser of the annuity, is 80; and m , the number of years before the annuity is to commence, is 5; and consequently $N+m$, or the number of years in the age of the annuitant at the time the annuity is to commence, is 85. E , the greatest number of years through which human life can be extended, is, according to Monsieur de Parcieux's table of probabilities, 94 years; and consequently $E-N-m$, or the greatest number of years through which it

is possible that the life of the proposed annuitant can be extended after the annuity shall have commenced, is 94—80—5, or 94—85, or 9, years. And, n , or the number of years during which it is possible the annuity may continue, will consequently be equal to 9 years, because the annuity, when once it has taken place, is supposed to continue during the whole remaining part of the annuitant's life; and consequently the series

$$\frac{P_{m+1}}{r^{m+1}} + \frac{P_{m+1}}{r^{m+2}} + \frac{P_{m+1}}{r^{m+3}} + \frac{P_{m+1}}{r^{m+4}} + \frac{P_{m+1}}{r^{m+5}} + \&c.$$

will consist of nine terms. These terms may be computed as follows.

Since m is = 5, we shall have $m + 1 = vi$, and $m + 11 = vii$, and
 $m + 111 = viii$, and $m + iv = ix$, and $m + v = x$, and $m + vi = xi$,
and $m + vii = xii$, and $m + viii = xiii$, and $m + ix = xiv$; and, in
like manner, $m + i = 6$, and $m + 2 = 7$, and $m + 3 = 8$, and $m + 4 = 9$,
and $m + 5 = 10$, and $m + 6 = 11$, and $m + 7 = 12$, and $m + 8 = 13$,
and $m + 9 = 14$. Therefore the series

$$\frac{P_{m+i}}{r^{m+1}} + \frac{P_{m+11}}{r^{m+2}} + \frac{P_{m+111}}{r^{m+3}}$$

$$+ \frac{P_{m+iv}}{r^{m+5}} + \frac{P_{m+vi}}{r^{m+6}} + \frac{P_{m+vii}}{r^{m+7}} + \frac{P_{m+viii}}{r^{m+8}} + \frac{P_{m+ix}}{r^{m+9}}$$

$$+ \frac{P_{m+x}}{r^6} + \frac{P_{m+xi}}{r^7} + \frac{P_{m+xii}}{r^8} + \frac{P_x}{r^5} + \frac{P_{xi}}{r^{11}} + \frac{P_{xii}}{r^{12}} + \frac{P_{xiii}}{r^{13}}$$

$$+ \frac{P_{xiv}}{r^{14}} + \dots$$

But r is, as before, $= 1.03$; and consequently $\frac{1}{r}$ is $= \frac{1}{1.03} = .9708$, and $\frac{1}{r^6}$ is $= .8374$, and $\frac{1}{r^7}$ is $= .8130$, and $\frac{1}{r^8}$ is $= .7894$, and $\frac{1}{r^9}$ is $= .7664$, and $\frac{1}{r^{10}}$ is $= .7440$, and $\frac{1}{r^{11}}$ is $= .7224$, and $\frac{1}{r^{12}}$ is $= .7013$, and $\frac{1}{r^{13}}$ is $= .6809$, and $\frac{1}{r^{14}}$ is $= .6611$. And P is $= 118$, and $P_{vi} = 38$, and $P_{vii} = 29$, and $P_{viii} = 22$, and $P_{ix} = 16$, and $P_x = 11$, and $P_{xi} = 7$, and $P_{xii} = 4$, and $P_{xiii} = 2$, and $P_{xiv} = 1$. Therefore

$$\frac{1}{P} \times \text{the series } \frac{P_{vi}}{r^6} + \frac{P_{vii}}{r^7} + \frac{P_{viii}}{r^8} + \frac{P_{ix}}{r^9} + \frac{P_x}{r^{10}} + \frac{P_{xi}}{r^{11}} + \frac{P_{xii}}{r^{12}} + \frac{P_{xiii}}{r^{13}} + \frac{P_{xiv}}{r^{14}} \text{ is equal to } \frac{1}{118} \times \text{the series } 38 \times .8374 + 29 \times .8130$$

$+29 \times .8130 + 22 \times .7894 + 16 \times .7664 + 11 \times .7440 + 7 \times .7214$
 $+ 4 \times .7013 + 2 \times .6809 + 1 \times .6611 = \frac{1}{118} \times \text{the series } 31.8211$
 $+ 23.5770 + 17.3668 + 12.2624 + 8.1840 + 5.0568 + 2.8052$
 $+ 1.3618 + .6611 = \frac{1}{118} \times 103.0963 = \text{£}1 \times \frac{103.0963}{118} = \text{£}1 \times .8736$
 $= \text{£}.8736 = 17s. 5d. \frac{1}{2}$. Therefore $17s. 5d. \frac{1}{2}$ is the value of an annuity of one pound *per annum* for the life of a man of four score years of age, to commence at the distance of five years, or when he shall be four score and five years old, so that the first payment of it shall be made to him when he shall be 86 years old, according to Monsieur de Parcieux's table of the probabilities of life and when the interest of money is 3 per cent. Q.EI.

XCII. N.B. This value of this annuity is a little greater than that which is found for it above in Art. 37, page 34, which is $17s. 3d. \frac{1}{4}$. The reason of the difference is a small mistake which I have discovered to have been made in one of the arithmetical operations in Art. 32, page 30. The quotient of the division of 31.8212 by 118 is there made to be $.2612$, whereas it ought to be $.2696$, which is greater than $.2612$ by $.0084$. This difference $.0084$, being added to the number $.8650$ in Art. 37, page 34, will make it equal to $.8734$, which agrees in its three highest figures with the number $.8736$, just now found for the value of this annuity. The discovery of this mistake, if I had made it sooner, would have prevented the insertion of the note at the end of Art. 33, page 32, which now appears to be ill-grounded.

XCIII. The examples that have been given, in the several foregoing articles, of the computation of the values of life-annuities, both immediate and remote, are the same which were given above in the several articles that come after the solution of Problem 11. The reason of repeating them in this place was that they might serve to illustrate the short and general expressions of those values which have been set forth in this latter part of the present tract, and might thereby enable the reader, if he chose it, to become more familiarly acquainted with the practice of computing the values of life-annuities by means of those expressions. With the same view I shall here subjoin another example of the computation of the values of life-annuities by means of those general expressions, and shall choose for that purpose a younger life than those in the foregoing examples. This will occasion a long and tedious calculation, which would, I thought, have too much interrupted the chain of the reasonings used in Problem 11 and its corollaries,

corollaries, which are intended to explain the theory, of the grounds and reasons, of these computations; but in this part of the work, which is intended to illustrate the practice of these computations, I conceive that the exhibition of such a calculation may be of use.

A calculation of the value of an annuity of one pound sterling for the life of a person of 10 years of age; upon a supposition that the interest of money is 3 and an half per cent. and that the probabilities of the duration of human life are such as they are represented to be in *Monsieur de Parcieux's table*.

XCIV. The number of persons of the age of 10 years, represented in *Monsieur de Parcieux's table* to be all living at the same time, is 880; that is, according to the notation of Art. 86, P is = 880. And it appears by the same table that the number living at 11 years of age is 872; and the number living at 12 years of age is 866, and those living at 13, 14, 15, and all the following years of age up to 94 years, are 860, 854, 848, 842, 835, 828, 821, 814, 806, 798, 790, 782, 774, 766, 758, 750, 742, 734, 726, 718, 710, 702, 694, 686, 678, 671, 664, 657, 650, 643, 636, 629, 622, 615, 607, 599, 590, 581, 571, 560, 549, 538, 526, 514, 503, 489, 476, 463, 450, 437, 423, 409, 395, 380, 364, 347, 329, 310, 291, 271, 251, 231, 211, 192, 173, 154, 136, 118, 101, 85, 71, 59, 48, 38, 29, 22, 16, 11, 7, 4, 2, and 1; that is, according to the notation of Art. 86, P is = 872, and P' is = 866, and P'' is = 860, &c. are equal to 860, 854, 848, and the other numbers just now mentioned, respectively.

And it appears by *Mr. Smar's second table of Compound Interest*, pages 60—67, that, when the interest of money is $3\frac{1}{4}$ per cent. r is = 1.035,

$$\text{and } \frac{1}{r} = \frac{1}{1.035} = .96618, \text{ and } \frac{1}{r^2} = .93351, \text{ and } \frac{1}{r^3} = .90194, \text{ and}$$

$$\frac{1}{r^4} = .87144, \text{ and } \frac{1}{r^5} = .84197, \text{ and } \frac{1}{r^6} = .81350, \text{ and } \frac{1}{r^7} = .78599,$$

$$\text{and } \frac{1}{r^8} = .75941, \text{ and } \frac{1}{r^9} = .73373, \text{ and } \frac{1}{r^{10}} = .70891, \text{ and } \frac{1}{r^{11}} =$$

$$.68494, \text{ and } \frac{1}{r^{12}} = .66178, \text{ and } \frac{1}{r^{13}} = .63940, \text{ and } \frac{1}{r^{14}} = .61778,$$

$$\text{and } \frac{1}{r^{15}} = .59689, \text{ and } \frac{1}{r^{16}} = .57670, \text{ and } \frac{1}{r^{17}} = .55720, \text{ and } \frac{1}{r^{18}} = .53836,$$

$=.53836$, and $\frac{1}{r_{11}^2} = .52015$, and $\frac{1}{r_{20}^2} = .50256$, and $\frac{1}{r_{31}^2} = .48557$,
 and $\frac{1}{r_{42}^2} = .46915$, and $\frac{1}{r_{53}^2} = .45328$, and $\frac{1}{r_{64}^2} = .43795$, and $\frac{1}{r_{75}^2}$
 $=.42314$, and $\frac{1}{r_{86}^2} = .40883$, and $\frac{1}{r_{97}^2} = .39501$, and $\frac{1}{r_{108}^2} = .38165$,
 and $\frac{1}{r_{119}^2} = .36874$, and $\frac{1}{r_{120}^2} = .35627$, and $\frac{1}{r_{131}^2} = .34423$, and $\frac{1}{r_{142}^2}$
 $=.33258$, and $\frac{1}{r_{153}^2} = .32134$, and $\frac{1}{r_{164}^2} = .31047$, and $\frac{1}{r_{175}^2} = .29997$,
 and $\frac{1}{r_{186}^2} = .28983$, and $\frac{1}{r_{197}^2} = .28003$, and $\frac{1}{r_{208}^2} = .27056$, and $\frac{1}{r_{219}^2}$
 $=.26141$, and $\frac{1}{r_{220}^2} = .25257$, and $\frac{1}{r_{231}^2} = .24403$, and $\frac{1}{r_{242}^2} = .23577$,
 and $\frac{1}{r_{253}^2} = .22780$, and $\frac{1}{r_{264}^2} = .22010$, and $\frac{1}{r_{275}^2} = .21265$, and $\frac{1}{r_{286}^2}$
 $=.20546$, and $\frac{1}{r_{297}^2} = .19851$, and $\frac{1}{r_{308}^2} = .19180$, and $\frac{1}{r_{319}^2} = .18532$,
 and $\frac{1}{r_{320}^2} = .17905$, and $\frac{1}{r_{331}^2} = .17299$, and $\frac{1}{r_{342}^2} = .16714$, and $\frac{1}{r_{353}^2}$
 $=.16149$, and $\frac{1}{r_{364}^2} = .15603$, and $\frac{1}{r_{375}^2} = .15075$, and $\frac{1}{r_{386}^2} = .14566$,
 and $\frac{1}{r_{397}^2} = .14073$, and $\frac{1}{r_{408}^2} = .13597$, and $\frac{1}{r_{419}^2} = .13137$, and $\frac{1}{r_{420}^2}$
 $=.12693$, and $\frac{1}{r_{431}^2} = .12264$, and $\frac{1}{r_{442}^2} = .11849$, and $\frac{1}{r_{453}^2} = .11448$,
 and $\frac{1}{r_{464}^2} = .11061$, and $\frac{1}{r_{475}^2} = .10687$, and $\frac{1}{r_{486}^2} = .10326$, and $\frac{1}{r_{497}^2}$
 $=.09976$, and $\frac{1}{r_{508}^2} = .09639$, and $\frac{1}{r_{519}^2} = .09313$, and $\frac{1}{r_{520}^2} = .08998$,
 and $\frac{1}{r_{531}^2} = .08694$, and $\frac{1}{r_{542}^2} = .08400$, and $\frac{1}{r_{553}^2} = .08116$, and $\frac{1}{r_{564}^2}$
 $=.07841$, and $\frac{1}{r_{575}^2} = .07576$, and $\frac{1}{r_{586}^2} = .07320$, and $\frac{1}{r_{597}^2} = .07072$,
 and $\frac{1}{r_{608}^2} = .06833$, and $\frac{1}{r_{619}^2} = .06602$, and $\frac{1}{r_{620}^2} = .06379$, and $\frac{1}{r_{631}^2}$
 $=.06163$,

$$= .06163, \text{ and } \frac{1}{r^{82}} = .05955, \text{ and } \frac{1}{r^{83}} = .05753, \text{ and } \frac{1}{r^{84}} = .05559.$$

Therefore the expression $\frac{1}{P} \times$ the series $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4}$

$\frac{P^v}{r^5} + \frac{P^{vi}}{r^6} + \frac{P^{vii}}{r^7} + \&c.$ to $\frac{P_{LXXXIV}}{r^{84}}$ is in this case equal to $\frac{1}{880}$

$\times$ the series	872 $\times$.96618	+	866 $\times$.93351	+	860 $\times$.90194	
854 $\times$.87144	+	848 $\times$.84197	+	842 $\times$.81350	+	835 $\times$.78599
828 $\times$.75941	+	821 $\times$.73373	+	814 $\times$.70891	+	806 $\times$.68494
798 $\times$.66178	+	790 $\times$.63940	+	782 $\times$.61778	+	774 $\times$.59689
766 $\times$.57670	+	758 $\times$.55720	+	750 $\times$.53836	+	742 $\times$.52015
734 $\times$.50256	+	726 $\times$.48557	+	718 $\times$.46915	+	710 $\times$.45328
702 $\times$.43795	+	694 $\times$.42314	+	686 $\times$.40883	+	678 $\times$.39501
671 $\times$.38165	+	664 $\times$.36874	+	657 $\times$.25627	+	650 $\times$.34423
643 $\times$.33258	+	636 $\times$.32134	+	629 $\times$.31047	+	622 $\times$.29997
615 $\times$.28983	+	607 $\times$.28003	+	599 $\times$.27056	+	590 $\times$.26141
581 $\times$.25257	+	571 $\times$.24403	+	560 $\times$.23577	+	549 $\times$.22780
538 $\times$.22010	+	526 $\times$.21265	+	514 $\times$.20546	+	502 $\times$.19851
489 $\times$.19180	+	476 $\times$.18532	+	463 $\times$.17905	+	450 $\times$.17299
437 $\times$.16714	+	423 $\times$.16149	+	409 $\times$.15603	+	395 $\times$.15075
380 $\times$.14566	+	364 $\times$.14073	+	347 $\times$.13597	+	329 $\times$.13137
310 $\times$.12693	+	291 $\times$.12264	+	271 $\times$.11849	+	251 $\times$.11448
231 $\times$.11061	+	211 $\times$.10687	+	192 $\times$.10326	+	173 $\times$.09976
154 $\times$.09639	+	136 $\times$.09313	+	118 $\times$.08998	+	101 $\times$.08694
85 $\times$.08400	+	71 $\times$.08116	+	59 $\times$.07841	+	48 $\times$.07576
38 $\times$.07320	+	29 $\times$.07072	+	22 $\times$.06833	+	16 $\times$.06602
11 $\times$.06379	+	7 $\times$.06163	+	4 $\times$.05955	+	2 $\times$.05753
1 $\times$.05559	=	$\frac{1}{880}$ $\times$ the series	842 .50896	+	808 .41966	
775 .66840	+	744 .20976	+	713 .99056	+	684 .96700
656 .30165	+	628 .79148	+	602 .39133	+	577 .05274
552 .06164	+	528 .10044	+	505 .12600	+	483 .10396
461 .99286	+	441 .75220	+	422 .35760	+	403 .77000
385 .95130	+	368 .87904	+	352 .52382	+	336 .84970
		P			+	321 .82880

+	321.82880	+	307.44090	+	293.65916	+	280.45738
+	267.81678	+	256.08715	+	244.84336	+	234.06939
+	223.74950	+	213.84894	+	204.37224	+	195.28563
+	186.58134	+	178.24545	+	169.97821	+	162.06544
+	154.23190	+	146.74317	+	139.34113	+	132.03120
+	125.06220	+	118.41380	+	111.85390	+	105.60644
+	99.65202	+	93.79020	+	88.21232	+	82.90015
+	77.84550	+	73.04018	+	68.31027	+	63.81627
+	59.54625	+	55.35080	+	51.22572	+	47.18159
+	43.22073	+	39.34830	+	35.68824	+	32.11073
+	28.73448	+	25.55091	+	22.54957	+	19.82592
+	17.25848	+	14.84406	+	12.66568	+	10.61764
+	8.78094	+	7.14000	+	5.76236	+	4.62619
+	3.63628	+	2.78160	+	2.05088	+	1.50375
+	1.05632	+	.70169	+	.43141	+	.23810
+	.11506	+	.05559	=	$\frac{1}{880}$	x	18250.54036 = λ_1 x

$$\frac{18250.54036}{880} = £1 \times 20.73925 = £20.73925, \text{ or } 20\text{ s. } 14\text{ s. } 9\text{ d. } \frac{1}{2}$$

Therefore the value of an annuity of one pound *per annum* for the life of a person of the age of 10 years, when the interest of money is $3\frac{1}{2}$ per cent. is, according to Monsieur de Parcieux's table of probabilities, 20*l.* 14*s.* 9*d.* $\frac{1}{2}$. Q.E.I.

XCV. The addition of the terms of the foregoing series may be performed as follows.

842.50896	552.06164	352.52382	223.74950	139.34113
808.41966	528.10044	336.84970	213.84894	132.03110
775.66840	505.12600	321.82880	204.37224	125.06210
744.20976	483.10396	307.44090	195.28563	118.41384
713.99056	461.99286	293.65916	186.58134	111.85391
684.96700	441.75220	280.45738	178.24545	105.60604
656.30165	422.35760	267.81678	169.97821	99.65202
628.79148	403.77000	256.08715	162.06544	93.79014
602.39233	385.95130	244.84336	154.23190	88.21231
577.05274	368.87904	234.06939	146.74317	82.90011
4553.09504		2895.57644	1835.10182	1096.8633

77.84550	35.68824	8.78094	7034.30254
73.04018	32.11079	7.14000	4553.09504
68.31027	28.73448	5.76236	2895.57644
63.81627	25.55091	4.62619	1835.10182
59.54625	22.54957	3.63628	1096.86336
53.35080	19.82592	2.78160	576.88561
51.22572	17.25848	2.05088	219.84577
47.18159	14.84406	1.50326	38.86978
43.22073	12.66568	1.05632	
39.34830	10.61764	.70169	18250.54036
576.88561	219.84577	.43141	
		.23820	
		.11506	
		.05559	
		38.86978	

XCVI. It appears from the foregoing calculation that, when the interest of money is $3\frac{1}{2}$ per cent. an annuity for the life of a person of 10 years of age is worth near 21 years purchase: and, if the annuity, or rather in this case we should say, the *pension*, was to be paid every half-year, instead of every year, so that the first payment of it should be made at the end of half a year after the time of granting it, or when the grantee is ten years and a half old, the value of it would be greater than before by almost one half a year's payment, and consequently would be worth more than 21 years purchase. And, if we were to make the like calculation upon a supposition that the interest of money was only 3 per cent. we should find that such an annuity would be worth about 23 years purchase. But these are greater prices than, I believe, any life-annuities have ever been sold for.

XCVII. It appears from the operations in the foregoing calculation, that, when the interest of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for 30 years depending on the life of a person of 10 years of age, or that shall continue for 30 years in case the said person shall so long live, but shall cease upon his death, is, according to Monsieur de Parcieux's table of probabilities of the duration of human life, equal to

$$\begin{aligned} & \frac{1}{885} \times \text{the series } 7034.30254 + 4553.09504 + 2895.57644 = \\ & \frac{1}{880} \times 14482.97402 = \text{£}1 \times \frac{14482.97402}{880} = \text{£}1 \times 16.45792 = \\ & \text{P } 2 \quad \text{£}16.45792, \end{aligned}$$

Of the value £16.45792, or 16*l.* 9*s.* 2*d.* And it appears likewise that the value of a remote annuity for the life of a person of ten years of age, to commence at the end of 30 years, (or whereof the first payment shall be made at the end of 31 years, or when the said person shall be completely 41 years of age,) and to continue during the whole remainder of his life, will, (upon the same suppositions of the interest of money and the probabilities of the duration of human life,) be equal to

$$\frac{1}{880} \times \text{the series } 1835.10182 + 1096.86336 + 576.88561 + 219.84577 \\ + 38.86978 = \frac{1}{880} \times 3768.46634 = £1 \times \frac{3768.46634}{880} = £1 \times 4.28234$$

= 4.28234, or 4*l.* 5*s.* 7*d.* $\frac{2}{3}$. Therefore any life-annuity for the life of a person of 10 years of age, to commence at the distance of 30 years, is (upon the suppositions here made,) worth about four years and a quarter's purchase.

Of the different interests that might be made of money by vesting it in the publick funds in the years 1773 and 1780.

XCVIII. The interest that might be made of money by vesting it in the publick funds in January, 1773, was something less than $3\frac{1}{2}$ per cent. For 100 pounds stock in the consolidated 3 per cent. annuities might then be bought for 87 pounds; and, when it is worth 85*l.* 14*s.* 3*d.* $\frac{1}{2}$, or £85.7142, or, when £85.7142 will purchase a perpetual annuity of 3 pounds

per annum, £100 will purchase a perpetual annuity of $\frac{3 \times 100}{85.7142}$, or $3\frac{1}{2}$

pounds, *per annum*. Therefore, if the Government had then wanted to borrow a moderate sum of money of the people, (as, for instance, a million or two of pounds sterling,) they could, probably, have borrowed it at the rate of $3\frac{1}{2}$ per cent. This therefore was at that time the most useful rate of interest, and the first to be attended to and made the ground of calculations of the values of annuities either for terms of years or for lives. But now, in October, 1780, £100 stock in the 3 per cent. consolidated annuities, or a perpetual annuity of 3 pounds *per annum*, is worth only about 60 pounds; and consequently £100, employed in the purchase of those annuities, would purchase a perpetual annuity of 5 pounds a year. Therefore 5 per cent. or, perhaps, $5\frac{1}{2}$, or even 6, per cent. is now the rate of interest which most deserves to be considered in the estimation of the value of annuities, and to be made the foundation of our calculations concerning them. In order therefore to make proper estimates of the values of annuities in different periods of national wealth and prosperity, it will be most convenient to have tables of the values of annuities adapted

Expedience of having several different tables of life-annuities adapted to different rates of interest,

to several different rates of interest; as we have in Mr. Smart's excellent book above-mentioned for the values of annuities for terms of years at the twelve different rates of interest mentioned above in Art. 24, page 23, to wit, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, 8, 9, and 10, per cent. And accordingly I shall insert in the following part of this work the like tables for annuities for single lives of all ages from 3 years to 93 years, at the same twelve different rates of interest and according to Monsieur de Parcieux's table of the probabilities of the duration of human life; which have all been faithfully calculated, under the inspection and direction of the learned and worthy Dr. Price above-mentioned, upon the principles that have been above explained.

XCIX. In perusing the foregoing calculation of the value of an annuity for the life of a person of 10 years of age, in Art. 94, the reader will, I doubt not, have entertained a high idea of the difficulty of composing a whole table of the values of life-annuities for every year of human life as far as 93 years, if the same method is to be taken for the computation of every one of those values as is there pursued for that of an annuity for a life of the age of ten years, He will therefore, probably, be glad to be informed that the repetition of this laborious process for every year of human life is not necessary, but that the value of an annuity for a life of any age may be deduced from the value of an annuity for a life that is one year older by a very short and easy process, inasmuch that a whole table of the values of life-annuities for every year of human life may be computed with very little more labour than has been employed above, in Art. 94, in computing the value of only that one annuity for the life of a person of ten years of age. The method of doing this was communicated to me by the learned Dr. Price above-mentioned, and may be explained as follows.

A short and easy method of deducing from the value of an annuity of one pound for any given life the value of a like annuity for a life one year younger than the former.

C. We have seen in Art. 94 that, if the interest of money be $3\frac{1}{4}$ per cent. and the probabilities of the duration of human life such as they are represented in Monsieur de Parcieux's table, the value of an annuity of one pound for the life of a person of 10 years of age is £20.73925, or 20l. 14s. 9d. $\frac{1}{2}$. Now from this value we may derive that of a like annuity of one pound for the life of a person of the age of 9 years in the following manner.

CI. In Monsieur de Parcieux's table of probabilities there are 890 persons represented to be living at the age of 9 years. Now let us suppose that a man were to make 890 grants of annuities, of one pound a year each, to all these 890 persons of the age of 9 years, for their respective lives. And let the payments which the said grantor would thereby oblige himself to make to the said grantees be divided into the two following parts, or classes, to wit, first, the payments he is to make at the end of the first year to such of the 890 grantees as shall be then alive, and, secondly, all the other payments which he will be obliged to make to such of them as shall survive beyond the said first year, and which will become due at the ends of the second, third, fourth, fifth, and other following years respectively, till all the said grantees shall be dead. And let us investigate separately the prices he ought to receive from the said grantees for these two sets, or classes, of payments which he thus binds himself to make.

Now it appears from Monsieur de Parcieux's table that of these 890 grantees, all of the age of 9 years, 880 will live to the end of one year, or till they are 10 years old. Therefore at the end of the said first year the said grantor will have 880 payments, of one pound each, to make to the said 880 surviving grantees. He ought therefore to receive, as the fair price of those payments, 880 times the present value of one pound to be received at the end of a year, or 880 times $\frac{1}{r}$ of a pound, or $880 \times \frac{1}{r}$, or $\frac{1}{r} \times 880 \times £1$.

Secondly, it is evident that the subsequent payments which the said grantor will have to make, at the ends of the second, third, fourth, fifth, and other following years, to such of the said 880 grantees (who are living at the end of the first year,) as shall further survive to the ends of the said second, third, fourth, fifth, and other following years, will be precisely the same as he would have had to make to them at the same times respectively if he had post-poned making any of these grants for the space of a year, or till the said 880 persons were 10 years old, and had then made them 880 grants, of one pound a year each, for their respective lives. Thus, for example, the payment he will have to make at the end of the second year, or when the grantees are 11 years old, will be in both cases $£872$, because there will be 872 of the said grantees then alive; and the payment he will have to make at the end of the third year, or when the grantees are 12 years old, will be in both cases $£866$, because there will be 866 of the said grantees alive at that time; and the same is true of the payments at the ends of all the following years, to wit, that they will be the same in both cases. If therefore the prices of these several payments were to be paid when the said grantees were 10 years old, they must be precisely the same as if the grants had been made when the grantees were 10 years old; and

and consequently the sum total of all these prices would be precisely equal to the value of 880 life-annuities of one pound a year granted to 880 persons of the age of 10 years, that is, (as appears by Art. 94) to 880 times £20.73925. But the prices of these several payments of £872, £866, &c. are supposed in the present case to be paid to the grantor when the grantees are only 9 years old, or a year sooner than on the last supposition. Therefore (by Problem 1, Art. xxiii, page 21,) they must be less than those other prices of the same payments, respectively, in the proportion of $\frac{1}{r}$ to 1. Therefore the sum total of all the said prices must be less than the sum total of all those other prices of the same payments in the same proportion of $\frac{1}{r}$ to 1, and consequently must be equal to $\frac{1}{r} \times 880 \times £20.73925$; that is, the price to be paid to the grantor by all the 890 grantees of the age of 9 years, on account of all the said future payments of the second class, which are to be made at the ends of the second, third, fourth, fifth, and other following years from the time of making the grants, is $\frac{1}{r} \times 880 \times £20.73925$.

But it was before shewn that the price to be paid to the said grantor by the said 890 grantees on account of the payments to be made to them at the end of the first year, is $\frac{1}{r} \times 880 \times £1$.

Therefore the price to be paid to the said grantor by the said 890 grantees on both the said accounts, or for the whole life-annuities of one pound that are thus granted to them at the age of 9 years, is $\frac{1}{r} \times 880 \times £1 + \frac{1}{r} \times 880 \times £20.73925$. And consequently the price that ought to be paid by each of the said 890 grantees for his annuity is the 890th part of the said sum, or $\frac{1}{r} \times \frac{880}{890} \times £1 + \frac{1}{r} \times \frac{880}{890} \times £20.73925$, or $\frac{1}{r} \times \frac{880}{890} \times [£1 + £20.73925]$. Therefore by Art. 21, if only one such grant is made to a person of the age of 9 years, the price of the said single annuity ought also to be $\frac{1}{r} \times \frac{880}{890} \times [£1 + £20.73925]$; that is, (because $\frac{1}{r}$ is in this case = $\frac{1}{1.035}$)

the price of the said annuity will be $\frac{1}{1.035} \times \frac{880}{890} \times [£1 + 20.73925 £]$
 $(= \frac{1}{1.035} \times \frac{880}{890} \times £21.73925 = \frac{1}{1.035} \times £21.494988) = £20.76810$,
 or 20l. 15s. 4d. $\frac{1}{2}$. Q.E.I.

CII. Thus

CII. Thus it appears that from the value of an annuity of one pound for a life of the age of 10 years we may, by very easy arithmetical operations, deduce that of a like annuity for a life of the age of 9 years, to wit, by only adding £1 to the former value, and then multiplying the sum thence arising, first, into the fraction $\frac{880}{890}$, (of which the numerator expresses the number of persons represented in Monsieur de Parcieux's table of probabilities to be living at the age of 10 years, and the denominator expresses the number of persons that are therein represented to be living at the age of 9 years,) and, secondly, into the fraction $\frac{1}{r}$, which bears the same proportion to 1 as the present value of one pound to be received at the end of a year bears to one pound. And, as the reasonings employed in making this deduction are not peculiar to the ages of 9 and 10 years, but may be applied to any other two ages of human life that differ from each other only by a year, we may safely conclude in general terms, that, if V denotes the value of an annuity of one pound a year for the life of a person of the age of N years, and P is the number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life, (or in such other table of those probabilities as is adopted as the ground of the calculation,) as living at the age of N years, and $P+1$ is the number of persons represented there as living at the age of $N-1$ years, the value of an annuity of one pound a year for the life of a person of the age of $N-1$ years will be equal to $\frac{1}{r} \times \frac{P}{P+1} \times \frac{1}{1+V} \times \text{QEL.}$

Other examples of the foregoing method of deducing the value of an annuity for any proposed life from that of a like annuity for a life that is one year older.

CIII. Let it be supposed that the value of an annuity of one pound a year for the life of a person of 9 years of age is already known, and that it is (as we have just now found it to be by the computation of it made above in Art. 101,) equal to £20.76810; the interest of money being $3\frac{1}{4}$ per cent. and the probabilities of the duration of human life such as they are represented in Monsieur de Parcieux's table. And let it be required to deduce from this value of an annuity of one pound for a life of a person of the age of 9 years the value of a like annuity for the life of a person of the age of 8 years, in the method above explained, or by means of the expression

$$\frac{1}{r} \times \frac{P}{P+1} \times \frac{1}{1+V} \times \text{QEL.}$$

Here

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Here P is = 890, and $P+d$ is = 902, and $\dot{V}$ is = £20.76810, and consequently $\frac{1}{1+\dot{V}}|£$ is = £21.76810; and $\frac{1}{r}$ is (as before) = $\frac{1}{1.035}$. therefore $\frac{1}{r} \times \frac{P}{P+d} \times \frac{1}{1+\dot{V}}|£$ is = $\frac{1}{1.035} \times \frac{890}{902} \times £21.76810 = \frac{1}{1.035} \times £21.47850 = £20.75217$. Consequently the value of an annuity of one pound for the life of a person of the age of 8 years is £20.75217, or 20*l.* 15*s.* $\frac{1}{2}$ *d.* Q.E.I.

CIV. In like manner by putting $\dot{V} = £20.75217$, and $P = 902$, and $P+d = 915$, and computing the expression $\frac{1}{r} \times \frac{P}{P+d} \times \frac{1}{1+\dot{V}}|£$, or $\frac{1}{1.035} \times \frac{902}{915} \times £21.75217$, we shall find the value of an annuity of one pound for a life of 7 years to be equal to $\left(\frac{1}{1.035} \times £21.44312\right)$, or £20.71799, or 20*l.* 14*s.* 4*d.* $\frac{1}{2}$.

CV. And in like manner an annuity of one pound for a life of the age of 6 years will be found to be worth $\frac{1}{1.035} \times \frac{915}{930} \times £21.71799 = \frac{1}{1.035} \times £21.367,699,8 = £20.64512$, or 20*l.* 12*s.* 10*d.* $\frac{1}{2}$.

And an annuity of one pound for a life of the age of 5 years will be worth $\frac{1}{1.035} \times \frac{930}{948} \times £21.64512 \left(= \frac{1}{1.035} \times £21.23413\right) = £20.51606$, or 20*l.* 10*s.* 3*d.* $\frac{1}{2}$.

And an annuity of one pound for a life of the age of 4 years will be worth $\frac{1}{1.035} \times \frac{948}{970} \times £21.51606 \left(= \frac{1}{1.035} \times £21.028,06\right) = £20.31696$, or 20*l.* 6*s.* 4*d.*

Q

And

And an annuity of one pound for a life of the age of 3 years will be worth
 $\frac{1}{1.035} \times \frac{970}{1000} \times £21.31696 (= \frac{1}{1.035} \times £20.67745) = £19.97821,$
 or 19l. 19s. 6d. $\frac{2}{3}$.

CVI. Therefore, if no mistakes have been made in the foregoing calculations, we may conclude that the values of an annuity of one pound for the lives of children of 3, 4, 5, 6, 7, 8, 9, and 10 years of age, when the interest of money is $3\frac{1}{4}$ per cent. are (according to Monsieur de Parcieux's table of the probabilities of the duration of human life,) respectively, equal to the following sums of money, to wit,

£ 19.97821,	or	£ 19.	19s.	6d. $\frac{2}{3}$.
And £ 20.31696,	or	£ 20.	6s.	4d.
And £ 20.51606,	or	£ 20.	10s.	3d. $\frac{1}{2}$.
And £ 20.64512,	or	£ 20.	12s.	10d. $\frac{2}{3}$.
And £ 20.71799,	or	£ 20.	14s.	4d. $\frac{2}{3}$.
And £ 20.75217,	or	£ 20.	15s.	$\frac{1}{2}$ d.
And £ 20.76810,	or	£ 20.	15s.	4d. $\frac{1}{2}$.
And £ 20.73925,	or	£ 20.	14s.	9d. $\frac{1}{2}$.

CVII. All the other values of a compleat table of life-annuities (grounded upon the foregoing suppositions that the interest of money is $3\frac{1}{4}$ per cent. and that the life of man decays in the manner represented in Monsieur de Parcieux's table of probabilities,) may be computed in the same manner, one from another, by beginning with the oldest life in the table and proceeding downwards to the next younger life till we have obtained the value of an annuity of one pound for a life of 10 years of age, which we have already computed in Art. 94, and found to be £20.73925. And the trouble of computing the values of the annuities of one pound belonging to all these different ages will not be very much greater than that of computing, by the method used in Art. 94, the value of only that one annuity for the life of a person of the age of 10 years, which is computed in that article. The operations necessary for this purpose are as follows.

A computation of the values of an annuity of one pound for every different year of human life from the age of 93 years to the age of 10 years, inclusively; upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. and that the probabilities of the duration of human life are such as they are represented to be in Monsieur de Parcieux's table.

CVIII. It appears by Monsieur de Parcieux's table that of two persons aged 93 years, living at the same time, only one will live another year, or to the age of 94, and that the said survivor at 94 years of age will die before he is 95. If therefore a man were to grant to 2 old persons of the age of 93 years annuities of one pound a year each for their lives, he would, at the end of the first year, have only one payment of one of the said annuities to make, to wit, that which would be then due to the survivor of the said two grantees; and he would never after be obliged to make another payment of either of the annuities, because both the said grantees would be dead before the end of the second year, or before the second payment would become due. To the end therefore that the grantor of the said annuities might be neither a gainer nor a loser by granting them, it would be necessary that he should receive from both the said grantees, at the time of making the said grants, the present value of the said only payment of one pound which he would have to make to the survivor of the said two grantees at the end of the first year; which present value is the sum of

$\frac{1}{r}$ of a pound, or $\frac{1}{r}$. And consequently he ought to receive from each of

the said two grantees one half of that sum, or $\frac{1}{2} \times \frac{1}{r}$, or (because $\frac{1}{r}$ is

in this case $= \frac{1}{1.035}$, the interest of money being $3\frac{1}{2}$ per cent.) $\frac{1}{2} \times \frac{1}{1.035}$,

or $\frac{1}{2} \times \text{£.96618}$, or £.48309 . Therefore, by Art. 21, if the grantor were to make only one such grant of an annuity of one pound a year to a person of the age of 93 years for his life, he ought to receive, as the price of such single grant, the same sum of £.48309 ; or, in other words, the value of an annuity of one pound a year for a life of 93 years is £.48309 , or *gr. 8d.* Q.E.I.

CIX. Having thus found the value of an annuity of one pound a year for a life of 93 years, the values of the like annuities for all younger lives may be found by the continual application of the expression $\frac{1}{P} \times \frac{P}{P+d} \times \frac{1}{1+V}$, or $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+V}$, in the manner following.

CX. To find the value of an annuity of one pound for a life of 92 years, we must put $V = £.48309$, and $P = 2$, and $P+d = 4$; because there are 2 persons living, according to *Monf. de Parcieux's* table, at the age of 93 years, and 4 persons living at the age of 92 years. And we shall have $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+V} | £ = \frac{1}{1.035} \times \frac{2}{4} \times \frac{1}{1+.48309} | £ = \frac{1}{1.035} \times \frac{2}{4} \times £1.48309 = \frac{1}{1.035} \times £.741,545 = £.716,468$. Therefore the value of an annuity of one pound for a life of 92 years is $£.716,468$, or 14s. 4d. QEI.

CXI. To find the value of an annuity of one pound for a life of 91 years, we must put $V = £.716,468$, and consequently $\frac{1}{1+V} | £ = £1.716,468$. And, by the table of probabilities, the numbers of persons living at the ages of 91 and 92 years are 7 and 4; that is, $P = 4$, and $P+d = 7$. Therefore $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+V} | £ = \frac{1}{1.035} \times \frac{4}{7} \times £1.716,468 = \frac{1}{1.035} \times £.980,838 = .947,669$. Consequently the value of an annuity of one pound for the life of a person of the age of 91 years is .947,669, or 18s. 11d. $\frac{1}{2}$.

CXII. Again, put $V = £.947,669$, and consequently $\frac{1}{1+V} | £ = £1.947,669$; and let P be = 7, and $P+d = 11$, which are the numbers of persons represented in the table as living at the ages of 91 and 90 years. And we shall have $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+V} | £ = \frac{1}{1.035} \times \frac{7}{11} \times £1.947,669 = \frac{1}{1.035} \times £1.239,425 = £1.197,512$. Therefore the value of an annuity of one pound for a life of the age of 90 years is = $£1.197,512$, or 1l. 3s. 11d. $\frac{1}{2}$.

CXIII. And

And that the value of the like annuity for a life of 82 years is

$$\left(= \frac{1}{1.035} \times \frac{71}{85} \times £3.984,473 = \frac{1}{1.035} \times £3.328,206 = \right) £3.215,658,$$

 or 3*l.* 4*s.* 3*d.* $\frac{3}{4}$.

And that for a life of 81 years it is $\left(= \frac{1}{1.035} \times \frac{85}{101} \times £4.215,658 \right.$

$$\left. = \frac{1}{1.035} \times £3.547,831 = \right) £3.427,856, \text{ or } 3*l.* 8*s.* 6*d.* $\frac{3}{4}$.$$

And that for a life of 80 years it is $\left(= \frac{1}{1.035} \times \frac{101}{118} \times £4.427,856 \right.$

$$\left. = \frac{1}{1.035} \times £3.789,944 = \right) £3.661,781, \text{ or } 3*l.* 13*s.* 2*d.* $\frac{3}{4}$.$$

CXIV. And for a life of 79 years it is $\left(= \frac{1}{1.035} \times \frac{118}{136} \times \right.$

$$\left. £4.661,781 = \frac{1}{1.035} \times £4.044,780 = \right) £3.908,000, \text{ or } 3*l.* 18*s.* 2*d.*$$

And for a life of 78 years it is $\left(= \frac{1}{1.035} \times \frac{136}{154} \times £4.908,000 \right.$

$$\left. = \frac{1}{1.035} \times £4.334,337 = \right) £4.187,758, \text{ or } 4*l.* 3*s.* 9*d.*$$

And for a life of 77 years it is $\left(= \frac{1}{1.035} \times \frac{154}{173} \times £5.187,758 \right.$

$$\left. = \frac{1}{1.035} \times £4.618,004 = \right) £4.461,839, \text{ or } 4*l.* 9*s.* 2*d.* $\frac{3}{4}$.$$

And for a life of 76 years it is $\left(= \frac{1}{1.035} \times \frac{173}{192} \times £5.461,839 = \right.$

$$\left. \frac{1}{1.035} \times £4.921,344 = \right) £4.754,921, \text{ or } 4*l.* 15*s.* 1*d.* $\frac{3}{4}$.$$

And

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And for a life of 75 years it is $\left(= \frac{1}{1.035} \times \frac{192}{211} \times £5.754,921 \right.$

$$\left. = \frac{1}{1.035} \times £5.236,705 = \right) £5.059,623, \text{ or } 5\text{ l. } 1\text{ s. } 2\text{ d. } \frac{1}{4}.$$

And for a life of 74 years it is $\left(= \frac{1}{1.035} \times \frac{211}{231} \times £6.059,623 \right.$

$$\left. = \frac{1}{1.035} \times £5.534,980 = \right) £5.347,806, \text{ or } 5\text{ l. } 6\text{ s. } 11\text{ d. } \frac{1}{4}.$$

And for a life of 73 years it is $\left(= \frac{1}{1.035} \times \frac{231}{251} \times £6.347,806 \right.$

$$\left. = \frac{1}{1.035} \times £5.842,004 = \right) £5.644,448, \text{ or } 5\text{ l. } 12\text{ s. } 10\text{ d. } \frac{1}{2}.$$

And for a life of 72 years it is $\left(= \frac{1}{1.035} \times \frac{251}{271} \times £6.644,448 \right.$

$$\left. = \frac{1}{1.035} \times £6.154,082 = \right) £5.945,972, \text{ or } 5\text{ l. } 18\text{ s. } 11\text{ d.}$$

And for a life of 71 years it is $\left(= \frac{1}{1.035} \times \frac{271}{291} \times £6.945,972 \right.$

$$\left. = \frac{1}{1.035} \times £6.468,585 = \right) £6.249,840, \text{ or } 6\text{ l. } 5\text{ s. } 0\text{ d.}$$

And for a life of 70 years it is $\left(= \frac{1}{1.035} \times \frac{291}{310} \times £7.249,840 \right.$

$$\left. = \frac{1}{1.035} \times £6.805,495 = \right) £6.575,357, \text{ or } 6\text{ l. } 11\text{ s. } 6\text{ d.}$$

And for a life of 69 years it is $\left(= \frac{1}{1.035} \times \frac{310}{329} \times £7.575,357 \right.$

$$\left. = \frac{1}{1.035} \times £7.137,874 = \right) £6.896,496, \text{ or } 6\text{ l. } 17\text{ s. } 11\text{ d. } \frac{1}{4}.$$

And

$$\text{And for a life of 68 years it is } \left(= \frac{1}{1.035} \times \frac{329}{347} \times £7.896,496 \right. \\ \left. = \frac{1}{1.035} \times £7.486,879 = \right) £7.233,699, \text{ or } 7l. 4s. 8d. \frac{1}{2}$$

$$\text{And for a life of 67 years it is } \left(= \frac{1}{1.035} \times \frac{347}{364} \times £8.233,699 \right. \\ \left. = \frac{1}{1.035} \times £7.849,158 = \right) £7.583,727, \text{ or } 7l. 11s. 8d. \frac{1}{2}$$

$$\text{And for a life of 66 years it is } \left(= \frac{1}{1.035} \times \frac{364}{380} \times £8.583,727 \right. \\ \left. = \frac{1}{1.035} \times £8.222,307 = \right) £7.944,258, \text{ or } 7l. 18s. 10d. \frac{1}{2}$$

$$\text{And for a life of 65 years it is } \left(= \frac{1}{1.035} \times \frac{380}{395} \times £8.944,258 \right. \\ \left. = \frac{1}{1.035} \times £8.604,602 = \right) £8.313,625, \text{ or } 8l. 6s. 3d. \frac{1}{2}$$

$$\text{And for a life of 64 years it is } \left(= \frac{1}{1.035} \times \frac{395}{409} \times £9.313,625 \right. \\ \left. = \frac{1}{1.035} \times £8.994,821 = \right) £8.690,648, \text{ or } 8l. 13s. 9d. \frac{1}{2}$$

$$\text{And for a life of 63 years it is } \left(= \frac{1}{1.035} \times \frac{409}{423} \times £9.690,648 \right. \\ \left. = \frac{1}{1.035} \times £9.369,917 = \right) £9.053,059, \text{ or } 9l. 1s. 0d. \frac{1}{2}$$

$$\text{And for a life of 62 years it is } \left(= \frac{1}{1.035} \times \frac{423}{437} \times £10.053,059 \right. \\ \left. = \frac{1}{1.035} \times £9.730,993 = \right) £9.401,925, \text{ or } 9l. 8s. 0d. \frac{1}{2}$$

And

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And for a life of 61 years it is $\left(= \frac{1}{1.035} \times \frac{437}{450} \times £10.401925 \right.$

$= \frac{1}{1.035} \times £10.101424 =) £9.759.889, \text{ or } 9\text{ l. } 15\text{ s. } 3\text{ d. } \frac{1}{2}.$

And for a life of 60 years it is $\left(= \frac{1}{1.035} \times \frac{450}{463} \times £10.759,829 \right.$

$= \frac{1}{1.035} \times £10.457,717 =) £10.164,074, \text{ or } 10\text{ l. } 2\text{ s. } 1\text{ d. } \frac{1}{2}.$

And for a life of 59 years it is $\left(= \frac{1}{1.035} \times \frac{463}{476} \times £11.104,074 \right.$

$= \frac{1}{1.035} \times £10.800,811 =) £10.435,566, \text{ or } 10\text{ l. } 8\text{ s. } 8\text{ d. } \frac{1}{2}.$

And for a life of 58 years it is $\left(= \frac{1}{1.035} \times \frac{476}{489} \times £11.435,566 \right.$

$= \frac{1}{1.035} \times £11.131,553 =) £10.755,123, \text{ or } 10\text{ l. } 15\text{ s. } 1\text{ d. } \frac{1}{2}.$

And for a life of 57 years it is $\left(= \frac{1}{1.035} \times \frac{489}{502} \times £11.755,123 \right.$

$= \frac{1}{1.035} \times £11.450,707 =) £11.063,485, \text{ or } 11\text{ l. } 1\text{ s. } 3\text{ d. } \frac{1}{2}.$

And for a life of 56 years it is $\left(= \frac{1}{1.035} \times \frac{502}{514} \times £12.063,485 \right.$

$= \frac{1}{1.035} \times £11.781,847 =) £11.383,427, \text{ or } 11\text{ l. } 7\text{ s. } 8\text{ d. } \frac{1}{2}.$

And for a life of 55 years it is $\left(= \frac{1}{1.035} \times \frac{514}{526} \times £12.383,427 \right.$

$= \frac{1}{1.035} \times £12.100,915 =) £11.691,801, \text{ or } 11\text{ l. } 13\text{ s. } 10\text{ d. } \frac{1}{2}.$

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And

And for a life of 54 years it is $\left(= \frac{1}{1.035} \times \frac{526}{538} \times £12.691,801 = \frac{1}{1.035} \times £12.408,712 = \right) £11.989,093$, or 11l. 19s. 9d. $\frac{1}{4}$.

And for a life of 53 years it is $\left(= \frac{1}{1.035} \times \frac{538}{549} \times £12.989,093 = \frac{1}{1.035} \times £12.728,838 = \right) £12.298,386$, or 12l. 5s. 11d. $\frac{1}{4}$.

And for a life of 52 years it is $\left(= \frac{1}{1.035} \times \frac{549}{560} \times £13.298,386 = \frac{1}{1.035} \times £13.037,167 = \right) £12.596,296$, or 12l. 11s. 11d.

And for a life of 51 years it is $\left(= \frac{1}{1.035} \times \frac{560}{571} \times £13.596,296 = \frac{1}{1.035} \times £13.334,370 = \right) £12.883,449$, or 12l. 17s. 8d.

And for a life of 50 years it is $\left(= \frac{1}{1.035} \times \frac{571}{581} \times £13.883,449 = \frac{1}{1.035} \times £13.644,491 = \right) £13.183,083$, or 13l. 3s. 8d.

And for a life of 49 years it is $\left(= \frac{1}{1.035} \times \frac{581}{590} \times £14.183,083 = \frac{1}{1.035} \times £13.966,730 = \right) £13.494,425$, or 13l. 9s. 10d. $\frac{1}{4}$.

And for a life of 48 years it is $\left(= \frac{1}{1.035} \times \frac{590}{599} \times £14.494,425 = \frac{1}{1.035} \times £14.276,645 = \right) £13.793,859$, or 13l. 15s. 10d. $\frac{1}{4}$.

And

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$$\begin{aligned} \text{And for a life of 47 years it is } \left(= \frac{1}{1.035} \times \frac{599}{607} \times £14.793,859 \right. \\ \left. = \frac{1}{1.035} \times £14,598,882 = \right) £14.105,200, \text{ or } 14l. 24s. 1d. \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} \text{And for a life of 46 years it is } \left(= \frac{1}{1.035} \times \frac{607}{615} \times £15.105,200 \right. \\ \left. = \frac{1}{1.035} \times £14.908,709 = \right) £14.404,549, \text{ or } 14l. 8s. 1d. \end{aligned}$$

$$\begin{aligned} \text{And for a life of 45 years it is } \left(= \frac{1}{1.035} \times \frac{615}{622} \times £15.404,549 \right. \\ \left. = \frac{1}{1.035} \times £15.231,185 = \right) £14.716,120, \text{ or } 14l. 14s. 3d. \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} \text{And for a life of 44 years it is } \left(= \frac{1}{1.035} \times \frac{622}{629} \times £15.716,120 \right. \\ \left. = \frac{1}{1.035} \times £15.541,218 = \right) £15.015,669, \text{ or } 15l. 0s. 3d. \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} \text{And for a life of 43 years it is } \left(= \frac{1}{1.035} \times \frac{629}{636} \times £16.015,669 \right. \\ \left. = \frac{1}{1.035} \times £15.839,395 = \right) £15.303,763, \text{ or } 15l. 6s. 1d. \end{aligned}$$

$$\begin{aligned} \text{And for a life of 42 years it is } \left(= \frac{1}{1.035} \times \frac{636}{643} \times £16.303,763 \right. \\ \left. = \frac{1}{1.035} \times £16.126,272 = \right) £15.580,939, \text{ or } 15l. 14s. 7d. \frac{1}{2}. \end{aligned}$$

$$\begin{aligned} \text{And for a life of 41 years it is } \left(= \frac{1}{1.035} \times \frac{643}{650} \times £16.580,939 \right. \\ \left. = \frac{1}{1.035} \times £16.448,528 = \right) £15.892,297, \text{ or } 15l. 17s. 10d. \end{aligned}$$

R 2

And

$$\text{And for a life of 40 years it is } \left(= \frac{1}{1.035} \times \frac{650}{657} \times £16.892,297 \right. \\ \left. = \frac{1}{1.035} \times £16.712,318 = \right) £16.147,167, \text{ or } 16l. 2s. 11d. \frac{1}{2}.$$

$$\text{And for a life of 39 years it is } \left(= \frac{1}{1.035} \times \frac{657}{664} \times £17.147,167 \right. \\ \left. = \frac{1}{1.035} \times £16.966,398 = \right) £16.392,655, \text{ or } 16l. 7s. 10d. \frac{1}{2}.$$

$$\text{And for a life of 38 years it is } \left(= \frac{1}{1.035} \times \frac{664}{671} \times £17.392,655 \right. \\ \left. = \frac{1}{1.035} \times £17.211,211 = \right) £16.629,189, \text{ or } 16l. 12s. 7d.$$

$$\text{And for a life of 37 years it is } \left(= \frac{1}{1.035} \times \frac{671}{678} \times £17.629,189 \right. \\ \left. = \frac{1}{1.035} \times £17.447,176 = \right) £16.857,174, \text{ or } 16l. 17s. 1d. \frac{1}{2}.$$

$$\text{And for a life of 36 years it is } \left(= \frac{1}{1.035} \times \frac{678}{686} \times £17.857,174 \right. \\ \left. = \frac{1}{1.035} \times £17.648,927 = \right) £17.052,103, \text{ or } 17l. 6s. 11d. \frac{1}{2}.$$

$$\text{And for a life of 35 years it is } \left(= \frac{1}{1.035} \times \frac{686}{694} \times £18.052,103 \right. \\ \left. = \frac{1}{1.035} \times £17.844,009 = \right) £17.240,588, \text{ or } 17l. 4s. 9d. \frac{1}{2}.$$

$$\text{And for a life of 34 years it is } \left(= \frac{1}{1.035} \times \frac{694}{702} \times £18.240,588 \right. \\ \left. = \frac{1}{1.035} \times £18.032,718 = \right) £17.422,915, \text{ or } 17l. 8s. 5d. \frac{1}{2}.$$

And

And for a life of 33 years it is $(= \frac{1}{1.035} \times \frac{70^2}{710} \times £18,402,915$

$= \frac{1}{1.035} \times £18,215,332 (=) £17,599,354$ or $17l. 11s. 11d. \frac{1}{2}$.

And for a life of 32 years it is $(= \frac{1}{1.035} \times \frac{71^2}{718} \times £18,599,354$

$= \frac{1}{1.035} \times £18,392,118 (=) £17,776,162$, or $17l. 15s. 2d. \frac{1}{2}$.

And for a life of 31 years it is $(= \frac{1}{1.035} \times \frac{71^2}{726} \times £18,770,162$

$= \frac{1}{1.035} \times £18,563,328 (=) £17,935,582$, or $17l. 18s. 8d. \frac{1}{2}$.

And for a life of 30 years it is $(= \frac{1}{1.035} \times \frac{72^2}{734} \times £18,935,582$

$= \frac{1}{1.035} \times £18,729,199 (=) £18,095,844$ or $18l. 1s. 11d.$

And for a life of 29 years it is $(= \frac{1}{1.035} \times \frac{73^2}{742} \times £19,095,844$

$= \frac{1}{1.035} \times £18,889,958 (=) £18,251,167$, or $18l. 5s. 0d. \frac{1}{2}$.

And for a life of 28 years it is $(= \frac{1}{1.035} \times \frac{74^2}{750} \times £19,251,167$

$= \frac{1}{1.035} \times £19,045,821 (=) £18,401,759$, or $18l. 8s. \frac{1}{2}d.$

And for a life of 27 years it is $(= \frac{1}{1.035} \times \frac{75^2}{758} \times £19,401,759$

$= \frac{1}{1.035} \times £19,196,991 (=) £18,547,817$, or $18l. 10s. 11d. \frac{1}{2}$.

And

$$\text{And for a life of 26 years it is } (= \frac{1}{1.035} \times \frac{758}{766} \times £19.547,81) \\ = \frac{1}{1.035} \times £19.343,662 (=) £18.689,528, \text{ or } 18l. 13s. 9d. \frac{1}{4}.$$

$$\text{And for a life of 25 years it is } (= \frac{1}{1.035} \times \frac{766}{774} \times £19.689,51) \\ = \frac{1}{1.035} \times £19.486,018 (=) £18.827,070, \text{ or } 18l. 16s. 6d. \frac{1}{4}.$$

$$\text{And for a life of 24 years it is } (= \frac{1}{1.035} \times \frac{774}{782} \times £19.827,07) \\ = \frac{1}{1.035} \times £19.624,235 (=) £18.960,613, \text{ or } 18l. 19s. 2d. \frac{1}{4}.$$

$$\text{And for a life of 23 years it is } (= \frac{1}{1.035} \times \frac{782}{790} \times £19.960,61) \\ = \frac{1}{1.035} \times £19.758,480 (=) £19.090,318, \text{ or } 19l. 1s. 9d. \frac{1}{4}.$$

$$\text{And for a life of 22 years it is } (= \frac{1}{1.035} \times \frac{790}{798} \times £20,090,31) \\ = \frac{1}{1.035} \times £19.888,911 (=) £19.216,339, \text{ or } 19l. 4s. 4d.$$

$$\text{And for a life of 21 years it is } (= \frac{1}{1.035} \times \frac{798}{806} \times £20.216,33) \\ = \frac{1}{1.035} \times £20.015,680 (=) £19.338,821, \text{ or } 19l. 6s. 9d. \frac{1}{4}.$$

$$\text{And for a life of 20 years it is } (= \frac{1}{1.035} \times \frac{806}{814} \times £20.338,81) \\ = \frac{1}{1.035} \times £20.138,930 (=) £19.457,903, \text{ or } 19l. 9s. 1d. \frac{1}{4}.$$

And

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And for a life of 19 years it is $(= \frac{1}{1.035} \times \frac{814}{821} \times £20.457,903$

$= \frac{1}{1.035} \times £20.283,476 =) £19.597,560,$ or 19l. 11s. 11d. $\frac{1}{4}$.

And for a life of 18 years it is $(= \frac{1}{1.035} \times \frac{821}{828} \times £20.597,560$

$= \frac{1}{1.035} \times £20.423,426 =) £19.732,778,$ or 19l. 14s. 7d. $\frac{2}{3}$.

And for a life of 17 years it is $(= \frac{1}{1.035} \times \frac{828}{835} \times £20.732,778$

$= \frac{1}{1.035} \times £20.558,970 =) £19.863,739,$ or 19l. 17s. 3d. $\frac{1}{2}$.

And for a life of 16 years it is $(= \frac{1}{1.035} \times \frac{835}{842} \times £20.863,739$

$= \frac{1}{1.035} \times £20.690,287 =) £19.990,615,$ or 19l. 19s. 9d. $\frac{3}{4}$.

And for a life of 15 years it is $(= \frac{1}{1.035} \times \frac{842}{848} \times £20.990,615$

$= \frac{1}{1.035} \times £20.847,096 =) £20.137,194,$ or 20l. 21s. 9d.

And for a life of 14 years it is $(= \frac{1}{1.035} \times \frac{848}{854} \times £21.137,194$

$= \frac{1}{1.035} \times £20.988,677 =) £20.278,914,$ or 20l. 5s. 7d.

And for a life of 13 years it is $(= \frac{1}{1.035} \times \frac{854}{860} \times £21.278,914$

$= \frac{1}{1.035} \times £21.130,456 =) £20.415,899,$ or 20l. 8s. 3d. $\frac{2}{3}$.

And

And for a life of 12 years it is $(= \frac{1}{1.035} \times \frac{869}{866} \times £21.415)$

$$= \frac{1}{1.035} \times £21.267,520 (=) £20.540,338, \text{ or } 20\text{ l. } 10\text{ s. } 11\text{ d. } \frac{1}{2}.$$

And for a life of 11 years it is $(= \frac{1}{1.035} \times \frac{866}{872} \times £21.548,31)$

$$= \frac{1}{1.035} \times £21.400,069 (=) £20.676,395, \text{ or } 20\text{ l. } 13\text{ s. } 6\text{ d. } \frac{1}{2}.$$

And for a life of 10 years it is $(= \frac{1}{1.035} \times \frac{872}{880} \times £21.676,33)$

$$= \frac{1}{1.035} \times £21.479,336 (=) £20.752,981, \text{ or } 20\text{ l. } 15\text{ s. } 0\text{ d. } \frac{1}{2}.$$

CXV. This value of an annuity of one pound for a life of 10 years is somewhat greater than that which was found for it in Art. 94, which was £20.73925. But the difference between them is but small, being only .013,731 of a pound, or $(240 \times .013,731, \text{ or } 3.295,440 \text{ of a penny, or about three pence, farthing, upon the sum of twenty pounds, fifteen shillings. They may therefore be considered as being equal to each other, and consequently as affording a proof of the agreement of the two different methods of calculation by which they have been obtained, and a confirmation of their truth. Which of these two values is the more exact, I do not know; but I am inclined to think it is this last, which we have just now obtained by means of the repeated application of the expression$

$$\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+\frac{d}{P}} \times £, \text{ or } £20.751,981. \text{ And, if this number}$$

does exhibit the value of the said annuity more exactly than the other number £20.73925, it is evident that the values of the remaining annuities for lives of the ages of 9, 8, 7, 6, 5, 4, and 3 years, will be somewhat greater than they are found to be in Art. 103, 104, 105, and 106. In order, therefore, to make the table of these values (which we have here computed,) uniform and regular throughout, I shall compute the value of an annuity of one pound for lives of 9, 8, 7, 6, 5, 4, and 3 years of age

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second time by means of the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{P}{1+V} \text{ £}$, upon supposition that the first value of $V \text{ £}$, or the value of an annuity of one pound for a life of the age of 10 years, is = £20.752,981. This may be done in the manner following.

CXVI. If V is = £20.752,981, we shall have $\frac{1}{1+V} \text{ £} = \text{£}21.752,981$. And in this case P is = 880, and $P+d$ is = 890. Therefore the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{P}{1+V} \text{ £}$ will be = $\frac{1}{1.035} \times \frac{880}{890} \times \text{£}21.752,981$
 $= \frac{1}{1.035} \times \text{£}21.508,565 = \text{£}20.781,222$; or the value of an annuity of one pound for a life of 9 years is £20.781,222, or 20*l.* 15*s.* 7*d.* $\frac{1}{2}$.

CXVII. And in like manner the value of an annuity of one pound for a life of 8 years will be $(= \frac{1}{1.035} \times \frac{890}{902} \times \text{£}21.781,222 = \frac{1}{1.035} \times \text{£}21.491,449 =) \text{£}20.764,685$, or 20*l.* 15*s.* 3*d.* $\frac{1}{2}$.

And for a life of 7 years it is $(= \frac{1}{1.035} \times \frac{902}{915} \times \text{£}21.764,685 = \frac{1}{1.035} \times \text{£}21.455,459 =) \text{£}20.729,912$, or 20*l.* 14*s.* 7*d.*

And for a life of 6 years it is $(= \frac{1}{1.035} \times \frac{915}{930} \times \text{£}21.729,912 = \frac{1}{1.035} \times \text{£}21.379,322 =) \text{£}20.666,349$, or 20*l.* 13*s.* 1*d.* $\frac{1}{2}$.

And for a life of 5 years it is $(= \frac{1}{1.035} \times \frac{930}{948} \times \text{£}21.666,349 = \frac{1}{1.035} \times \text{£}21.245,152 =) \text{£}20.526,716$, or 20*l.* 10*s.* 6*d.* $\frac{1}{2}$.

S

And

$$\text{And for a life of 4 years it is } \left(= \frac{1}{1.035} \times \frac{948}{970} \times £21.526,716 \right. \\ \left. = \frac{1}{1.035} \times £21.038,481 = \right) £20.327,034, \text{ or } 20\text{ l. } 6\text{ s. } 6\text{ d. } \frac{1}{3}.$$

$$\text{And for a life of 3 years it is } \left(= \frac{1}{1.035} \times \frac{970}{1000} \times £21.327,034 \right. \\ \left. = \frac{1}{1.035} \times £20.687,222 = \right) £19.987,654, \text{ or } 19\text{ l. } 19\text{ s. } 9\text{ d.}$$

CXVIII. Having now completed the computation of the values of an annuity of one pound a year for every year of human life from the age of 93 years down to the age of 3 years, inclusively, I shall proceed to range them in a regular series or table, and shall express them (as I have hitherto done) both in decimal parts of a pound sterling, and in pounds, shillings, and pence. And, that the rate, or degree, at which they increase, or decrease, in the different periods of human life, may appear the more readily, I shall subjoin to them a table of their differences, expressed likewise in a double manner, to wit, in decimal parts of a pound sterling and in pounds, shillings, and pence. This table is as follows.

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T A B L E III.

Containing the values of an annuity of one pound sterling a year for the lives of persons of all ages from the age of 3 years to the age of 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

Years of Age.	Values of an annuity of one pound, in decimal parts of a pound.	Differences of the said values.	Values of the same annuity, in pounds, shillings, and pence.	Differences of the said values.
3	£ 19.987,654	£ 0.339,380	£ 19 19 9	£ 0 6 9½
4	20.327,034	0.199,682	20 6 6½	0 4 0
5	20.526,716	0.129,633	20 10 6½	0 2 7
6	20.656,349	0.073,563	20 13 1½	0 1 5½
7	20.729,912	0.034,773	20 14 7	0 0 8½
8	20.764,685	0.016,537	20 15 3½	0 0 4
9	20.781,222	0.008,241	20 15 7½	0 0 6½
10	20.752,981	0.007,586	20 15 0¼	0 1 6½
11	20.676,395	0.128,057	20 13 6¼	0 2 6½
12	20.548,338	0.132,439	20 10 11½	0 2 7½
13	20.415,899	0.136,985	20 8 3¾	0 2 8¾
14	20.278,914	0.141,720	20 5 7	0 2 10
15	20.137,194	0.146,579	20 2 9	0 2 11½
		S 2		

<i>Years of Age.</i>	<i>Values of an annuity of one pound, in deci- mal parts of a pound.</i>	<i>Differences of the said va- lues.</i>	<i>Values of the same annuity, in pounds, shil- lings, and pence.</i>	<i>Differences of the said va- lues.</i>
16	£ 19.990,615	£ 0.126,876	£ s. d. 19 19 9½	£ s. d. 0 2 6½
17	19.863,739	0.130,961	19 17 3½	0 2 7½
18	19.732,778	0.135,218	19 14 7½	0 2 8½
19	19.597,560	0.138,657	19 11 11½	0 2 9½
20	19.457,903	0.119,082	19 9 1½	0 2 4½
21	19.338,821	0.122,482	19 6 9½	0 2 5½
22	19.216,339	0.126,021	19 4 4	0 2 6½
23	19.090,318	0.129,705	19 1 9½	0 2 7
24	18.960,613	0.133,543	18 19 2½	0 2 8
25	18.827,070	0.137,542	18 16 6½	0 2 9
26	18.689,528	0.141,711	18 13 9½	0 2 10
27	18.547,817	0.146,058	18 10 11½	0 2 11
28	18.401,759	0.150,592	18 8 0½	0 3 0½
29	18.251,167	0.155,323	18 5 0½	0 3 1½
30	18.095,844	0.160,262	18 1 11	0 3 2½
31	17.935,582	0.165,420	17 18 8½	0 3 5½
32	17.770,162	0.170,808	17 15 2¾	0 3 3
33	17.599,354	0.176,439	17 11 11¾	0 3 6½
34	17.422,915	0.182,327	17 8 5½	0 3 7¾
35	17.240,583	0.188,485	17 4 9½	0 3 9½

LIFE-ANNUITIES.

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Years of Age.	Value of an annuity of one pound, in deci- mal parts of a pound.	Differences of the said va- lues.	Value of the same annuity, in pounds, shil- lings, and pence.	Differences of the said va- lues.
36	£ 17.052,103	£ 0.194,929	£ 17 1 0½	£ 3 10½
37	16.857,174	0.227,985	16 17 1½	4 6½
38	16.629,189	0.236,534	16 12 7	4 8½
39	16.392,655	0.245,488	16 7 10½	4 11
40	16.147,167	0.254,870	16 2 11½	5 1½
41	15.892,297	0.311,358	15 17 10	6 2½
42	15.580,939	0.277,176	15 11 7½	5 6½
43	15.303,763	0.288,094	15 6 1	5 9½
44	15.015,669	0.299,549	15 0 3¾	6 0
45	14.716,120	0.311,571	14 14 3¾	6 2¾
46	14.404,549	0.299,349	14 8 1	5 11½
47	14.105,200	0.311,341	14 2 1½	6 2½
48	13.793,859	0.299,434	13 15 10½	6 0
49	13.494,425	0.311,342	13 9 10½	6 2½
50	13.183,083	0.299,634	13 3 8	6 0
51	12.883,449	0.287,153	12 17 8	5 9
52	12.596,296	0.297,910	12 11 11	5 11½
53	12.298,386	0.309,293	12 5 11½	6 2½
54	11.989,093	0.297,292	11 19 9½	5 11½
55	11.691,801	0.308,374	11 13 10	6 2

Years of Age.	Values of an annuity of one pound, in decimal parts of a hundred pound.	Differences of the said values.	Values of the same annuity, in pounds, shillings, and pence.	Differences of the said values.
56	£ 11.383,427	£ 0.319,942	£ 11 7 8 1.5	£ 0.71 6 4 1
57	11.063,485	0.308,362	11 1 13 7 7 8 2.01	0 6 4 1
58	10.755,123	0.319,557	10 15 6 1 7 8 2.01	0 6 2
59	10.435,566	0.331,492	10 8 8 1 7 8 2.01	0 6 4 1
60	10.104,074	0.344,243	10 2 1 1 7 8 2.01	0 6 7 1
61	9.759,829	0.357,904	9 15 2 7 8 2.01	0 6 10 1
62	9.401,925	0.348,866	9 8 0 1 7 8 2.01	0 6 11 1
63	9.053,059	0.362,411	9 1 0 1 7 8 2.01	0 6 11 1
64	8.690,648	0.377,023	8 13 9 1 7 8 2.01	0 7 3
65	8.313,625	0.369,367	8 6 3 1 7 8 2.01	0 7 6 1
66	7.944,258	0.360,531	7 18 10 1 7 8 2.01	0 7 4 1
67	7.583,727	0.350,028	7 11 8 1 7 8 2.01	0 7 2 1
68	7.233,699	0.337,203	7 4 8 1 7 8 2.01	0 7 0
69	6.896,496	0.321,139	6 17 11 1 7 8 2.01	0 6 8 1
70	6.575,357	0.315,517	6 11 6 1 7 8 2.01	0 6 5 1
71	6.249,840	0.303,868	6 5 0 1 7 8 2.01	0 6 2 1
72	5.945,972	0.301,524	5 18 11 1 7 8 2.01	0 6 1 1
73	5.644,448	0.296,642	5 12 10 1 7 8 2.01	0 5 11
74	5.347,806	0.288,183	5 6 11 1 7 8 2.01	0 5 9 1
75	5.059,623	0.304,702	5 1 2 1 7 8 2.01	0 5 7 1

LIFE-ANNUITIES.

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Years of Age.	Values of an annuity of one pound, in the last parts of a pound.	Differences of the said values.	Values of the same annuity, in pounds, shillings, and pence.	Differences of the said values.
76	4.754,921	0.293,082	4 15 1 $\frac{1}{2}$	0 5 10 $\frac{1}{2}$
77	4.461,839	0.274,081	4 29 2 $\frac{3}{4}$	0 5 10 $\frac{1}{2}$
78	4.187,758	0.247,979	4 43 3 39	0 5 7 39
79	3.908,000	0.246,219	3 18 2	0 4 11 $\frac{1}{2}$
80	3.661,781	0.233,925	3 8 6 $\frac{1}{2}$	0 4 8 39
81	3.427,856	0.212,198	3 4 3 $\frac{3}{4}$	0 4 7 $\frac{1}{2}$
82	3.215,658	0.231,185	2 19 8 $\frac{1}{4}$	0 5 4 $\frac{1}{2}$
83	2.984,473	0.267,285	2 14 4	0 5 2 $\frac{1}{2}$
84	2.717,188	0.260,414	2 4 2 $\frac{3}{4}$	0 4 10 $\frac{3}{4}$
85	2.456,774	0.244,863	2 9 1 $\frac{1}{2}$	0 4 2 $\frac{3}{4}$
86	2.211,911	0.212,101	2 0 0	0 5 5 $\frac{1}{4}$
87	1.999,810	0.271,432	1 14 6 $\frac{3}{4}$	0 5 4 $\frac{1}{2}$
88	1.728,378	0.268,679	1 9 2 $\frac{1}{2}$	0 5 2 $\frac{3}{4}$
89	1.459,699	0.262,187	1 3 11 $\frac{1}{2}$	0 5 0
90	1.197,512	0.249,843	0 18 11 $\frac{1}{2}$	0 4 7 $\frac{1}{2}$
91	0.947.669	0.231,201	0 14 4	0 4 8
92	0.716,468	0.233,378	0 9 8	
93	0.48309			

CXIX. In the foregoing table it may be observed that the difference of the values of an annuity of one pound for the several successive years of age in human life, (which are set down in pounds, shillings, and pence in the fifth column of the said table,) vary gradually, by increasing or decreasing, within the limits of 4 pence on the one hand, and 7 shillings and 6 pence, half-penny, on the other; the least of them, (which is the difference of the said values for the ages of 8 and 9 years,) being 4 pence, and the greatest of them (which is the difference of the said values for the ages of 64 and 65 years,) being 7s. 6d. $\frac{1}{2}$. And, between these limits, they vary, in some places by increasing, and in others by decreasing, (while the years of life increase,) by the small differences of a penny, two pence, or three pence, for the most part, and in a few places by six pence, or seven pence, or, at most, eleven pence, excepting only the beginning of the table, between the ages of 3 and 12 years, in which they decrease (while the years of life increase,) by differences of 2s. 9d. $\frac{1}{2}$, seventeen pence, and 13 pence, halfpenny.

CXX. This observation may be of use in computing other tables of this kind according to other rates of interest, by enabling the computer to detect in good time any error he may happen to have fallen into in the course of his arithmetical operations. Thus, for example, if in computing the value of an annuity for a life of 60 years by means of the expression

$$\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+V} £, \text{ or } \frac{1}{1.035} \times \frac{450}{463} \times \frac{£1 + £9.759,829}{1.035}, \text{ or}$$

$$\frac{1}{1.035} \times \frac{450}{463} \times £10.759,829, \text{ I had found the said value to be}$$

£10.154,074, instead of £10.104,074, that is, 10l. 3s. 1d. instead of 10l. 2s. 1d. I might have soon discovered that I had made a mistake in the calculation, by subtracting from this value the value of the like annuity of one pound for a life of 61 years, which I had before found to be £9.759,829, or 9l. 15s. 2d. $\frac{1}{4}$, and then comparing the remainder, or difference, with the foregoing differences between the values of the same annuity for lives of 61, 62, 63 and 64 years, all which I had before computed. For I should then have found that this last difference, (or the excess of 10l. 3s. 1d. above 9l. 15s. 2d. $\frac{1}{4}$) was 7s. 10d. $\frac{3}{4}$; which exceeds the difference next preceding it, or that between the values of two annuities of one pound each for lives of 61 and 62 years, (which is 7s. 1d. $\frac{3}{4}$) by no less a sum than 9 pence: whereas that difference (7s. 1d. $\frac{3}{4}$) exceeds the next preceding difference (which is 6s. 11d. $\frac{2}{3}$) by only 2 pence; and that difference (6s. 11d. $\frac{2}{3}$) is exceeded by the next preceding difference, to wit, 7s. 3d. by only 3d. $\frac{1}{4}$; and this last difference, (7s. 3d) is exceeded by the next preceding difference (which is 7s. 4d. $\frac{1}{2}$) by only 3d. $\frac{1}{2}$. Now each of these three differences of the differences

differences set down in the table, to wit, $2d. 3d. \frac{1}{2}$, and $3d. \frac{1}{2}$, is much less than 9 pence, which is the excess of the new difference, $7s. 10d. \frac{2}{3}$, above the next preceding difference, $7s. 1d. \frac{1}{2}$. I should therefore, upon observing the uncommon magnitude of this last-mentioned excess, have been led to suspect that the last-found difference, $7s. 10d. \frac{2}{3}$, was greater than the truth, and consequently that the number found for the value of an annuity of one pound for a life of 60 years, to wit, $10l. 3s. 1d.$ or $\pounds 10.154,074$, was too great. And upon this suspicion, to arising from the comparison of the said differences in the fifth column of the foregoing table with each other, I should have revised the calculation of the value of the said annuity by means of the expression $\frac{1}{1.035} \times \frac{450}{463} \times \pounds 10.759,829$, and should have discovered my mistake, and have found that the true value of the said annuity for a life of 60 years was not $\pounds 10.154,074$, but $\pounds 10.104,074$, or $10l. 2s. 1d.$. And this number there is no reason to suspect of being erroneous, because it exceeds the value of the like annuity for a life of 61 years (which is $9l. 15s. 2d. \frac{1}{2}$) by only $6s. 10d. \frac{2}{3}$, which is exceeded by the next preceding difference, to wit, $7s. 1d. \frac{1}{2}$, by only 3 pence. And in fact I did in this manner discover and set right some mistakes I had fallen into in computing the foregoing table.

CXXI. There is, however, another and more accurate method of discovering the errors that may have been made in computing such tables of the values of life-annuities by means of the expression above-mentioned, as soon as they arise, and consequently of confirming the truth of the computations, when they have been rightly performed. But it requires considerably more labour than is necessary to the making of such a table of differences as is above set down. Yet its usefulness in ascertaining the truth of these computations is so great, that, I think, all persons who undertake to compute tables of the values of life-annuities by means of the expression above-mentioned, would do well to undergo that labour, and to examine and correct their calculations by it. It is given, and explained, and illustrated by examples, by Mr. William Morgan, actuary to the Society of Equitable Assurances on Lives and Survivorships, in the 58th, 59th, 60th, &c.—73d, and the 212th and 213th, pages of his useful and learned treatise, intitled, *The Doctrine of Annuities and Assurances on Lives and Survivorships, stated and explained*; which was published in octavo in the year 1779, and printed for T. Cadell in the Strand. And I propose to make farther mention of it in the subsequent part of this discourse.

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CXXII. But,

Of certain irregularities in the increase and decrease of the differences in the foregoing table.

CXXII. But, though the observation made in Art. 119 concerning the differences set down in the fifth column of the foregoing table, to wit, that they increase, or decrease, in different parts of the table, by small and almost equal increments or decrements, be true in general, yet it is liable to several exceptions. For it may also be observed, that, in several places in the said table, the said differences, after having increased or decreased almost regularly, or by nearly equal increments or decrements, for several years together, do, on a sudden, increase, or decrease, by a much larger increment, or decrement, than before; after which they again increase, or decrease, as before, in a nearly regular manner, or by nearly equal increments, or decrements, for several more years together; and then again receive an uncommonly large increment, or decrement. Of this it will be proper to give some examples: and in examining these differences with this view, it will be most convenient to proceed in the order in which the values of the annuities set down in the foregoing table were computed, that is, from the older lives to the younger.

Now, if we begin this examination from the age of 41 years and continue it upwards to the age of 19 years, we shall find the variations of these differences to be as follows.

From the age of 41 years to the age of 37 years these differences are $5s. 1d. \frac{1}{4}$, $4s. 11d.$, $4s. 8d. \frac{1}{2}$, and $4s. 6d. \frac{1}{4}$; which differ from each other by only $2d. \frac{1}{4}$, $2d. \frac{1}{4}$ and $2d.$: and then, from the age of 37 years to the age of 36 years, the difference is $3s. 10d. \frac{1}{4}$; which is less than the next preceding difference, $4s. 6d. \frac{1}{2}$, by the much larger sum of 8 pence, instead of $2d.$ or $2d. \frac{1}{4}$. After this, from the age of 36 years to the age of 32 years, the differences are $3s. 9d. \frac{1}{4}$, $3s. 7d. \frac{1}{4}$, $3s. 6d. \frac{1}{4}$, and $3s. 3d.$, which differ from the next preceding difference, $3s. 10d. \frac{1}{4}$, and from each other by the small sums of $1d. \frac{1}{2}$, $1d. \frac{1}{2}$, $1d. \frac{1}{2}$, and $3d. \frac{1}{4}$. And thus far, that is, from the age of 41 years to the age of 32 years, the said differences have continually decreased from $5s. 1d. \frac{1}{4}$ to $3s. 3d.$. But here they increase for one year, to wit, from the age of 32 years to the age of 31 years, and then decrease again for the next 11 years, or from the age of 31 years to the age of 20 years, and then the said difference increases again from the age of 20 years to the age of 19 years. From the age of 32 years to the age of 31 years the said difference increases from $3s. 3d.$ to $3s. 5d. \frac{1}{4}$, and therefore differs from the value it would have had if it had gone on decreasing as before, (which would have been about $3s.$) by about $5d. \frac{1}{4}$. And from the age of 31 years to the age of 20 years, the said differences are $3s. 2d. \frac{1}{2}$, $3s. 1d. \frac{1}{4}$, $3s. 0d. \frac{1}{4}$, $2s. 11d.$, $2s. 10d.$, $2s. 9d.$, $2s. 8d.$, $2s. 7d.$, $2s. 6d. \frac{1}{2}$, $2s. 5d. \frac{1}{4}$, and $2s. 4d. \frac{1}{2}$; which decrease continually, and differ from the next preceding difference, $3s. 5d. \frac{1}{4}$, and from each other, by the moderate decrements of $3d. \frac{1}{4}$ from

$1d.\frac{1}{2}$, $1d.$ $1d.\frac{1}{2}$, $1d.$ $1d.$ $od.\frac{1}{2}$, $1d.\frac{1}{2}$, and $od.\frac{1}{2}$. And, from the age of 20 years to the age of 19 years, the said difference (instead of decreasing, as it had done for the preceeding 11 years;) increases from 2s. $4d.\frac{1}{2}$ to 2s. $9d.\frac{1}{2}$, or by an increment of $5d.\frac{1}{2}$; and therefore is greater than it would have been if it had gone on decreasing as before, by about 6 pence, or 7 pence.

And the like irregularities, or inequalities, in the variation of these differences may be observed in other parts of the said table.

CXXIII. This irregularity seems, at first sight, strange and unaccountable, and (agreeably to what has been observed in Art. 119, 120) may give room to a suspicion that the values which increase, or decrease, by these uncommonly large increments, or decrements, have been erroneously computed. And yet I am persuaded, from the care I took in the computation of them, that they are not erroneous. But, I believe, it will be generally found that this irregularity is owing to the variations in the decrease of the numbers of persons who are represented, in Monsieur de Parcieux's table above-mentioned, as living at the ends of the several successive years of human life. For it is remarkable that, in several parts of that table, the number of the persons who die in the space of a year continues the same for several years together, and then changes to some greater, or lesser, number. Thus, for example, from the age of 11 years to the age of 16 years 6 persons are represented in the said table as dying every year; the numbers of persons represented there as living at the ages of 11, 12, 13, 14, 15, and 16, years being 872, 866, 860, 854, 848, and 842, respectively; which numbers differ from each other by 6. And, in like manner, from the age of 16 years to the age of 20 years the number of those who die every year is 7; the numbers of persons represented to be living at the several ages of 16, 17, 18, 19, and 20, years being 842, 835, 828, 821, and 814, respectively; which numbers differ from each other by 7. And afterwards, for 17 years together, from the age of 20 years to the age of 37 years, the number of those who die every year is 8; the numbers of persons represented as living at the age of 20 years and the several subsequent ages of 21, 22, 23, 24, 25, &c. years, up to 37 years, being 814, 806, 798, 790, 782, 774, 766, 758, 750, 742, 734, 726, 718, 710, 702, 694, 686, and 678; which all differ from each other by 8. And for the next 9 years, or from the age of 37 years to the age of 46 years, the number of those who die every year is again only 7; the numbers of persons represented in the said table as living at the ages of 37, 38, 39, 40, 41, 42, 43, 44, 45, and 46, years being 678, 671, 664, 657, 650, 643, 636, 629, 622, and 615, respectively; which differ from each other by 7. And the same thing may be observed in other

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larities.

parts of Monsieur de Parcieux's said table, and likewise in Mr. Kerseboom's table above-mentioned and in Dr. Halley's Bressaw table, and in all other tables of the probabilities of the duration of human life that have ever been published.

Now it will be found that the extraordinary decrements and increments above-mentioned in the differences which are set down in the fifth column of the foregoing table, happen, for the most part, in those years of human life in which the number of persons who die in the said years, increases or decreases, or varies from the number of those who died in the year next preceding. Thus, for example, we have observed that, from the age of 41 years to the age of 37 years, the said differences decrease from 5s. 1d. $\frac{1}{2}$ to 4s. 6d. $\frac{3}{4}$ by the moderate decrements of 2d. $\frac{1}{2}$, 2d. $\frac{1}{2}$, and 2d. $\frac{1}{2}$; and during the same period we find in Monsieur de Parcieux's table of probabilities, that the number of those who die every year is always the same, to wit, 7. But in the next year, or from the age of 37 years to the age of 36 years, the corresponding difference in the fifth column of the aforesaid table is 3s. 10d. $\frac{3}{4}$; which is less than the next preceding difference, 4s. 6d. $\frac{3}{4}$, by the much greater sum of 8 pence: and in this same year the number of persons that die in the course of a year (which from the age of 46 years to the age of 37 years had been constantly 7 in a year,) is increased to 8, which occasions an uncommon increase in the value of d in the denominator of the fraction $\frac{P}{P+d}$ in the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+\sqrt{1.035}}$ which expresses the value of the next younger life. By this uncommon increase of d the whole denominator $P+d$ receives also a greater increase than usual, and consequently the whole fraction $\frac{P}{P+d}$, and therefore also the whole expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+\sqrt{1.035}}$, becomes less than it would have been if d , or the number of persons dying in the year, had not increased to the number 8, but had continued to be 7, as it had been for the foregoing 9 years, or from the age of 46 years to the age of 37 years. Therefore the value of the annuity of one pound for a life of 30 years (which is equal to the said expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+\sqrt{1.035}}$) will be less than it would have been if d had continued to be equal to 7 instead of becoming equal to 8. And hence it comes to pass that the excess of this value above the value of the like annuity for a life of 37 years, is less than it would have been if d had continued to be equal to 7, or that the difference 3s. 10d. $\frac{3}{4}$ is less than it would have been, if d had continued to be equal to 7, and consequently that it falls short of the preceding difference, 4s. 6d. $\frac{3}{4}$, by the uncommonly large decrement of 8 pence.

And

And in the same manner we have seen that from the age of 31 years to the age of 20 years the said differences continually decrease from $35.5d.\frac{1}{2}$ to $25.4d.\frac{1}{2}$ by the moderate decrements of $3d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, and $0d.\frac{1}{2}$; and during the same period of time we find in Monsieur de Parcieux's table of probabilities, that the number of those who die every year (and which is denoted by the letter d in the denominator of the fraction $\frac{P}{P+d}$ in the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times |1+V|K$) is always the same, to wit, 8. But in the next year, or from the age of 20 years to the age of 19 years, the corresponding difference in the fifth column of the aforesaid table is $25.9d.\frac{1}{2}$, which is, not only not less than the preceding difference, $25.4d.\frac{1}{2}$, by about a penny, (as might have been expected from a view of the aforesaid decrements $3d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, and $0d.\frac{1}{2}$ of the next preceding differences,) but exceeds it by $5d.\frac{1}{2}$, and consequently is greater than might have been expected by about 6 pence: and in this same year, or from the age of 20 years to the age of 19 years, the number of persons who die in the course of a year (and which from the age of 37 years to the age of 20 years had constantly been 8 in a year) is diminished to 7; which occasions a less diminution than usual in the denominator of the fraction $\frac{P}{P+d}$ in the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times |1+V|K$, by which every new value is derived from that of the annuity for the next preceding life which is one year older. Hence it comes to pass that the said expression $\frac{1}{1.035} \times \frac{P}{P+d}$ $\times |1+V|K$, (which is equal to the value of an annuity for a life of 19 years,) is greater than it would have been if d had not decreased to 7, but had continued to be equal to 8, as it had been for several years before; and consequently the said value of an annuity for a life of 19 years exceeds the value of an annuity for a life of 20 years by a greater difference than it would have done if d had continued to be equal to 8, and the said difference, to wit, $25.9d.\frac{1}{2}$, instead of being less than the preceding difference $25.4d.\frac{1}{2}$, by about a penny, exceeds it by $5d.\frac{1}{2}$.

CXXIV. We have seen in the foregoing article that the uncommonly great increments and decrements of the aforesaid differences, (which are set down in the fifth column of the foregoing table,) are often occasioned by correspondent changes in the number of persons who are represented in Monsieur de Parcieux's table of probabilities as dying in a year, or in the number

number d in the denominator of the fraction $\frac{P}{P+d}$ in the expression

$\frac{1}{1.035} \times \frac{P}{P+d} \times \sqrt{1+\frac{d}{P}} \times k$. But they also sometimes happen without

any such change in the value of d . Thus, for example, from the age of 36 years to the age of 32 years, the differences in the said fifth column are $35.9d.\frac{1}{2}$, $35.7d.\frac{1}{2}$, $35.6d.\frac{1}{2}$ and $35.3d$. which all decrease by the moderate decrements of $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, and $3d.\frac{1}{2}$. But in the next year, or from the age of 32 years to the age of 31 years, the difference is $35.5d.\frac{1}{2}$; which is not only not less than the next preceding difference, $35.3d$, by about $3d$. (as might have been expected from the course of the foregoing decrements,) but exceeds it by the sum of $2d.\frac{1}{2}$, and therefore is greater than it might have been naturally expected to be by about $5d.\frac{1}{2}$, or 6 pence. And yet the number of persons who die between the ages of 32 and 31 years is the same with that of those who die between the ages of 33 and 32 years, and between the ages of 34 and 33 years, and of 35 and 34 years, and of 36 and 35 years, and of 37 and 36 years, to wit, 8. How to account for this, and the like uncommonly great variations in the said difference, when there is no correspondent change in the number of persons represented in the table of probabilities as dying in the space of a year, I do not know. But thus much, I think, we may safely conclude, to wit, that, as it often happens that an uncommonly large decrement, or increment, of the differences in the said fifth column of the foregoing table, is occasioned by a change in the number of persons dying in the year, corresponding to such difference, there will always be reason, whenever such uncommonly large decrement, or increment, of the said differences is observed, and there is no change in the number of persons dying in the corresponding year of human life, to occasion it;—I say, there will always be reason in these cases to suspect that some error has crept into the calculation, and it will therefore be prudent to re-examine the operations of it with great care.

Of the decrements of human life.

CXXV. The numbers of persons represented, in a table of the probabilities of the duration of human life, as dying every year, are called by many writers on this subject *the decrements of human life*. And, because these decrements continue the same for several years together, in several different periods of human life, it is said by these writers that human life wastes, or *decreases, uniformly*, for several years together, in several of its stages, or periods; the numbers of the persons living at the ends of the several years in each of the said stages, or periods, (if reckoned from the younger ages to the older,) forming a regular, decreasing, arithmetical progression. Thus, for example, the numbers of persons living at the several

Several ages of 20 years, 21 years, 22 years, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, and 37, years according to Monsieur de Parcieux's table of probabilities, form the following regular, decreasing, arithmetical progression, in which the common difference of the terms is 8, to wit, $814 + 806 + 798 + 790 + 782 + 774 + 766 + 758 + 750 + 742 + 734 + 726 + 718 + 710 + 702 + 694 + 686 + 678$, or $814 + 814 - 8, + 814 - 2 \times 8, + 814 - 3 \times 8, + 814 - 4 \times 8, + 814 - 5 \times 8, + 814 - 6 \times 8, + 814 - 7 \times 8, + 814 - 8 \times 8, + 814 - 9 \times 8, + 814 - 10 \times 8, + 814 - 11 \times 8, + 814 - 12 \times 8, + 814 - 13 \times 8, + 814 - 14 \times 8, + 814 - 15 \times 8, + 814 - 16 \times 8, + 814 - 17 \times 8$.

CXXVI. To make the course of these decrements in the different periods of human life more apparent, it will be convenient to present again to the reader's view the above-mentioned tables of probabilities of Mr. Kerfeboom and Monsieur de Parcieux, and to set down in a separate column, adjoining to that which exhibits the numbers of the persons living at the ends of the several successive years of human life in each table, the numbers of the persons who have died in the next preceding year. These tables will then be as follows.

TABLE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	

TABLE IV.

Being the foregoing table of probabilities of Mr. Kerssboom, above set down in page 4, together with the numbers of the persons dying in every year, or the decrements of human life, set down in an adjoining column.

Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.	Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.
0	1400	275	15	856	7
1	1125	50	16	849	7
2	1075	45	17	842	7
3	1030	37	18	835	9
4	993	29	19	826	9
5	964	17	20	817	9
6	947	17	21	808	8
7	930	17	22	800	8
8	913	9	23	792	9
9	904	9	24	783	11
10	895	9	25	772	12
11	886	8	26	760	13
12	878	8	27	747	12
13	870	7	28	735	12
14	863	7	29	723	12

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Years of Age.	Persons living.	Persons dying in a year, or the decre- ments of hu- man life.	Years of Age.	Persons living.	Persons dying in a year, or the decre- ments of hu- man life.
30	711	12	50	507	12
31	699	12	51	495	13
32	687	12	52	482	12
33	675	10	53	470	12
34	665	10	54	458	12
35	655	10	55	446	12
36	645	10	56	434	13
37	635	10	57	421	13
38	625	10	58	408	13
39	615	10	59	395	13
40	605	9	60	382	13
41	596	9	61	369	13
42	587	9	62	356	13
43	578	9	63	343	14
44	569	9	64	329	14
45	560	10	65	315	14
46	550	10	66	301	14
47	540	10	67	287	14
48	530	8	68	273	14
49	518	11	69	259	14

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<i>Years of Age.</i>	<i>Persons living.</i>	<i>Persons dying in a year, or the decrements of human life.</i>	<i>Years of Age.</i>	<i>Persons living.</i>	<i>Persons dying in a year, or the decrements of human life.</i>
70	245	14	86	36	8
71	231	14	87	28	7
72	217	14	88	21	6
73	203	14	89	15	5
74	189	14	90	10	3
75	175	15	91	7	2
76	160	15	92	5	2
77	145	15	93	3	1
78	130	15	94	2	1
79	115	15	95	1	0.4
80	100	13	96	0.6	0.1
81	87	12	97	0.5	0.1
82	75	11	98	0.4	0.2
83	64	9	99	0.2	0.2
84	55	10	100	0.0	
85	45	9			

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T A B L E V.

Being the foregoing table of probabilities of Monsieur de Parcieux, above set down in page 6, together with the numbers of the persons dying in every year, or the decrements of human life, set down in an adjoining column.

Years of Age.	Persons living	Persons dying in a year, or the decre- ments of hu- man life.	Years of Age.	Persons living.	Persons dying in a year, or the decre- ments of hu- man life.
3	1000	30	18	828	7
4	970	22	19	821	7
5	948	18	20	814	8
6	930	15	21	806	8
7	915	13	22	798	8
8	902	12	23	790	8
9	890	10	24	782	8
10	880	8	25	774	8
11	872	6	26	766	8
12	866	6	27	758	8
13	860	6	28	750	8
14	854	6	29	742	8
15	848	6	30	734	8
16	842	7	31	726	8
17	835	7	32	718	8

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The Principles of the Doctrine of

<i>Years of Age.</i>	<i>Persons living.</i>	<i>Persons dying in a year, or the decrement of human life.</i>	<i>Years of Age.</i>	<i>Persons living.</i>	<i>Persons dying in a year, or the decrement of human life.</i>
33	710	8	52	560	11
34	702	8	53	549	11
35	694	8	54	538	11
36	686	8	55	526	12
37	678	7	56	514	12
38	671	7	57	502	12
39	664	7	58	489	13
40	657	7	59	476	13
41	650	7	60	463	13
42	643	7	61	450	13
43	636	7	62	437	13
44	629	7	63	423	14
45	622	7	64	409	14
46	615	8	65	395	15
47	607	8	66	380	16
48	599	9	67	364	17
49	590	9	68	347	18
50	581	10	69	329	19
51	571	11	70	310	19

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Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.	Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.
71	291	20	84	59	11
72	271	20	85	48	10
73	251	20	86	38	9
74	231	20	87	29	7
75	211	19	88	22	6
76	192	19	89	16	5
77	173	19	90	11	4
78	154	18	91	7	3
79	136	18	92	4	2
80	118	17	93	2	1
81	101	16	94	1	1
82	85	14	95	0	
83	71	12			

A remark

A remark on the decrements of human life in the foregoing tables of Mr. Kerseboom and Monsieur de Parcieux.

CXXXVII. It appears from the foregoing table of Mr. Kerseboom that in several different periods of human life, from the age of 12 years to the age of 80 years, the decrements of life, or the numbers of persons dying in a year, continue the same for five, six, or seven, years together, and in one period for twelve years together, namely, from the age of 63 years to the age of 75 years, in each of which years the number of persons dying is 14. And it appears in like manner from the foregoing table of Monsieur de Parcieux, that in several parts of human life, from the age of 11 years to the age of 80 years, the decrements of life, or the numbers of persons dying in a year, continue the same for four, five, or eight, years together, and in one period for no less than seventeen years together, namely, from the age of 20 years to the age of 37 years, in each of which years the number of persons dying is 8. Now by the help of this observation we may in some degree abridge the labour of computing the values of life-annuities, when those values are not derived one from the other by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times |1 + \sqrt{V}| \text{ £}$, but are computed separately in the manner described in Art. 86 and exemplified in Art. 94 by the computation of the value of an annuity of one pound for a life at the age of 10 years. For it is there shewn that the value of an annuity of one pound a year during n years, in case a person aged N years shall so

long live, is equal to $\frac{1}{P} \times$ the series $\frac{P^1}{r} + \frac{P^{11}}{r^2} + \frac{P^{111}}{r^3} + \frac{P^{1111}}{r^4} + \frac{P^{11111}}{r^5} + \frac{P^{111111}}{r^6} + \frac{P^{1111111}}{r^7} + \text{&c.}$ continued to n terms, or to the term $\frac{P^n}{r^n}$.

Now this series may be divided into several different serieses, in some of which the numerators of the terms will (as appears from the foregoing observation) decrease by equal differences, or form arithmetical progressions. And where this happens, the said lesser serieses may be summed in a more easy and expeditious manner than by, first, computing the values of every single term in them, and then adding them all together into one sum, as was done in Art. 94. For the denominators of the terms of these serieses are the terms of a geometrical progression, and decrease continually in the proportion of $\frac{1}{r}$ to 1, or of 1 to r : and where the terms of a series consist of fractions whose numerators form an arithmetical, and their denominators a geometrical, progression, the sum of the terms of such a series may be found in a compendious manner, without an actual computation and addition of them all together, by a method not unlike that explained above in Art. 80 and 81 for finding the sum of the terms of a simple

simple geometrical progression, (such as $\frac{1}{r^0} + \frac{1}{r^1} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$

$+ \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$) and which is derived from that method.

The manner of deriving such a method of summing these compound series, consisting of terms whose numerators form an arithmetical, and denominators a geometrical, progression, from the aforesaid method of summing a simple geometrical progression, I will now endeavour to explain.

Of the summation of a series of fractions, consisting of a given number of terms, whose numerators form a decreasing arithmetical progression, and denominators an increasing geometrical progression.

CXXVIII. Let $a, +a-d, +a-2d, +a-3d, +a-4d, +a-5d, +a-6d, +\&c.$ be a decreasing arithmetical progression, consisting of any number of terms denoted by the letter n . Then, it is evident, its last term will be $a-[n-1] \times d$. or $a-[nd-d]$, or $a-nd+d$.

In the next place let r be any quantity greater than 1, and let $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ be a geometrical progression consisting likewise of n terms, which, it is evident, will decrease in the common ratio of 1 to r . The last term of this series will evidently be $\frac{1}{r^n}$.

Lastly, let the terms of this latter, or geometrical, progression, $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$ be multiplied into the correspondent terms of the arithmetical progression, $a, +a-d, +a-2d, +a-3d, +a-4d, +a-5d, +a-6d, +\&c. +a-nd+d$; and we shall thereby form a third series, which will be as follows, to wit, $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} + \&c. + \frac{a-nd+d}{r^n}$. Now the sum of the

terms of a compound series of this form may be obtained without actually computing every separate term in it and then adding them all up together, by a method derived from that given in Art. 80 for finding the sum of the terms of the geometrical progression $A+B+C+D+E$. This method may be explained as follows.

the series $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} + \frac{a}{r^8} + \text{&c.} + \frac{1}{r^n}$
is $= \frac{a}{r-1} \times \frac{1}{1-\frac{1}{r^n}}$. QEI.

CXXXI. And, in like manner, if the same expression $\frac{AA-BE}{A-B}$

be applied to the second series $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \frac{d}{r^8} + \frac{d}{r^9} + \text{&c.}$

+ &c. $+\frac{d}{r^n}$, we shall have $A = \frac{d}{r^2}$, and $B = \frac{d}{r^3}$, and $E =$

$\frac{d}{r^n}$; and consequently $AA = \frac{dd}{r^4}$, and $BE = \frac{dd}{r^5}$, and $AA =$

$BE = \frac{dd}{r^4} - \frac{dd}{r^5} = \frac{r^5 dd - r^4 dd}{r^9} = \frac{r^4 dd}{r^9}$; and $A-B =$

$= \frac{d}{r^2} - \frac{d}{r^3} = \frac{r^3 d - r^2 d}{r^5} = \frac{rd - d}{r^3}$, and $\frac{AA-BE}{A-B} =$

$\frac{r^4 dd - dd}{rd - d} = \frac{r^4 dd - dd}{rd - d} \times \frac{r^3}{r^3} = \frac{r^7 dd - dd}{rd - d} = \frac{r^6 dd}{rd - d} = \frac{r^6 dd}{r^3} = \frac{r^3 dd}{1}$

$\times \frac{r^{n-1}-1}{r^n} = \frac{d}{r-1} \times \frac{1}{r} - \frac{1}{r^n}$. Therefore the sum of all the

terms of the said second series, $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \frac{d}{r^8} + \frac{d}{r^9} + \text{&c.}$

+ &c. $+\frac{d}{r^n}$, is $= \frac{d}{r-1} \times \frac{1}{r} - \frac{1}{r^n}$. QEI.

CXXXII. The third series $\frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \frac{d}{r^8} + \frac{d}{r^9} + \frac{d}{r^{10}} + \text{&c.}$

$+\frac{d}{r^n}$ is equal to the second series $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \frac{d}{r^8} + \frac{d}{r^9} + \text{&c.}$

$+\frac{d}{r^7} + \text{&c.} + \frac{d}{r^n}$ without the first term $\frac{d}{r^2}$, and therefore is $=$

$\frac{d}{r-1} \times \frac{1}{r} - \frac{1}{r^n}$

$= \frac{d}{r-1} \times \frac{1}{r} - \frac{1}{r^n}$

$= \frac{d}{r-1} \times \frac{1}{r} - \frac{1}{r^n}$

$$\begin{aligned} \frac{d}{r-1} \times \left[\frac{1}{r} - \frac{1}{r^n} - \frac{d}{r^2} \right] &= \frac{r^{n-1}d-d}{r^n \times r-1} - \frac{d}{r^2} = \frac{r^{n-1}d-d}{r^n \times r-1} \\ - \frac{r^{n-2} \times r-1 \times d}{r^n \times r-1} &= \frac{r^{n-1}d-d}{r^n \times r-1} - \frac{r^{n-2} \times r d-d}{r^n \times r-1} = \frac{r^{n-1}d-d}{r^n \times r-1} \\ - \frac{r^{n-1}d+r^{n-2}d}{r^n \times r-1} &= \frac{r^{n-2}d-d}{r^n \times r-1} = \frac{d}{r-1} \times \frac{r^{n-2}-1}{r^n} = \frac{d}{r-1} \\ \left[\frac{1}{r} - \frac{1}{r^n} \right] \cdot \text{Q.E.I.} \end{aligned}$$

CXXXIII. And the fourth series, $\frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c.$
 $+\frac{d}{r^n}$, is, in like manner, equal to the third series $\frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6}$
 $+\frac{d}{r^7} + \frac{d}{r^8} + \&c.$ without its first term $\frac{d}{r^3}$, and therefore
 $\frac{d}{r-1} \times \left[\frac{1}{r} - \frac{1}{r^n} - \frac{d}{r^2} \right] = \frac{r^{n-2}d-d}{r^n \times r-1} - \frac{d}{r^2} = \frac{r^{n-2}d-d}{r^n \times r-1}$
 $- \frac{r^{n-3} \times r-1 \times d}{r^n \times r-1} = \frac{r^{n-2}d-d}{r^n \times r-1} - \frac{r^{n-3} \times r d-d}{r^n \times r-1} = \frac{r^{n-2}d-d}{r^n \times r-1}$
 $- \frac{r^{n-2}d+r^{n-3}d}{r^n \times r-1} = \frac{r^{n-3}d-d}{r^n \times r-1} = \frac{d}{r-1}$
 $\times \frac{r^{n-3}-1}{r^n} = \frac{d}{r-1} \times \left[\frac{1}{r} - \frac{1}{r^n} \right] \cdot \text{Q.E.I.}$

CXXXIV. And by proceeding in the same manner we shall find that
the fifth series, $\frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \frac{d}{r^8} + \&c.$ is $= \frac{d}{r-1} \times$
 $\left[\frac{1}{r} - \frac{1}{r^n} \right]$ and that the sixth series, $\frac{d}{r^6} + \frac{d}{r^7} + \frac{d}{r^8} + \frac{d}{r^9} + \&c.$ is $=$
 $\frac{d}{r-1} \times \left[\frac{1}{r^2} - \frac{1}{r^n} \right]$ and that the seventh series, $\frac{d}{r^7} + \frac{d}{r^8} + \frac{d}{r^9} + \&c.$
 $+\frac{d}{r^n}$

$+\frac{d}{r_n}$, is $=\frac{d}{r-1} \times \left| \frac{1}{r^6} - \frac{1}{r^n} \right|$, and that, in general, the m th term,

$$\frac{d}{r^m} + \frac{d}{r^{m+1}} + \dots + \frac{d}{r^n}$$
 is $=\frac{d}{r-1} \times \left| \frac{1}{r^{m-1}} - \frac{1}{r^n} \right|$.

CXXXV. Consequently the value of all the serieses in Art. 129, except the first, that is, of all the serieses whose terms are marked with the sign — and are therefore to be subtracted from the first, or upper,

$$\begin{aligned} \text{series } \frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} + \&c. + \frac{a}{r^n} \\ \text{will be } &= \frac{d}{r-1} \times \left| \frac{1}{r} - \frac{1}{r^n} \right| + \frac{d}{r-1} \times \left| \frac{1}{r^2} - \frac{1}{r^n} \right| + \frac{d}{r-1} \times \left| \frac{1}{r^3} - \frac{1}{r^n} \right| \\ &+ \frac{d}{r-1} \times \left| \frac{1}{r^4} - \frac{1}{r^n} \right| + \frac{d}{r-1} \times \left| \frac{1}{r^5} - \frac{1}{r^n} \right| + \frac{d}{r-1} \times \left| \frac{1}{r^6} - \frac{1}{r^n} \right| \\ &+ \&c. \text{ continued to } n-1 \text{ terms, and consequently is } = \frac{d}{r-1} \times \text{the} \end{aligned}$$

geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \&c.$
continued to $n-1$ terms, or to the term $\frac{1}{r^{n-1}}$, $-\frac{d}{r-1} \times \frac{n-1}{r} \times \frac{1}{r}$
 $= \frac{d}{r-1} \times$ the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$
 $+ \&c. + \frac{1}{r^{n-1}}$, $-\frac{d}{r-1} \times \frac{n-1}{r^n}$. Now it has been shown above

in Art. 81, page 92, that the sum of all the terms of the geometrical series

$$1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \&c.$$

continued to the term

$\frac{1}{r^n}$ is $\frac{1}{r-1} - \frac{1}{r^n \times r-1}$. Therefore the sum of all the terms of the same geometrical series continued to the term $\frac{1}{r^{n-1}}$, or to one term less

than before, will be =

$$\frac{1}{r^n \times r-1} = \frac{1}{r-1} - \frac{r+1}{r^n \times r-1} = \frac{1}{r-1} - \frac{r}{r^n \times r-1}$$

$$= \frac{1}{r-1} - \frac{1}{r^{n-1} \times r-1}. \text{ Therefore } \frac{d}{r-1} \times \text{the geometrical series}$$

$$\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \&c. + \frac{1}{r^{n-1}} \text{ is } =$$

$$\frac{d}{r-1} \times \frac{1}{r-1} - \frac{d}{r-1} \times \frac{1}{r^{n-1} \times r-1}. \text{ Consequently the sum of all the}$$

series in Art. 129, except the first series, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4}$

$$+ \&c. + \frac{a}{r^n} \text{ is } = \frac{d}{r-1} \times \frac{1}{r-1} - \frac{d}{r-1} \times \frac{1}{r^{n-1} \times r-1} - \frac{d}{r-1} \times \frac{n-1}{r^n}$$

$$= \frac{d}{r-1} - \frac{d}{r^{n-1} \times r-1} - \frac{nd+d}{r^n \times r-1}. \text{ But it has been shewn above,}$$

in Art. 130, that the sum of all the terms of the first, or highest, series,

$$\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} + \&c. + \frac{a}{r^n},$$

$$\text{is } = \frac{a}{r-1} \times \left| \frac{1-\frac{1}{r^n}}{1-\frac{1}{r}} \right|. \text{ Therefore the excess of this first series above the}$$

$$\text{sum of all the rest is } = \frac{a}{r-1} \times \left| \frac{1-\frac{1}{r^n}}{1-\frac{1}{r}} - \frac{d}{r-1} \right| + \frac{d}{r^{n-1} \times r-1},$$

$$+ \frac{nd-d}{r^n \times r-1}. \text{ Therefore the sum of all the terms of the compound series}$$

$$\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7}$$

$$+ \&c. + \frac{a-nd+d}{r^n}, \text{ (which is equal to the said excess of the said first}$$

series in Art. 129 above the sum of all the rest,) is also equal to

$$\frac{a}{r-1} \times \left| \frac{1-\frac{1}{r^n}}{1-\frac{1}{r}} - \frac{d}{r-1} \right| + \frac{d}{r^{n-1} \times r-1} + \frac{nd-d}{r^n \times r-1}. \text{ Q.E.I.}$$

An example of the summation of a compound series of this form, to wit,

$$\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} + \text{\textit{\&c.}} \text{ continued to } n \text{ terms, or to the term } \frac{a-nd+d}{r^n},$$

by means of the expression $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$; in which the

capital letter G is put for $\frac{1}{r-1} \times \frac{1}{r^n \times r-1}$, or the sum of the

terms of the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6}$

$+ \frac{1}{r^7} + \frac{1}{r^8} + \text{\textit{\&c.}}$ continued to n terms, or to the term $\frac{1}{r^n}$.

CXXXVIII. The tenth, eleventh, twelfth, and other following terms

of the series $\frac{P^1}{r} + \frac{P^2}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{12}}{r^4} + \frac{P^v}{r^5} + \frac{P^{vi}}{r^6} + \frac{P^{vii}}{r^7}$

+ &c. in Art. 94, as far as the 27th term, or $\frac{P_{xxvii}}{r^{27}}$, are as follows,

$$\begin{aligned} & \text{to wit, } \frac{814}{r^{10}} + \frac{806}{r^{11}} + \frac{798}{r^{12}} + \frac{790}{r^{13}} + \frac{782}{r^{14}} + \frac{774}{r^{15}} + \frac{766}{r^{16}} \\ & + \frac{758}{r^{17}} + \frac{750}{r^{18}} + \frac{742}{r^{19}} + \frac{734}{r^{20}} + \frac{726}{r^{21}} + \frac{718}{r^{22}} + \frac{710}{r^{23}} + \frac{702}{r^{24}} \\ & + \frac{694}{r^{25}} + \frac{686}{r^{26}} + \frac{678}{r^{27}}, \text{ in which the numerators 814, 806, 798,} \end{aligned}$$

790, &c. decrease continually by the same common difference, 8. This series is equal to the product of $\frac{1}{r^9} \times$ the series $\frac{814}{r} + \frac{806}{r^2} + \frac{798}{r^3}$

$$\begin{aligned} & + \frac{790}{r^4} + \frac{782}{r^5} + \frac{774}{r^6} + \frac{766}{r^7} + \frac{758}{r^8} + \frac{750}{r^9} + \frac{742}{r^{10}} + \frac{734}{r^{11}} \\ & + \frac{726}{r^{12}} + \frac{718}{r^{13}} + \frac{710}{r^{14}} + \frac{702}{r^{15}} + \frac{694}{r^{16}} + \frac{686}{r^{17}} + \frac{678}{r^{18}}; \text{ and} \end{aligned}$$

this

this last series is of the same form with the series $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-d}{r^3}$
 $+ \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} + \text{&c.} + \frac{a-nd+d}{r^n}$,
 which is the subject of the foregoing articles; and consequently it may be
 summed by means of the expression $aG \frac{-dG}{r-1} + \frac{nd}{r^n \times r-1}$. This
 may be done in the following manner.

CXXXIX. Here r is = 1.035; and n , (or the number of terms in
 the series,) is 18; and a , (the numerator of the first term,) is 814;
 and d , (the common difference of the numerators,) is 8. Therefore the
 expression $aG \frac{-dG}{r-1} + \frac{nd}{r^n \times r-1}$ is = $814G \frac{-8G}{1.035} + \frac{18 \times 8}{1.035^1 \times .035}$
 $= 814G - 228.571,428G + \frac{144}{1.035^1 \times .035} = 585.428,572G$
 $+ \frac{4114.285,714}{1.035^1}$. Now it appears from Mr. Smar's first table of com-

pound interest, page 52, that the 18th power of 1.035 is 1.857,489;
 and it appears by his fourth table of compound interest, page 76, that G ,
 or the value of the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$
 $+ \frac{1}{r^6} + \frac{1}{r^7} + \text{&c.} + \frac{1}{r^{18}}$, when r is = 1.035, is 13.189,681.

Therefore $585.428,572G + \frac{4114.285,714}{1.035^1}$ is = $585.428,572 \times 13.189,681$
 $+ \frac{4114.285,714}{1.857,489} = 7721.616,112 + 2214.971,778 = 9936.587,890$.

Therefore the series $\frac{814}{r} + \frac{805}{r^2} + \frac{798}{r^3} + \frac{790}{r^4} + \frac{782}{r^5} + \frac{774}{r^6}$
 $+ \frac{766}{r^7} + \frac{758}{r^8} + \frac{750}{r^9} + \frac{742}{r^{10}} + \frac{734}{r^{11}} + \frac{726}{r^{12}} + \frac{718}{r^{13}} + \frac{710}{r^{14}}$
 $+ \frac{702}{r^{15}} + \frac{694}{r^{16}} + \frac{686}{r^{17}} + \frac{678}{r^{18}}$ is equal to 9936.587,890. Conf-

quently

$$\text{quently the series } \frac{814}{r^{10}} + \frac{806}{r^{11}} + \frac{798}{r^{12}} + \frac{790}{r^{13}} + \frac{782}{r^{14}} + \frac{774}{r^{15}} \\ + \frac{766}{r^{16}} + \frac{758}{r^{17}} + \frac{750}{r^{18}} + \frac{742}{r^{19}} + \frac{734}{r^{20}} + \frac{726}{r^{21}} + \frac{718}{r^{22}} + \frac{710}{r^{23}} \\ + \frac{702}{r^{24}} + \frac{694}{r^{25}} + \frac{686}{r^{26}} + \frac{678}{r^{27}} \text{ is } = 9936.587,890 \times \frac{1}{r^9} =$$

$$\frac{9936.587,890}{1.0351^9} = (\text{by Mr. Smart's first table of compound interest,}$$

$$\text{page 52,)} \frac{9936.587,890}{1.362,897} = 7290.784,182. \text{ QEI.}$$

CXL. If the terms of this last series be computed separately, (as they have been above in Art. 94) they will be equal to the following numbers, to wit, 577.05274 + 552.06164 + 528.10044 + 505.12600 + 483.10396 + 461.99286 + 441.75220 + 422.35760 + 403.77000 + 385.95130 + 368.87904 + 352.52382 + 336.84970 + 321.82880 + 307.44090 + 293.65916 + 280.45738 + 267.81678; the sum of which is 7290.72432, which agrees with the foregoing value of them found by means of the expression $aG - \frac{dG}{r-1} + \frac{n d}{r^n \times r-1}$ together with the

multiplication by $\frac{1}{r^9}$, to wit, 7290.784,182, to five places of figures, and thereby confirms the truth of the said expression. See these numbers, 577.05274 + 552.06164 + &c. above in Art. 94, pages 105, 106.

CXLI. When n , (or the numerator of the first and greatest term of A particular case of the series $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6}$ foregoing compound series, $+ \frac{a-6d}{r^7} + \&c. + \frac{a-nd+d}{r^n}$) is an exact multiple of d (the com-

mon difference of the numerators of the several terms,) and the series is continued so far as to make $a-nd+d$, or $a-|n-1| \times d$, (the numerator of its last term) equal to a , or $n-1| \times d$ equal to a , the series will be

Y

$$\begin{aligned} &\text{be as follows, to wit, } \frac{nd-d}{r} + \frac{nd-d-d}{r^2} + \frac{nd-d-d-2d}{r^3} + \frac{nd-d-3d}{r^4} \\ &+ \frac{nd-d-4d}{r^5} + \frac{nd-d-5d}{r^6} + \frac{nd-d-6d}{r^7} + \text{&c.} + \frac{nd-d-nd+1}{r^n} \\ &\text{or } \frac{nd-d}{r} + \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} \\ &+ \frac{nd-7d}{r^7} + \text{&c.} + \frac{nd-nd}{r^n}, \text{ or } + \frac{0}{r^n}; \text{ the last term } \frac{nd-nd}{r^n} \end{aligned}$$

being equal to 0, and being here set down only for the sake of analogy and uniformity, and to make the series appear to consist of n terms, as it did in the former case, though in reality it contains in this case only $n-1$ terms. For such a preservation of the general forms of expressions in cases that do not strictly come under them, is often found to be of use in these sciences.

The expression
of its sum in
that case.

CXLII. Now, when a is thus $= nd-d$, the expression $aG - \frac{dG}{r-1}$

$$\begin{aligned} &+ \frac{nd}{r^n \times r-1}, \text{ obtained in Art. 136, will be } = \frac{nd-d}{r-1} \times G - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1} \\ &= ndG - dG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1} = nd \times \left[\frac{1}{r-1} - \frac{1}{r^n \times r-1} \right] - dG \\ &- \frac{dG}{r-1} + \frac{nd}{r^n \times r-1} = \frac{nd}{r-1} - \frac{nd}{r^n \times r-1} - dG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1} \\ &= \frac{nd}{r-1} - dG - \frac{dG}{r-1} = \frac{nd}{r-1} - \frac{dG \times r-1}{r-1} - \frac{dG}{r-1} = \frac{nd}{r-1} \\ &- \frac{rdG + dG}{r-1} - \frac{dG}{r-1} = \frac{nd-rdG}{r-1}. \text{ Therefore the series } \frac{nd-d}{r} \\ &+ \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} + \frac{nd-7d}{r^7} \\ &+ \text{&c.} + \frac{nd-nd}{r^n}, \text{ or } + \frac{0}{r^n}, \text{ is equal to } \frac{nd-rdG}{r-1}, \text{ or to } d \times \left[\frac{n-rG}{r-1} \right], \end{aligned}$$

which is a very short and simple expression.

CXLIII. And

CXLIII. And, if d , the common difference of the numerators of the terms in the last-mentioned series $\frac{nd-d}{r} + \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} + \frac{nd-7d}{r^7} + \&c. + \frac{nd-nd}{r^n}$, A reduction of the series obtained in Art. 141 to a more simple form.

is = 1, both the said series itself and the expression $\frac{nd-rdG}{r-1}$ for the sum of all its terms, will be thereby still further simplified. For the series will then be $= \frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7} + \&c. + \frac{n-n}{r^n}$; and the expression $\frac{nd-rdG}{r-1}$ will be = $\frac{n-rG}{r-1}$. The expression of the sum of the said reduced series. And consequently the series $\frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7} + \&c. + \frac{n-n}{r^n}$ is $= \frac{n-rG}{r-1}$.

CXLIV. Therefore, if we divide both the last series $\frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7} + \&c. + \frac{n-n}{r^n}$ and the expression $\frac{n-rG}{r-1}$, (to which it is equal) by n , the quotients thence arising will be equal; that is, the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} + \&c. + \frac{n-n}{nr^n}$ is $\left(\frac{n-rG}{r-1} \right)$ An expression for the sum of a series derived from the series in the last article by dividing its terms by n .

Mr. De Moivre's expression for the sum of the terms of the last series.

CXLV. If the letter P be used, instead of the letter G , to denote the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \text{\&c.}$ the expression $\frac{1-rP}{1-r}$ will be equal to the sum of all the terms of the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} + \text{\&c.}$ $+ \frac{n-n}{nr^n}$.

CXLVI. This last expression $\frac{1-rP}{1-r}$ is that which Mr. De Moivre

has given us for the purpose of summing the last series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \text{\&c.}$ $+ \frac{n-n}{nr^n}$ in his treatise of Annuities on Lives, Part 2d, pages 310, 311, of the third edition published in quarto, at the end of his treatise on the doctrine of Chances, in the year 1756. And in the same place he has given us a synthetick demonstration of the truth of this expression, or of its equality with the said series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \text{\&c.}$ $+ \frac{n-n}{nr^n}$. But he has not informed us by what steps, or in what

manner, he discovered that this expression was equal to the said series, but has alledged as an excuse for this omission, that *the reasonings that led him to this expression require something more than an ordinary skill in the doctrine of series*. He probably therefore had deduced it from some more abstruse propositions in that doctrine than that which we have had recourse to for the same purpose in the foregoing articles, which is only the well-known theorem that was demonstrated in Art. 80, that the sum of the terms of a decreasing geometrical progression, $A+B+C+D+E$, is equal to the fraction $\frac{AA-BE}{A-B}$. His synthetick demonstration of the truth of this expression is for the most part very clear and satisfactory. But in the last

Observations on the said synthetick demonstration.

$$\begin{aligned} & + \frac{r}{r^1} + \&c. + \frac{r}{r^n} = 1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} \\ & + \&c. + \frac{1}{r^{n-1}}, \text{ and } \frac{r}{n} P = \frac{1}{n} + \frac{1}{nr} + \frac{1}{nr^2} + \frac{1}{nr^3} + \frac{1}{nr^4} + \frac{1}{nr^5} \\ & + \frac{1}{nr^6} + \&c. + \frac{1}{nr^{n-1}}. \end{aligned}$$

Therefore $1 - \frac{r^P}{n}$ is $1 - \frac{1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} - \&c. - \frac{1}{nr^{n-1}} = \frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} - \frac{1}{nr^7} + \&c. + \frac{1}{nr^{n-1}}$; and, dividing both sides by $r-1$,

$$\begin{aligned} \frac{-r^P}{n} & = \frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} \\ & + \frac{n-7}{nr^7} + \&c. + \frac{n-n}{nr^n}. \text{ Q.E.D.} \end{aligned}$$

CXLVIII. The division of the series $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} - \frac{1}{nr^7} + \&c. + \frac{1}{nr^{n-1}}$ by $r-1$ may be performed as follows,

$$(r-1)$$

The operation of division mentioned in the foregoing article.

$$\begin{aligned} & \frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \dots \\ & + \frac{n-1}{n} - \left| \frac{n-1}{nr} \right| \\ & * \frac{+ \frac{n-2}{nr} - \frac{1}{nr^2}}{+ \frac{n-2}{nr} - \left| \frac{n-2}{nr^2} \right|} \\ & * \frac{+ \frac{n-3}{nr^2} - \frac{1}{nr^3}}{+ \frac{n-3}{nr^2} - \left| \frac{n-3}{nr^3} \right|} \\ & * \frac{+ \frac{n-4}{nr^3}}{+ \frac{n-4}{nr^3}} + \dots \end{aligned}$$

Of the use made of the foregoing expression by Mr. De Moivre in the computation of life-annuities.

CXLIX. Mr. De Moivre, in his treatise of Life-annuities, makes great use of this expression $\frac{1-r^p}{1-r}$. For he employs it in the solution

of his first and most important problem, or the computation of the value of an annuity upon a life of any given age. And with this view he adopts an hypothesis, concerning the probabilities of the duration of human life, which is fitted to make that expression applicable to his purpose, and which he conceives to be so little different from the real estimate of those probabilities, as deduced from observations by Dr. Halley, Monsieur de Parcieux, and others, that it will make the values of life-annuities computed by it very nearly the same as they would be, if computed strictly from tables founded on observations in the manner that has been above explained. This hypothesis is as follows.

of Mr. De Moivre's Hypothesis concerning the decrements of human life.

CL. We have seen above, in Art. 126, that, both in Mr. Kerseboom's and Monsieur de Parcieux's tables of the probabilities of the duration of human life, the numbers of persons dying in a year, or the decrements of human life, (as they are called,) continue the same for several years together in several different stages, or periods, of life from the age of 11 or 12 years to the age of 75 or 80 years. And in this interval from the age of 11 or 12 years to the age of 75 or 80 years, the number of persons dying in a year during one of these stages, or periods, is not either constantly greater, or constantly less, than the number of those who die in a year during the next older period of life, but in some parts of human life is greater than the said next number, and in other parts of it is less. Thus, for example, in Monsieur de Parcieux's table of probabilities, it appears that from the age of 11 years to the age of 16 years the number of persons who die in every year is 6; and from the age of 16 years to the age of 20 years the number of persons who die in every year is 7; and from the age of 20 years to the age of 37 years, the number of persons who die in every year is 8: and thus far the number of persons dying every year in these three successive stages, or periods, of human life, is found to increase as the period of human life advances, or becomes older. But in the next period of life, namely, from the age of 37 years to the age of 46 years, the number of persons dying in a year is only 7; after which, in the interval between the age of 46 years and the age of 75 years, it again increases to 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20 persons in a year; and then it decreases again to 19 and 18 persons in the next five years, or from the age of 75 years to the age of 80 years. And the same variation, from a greater number to a lesser, and from a lesser number to a greater, may be observed in the decrements of human life, or the number of persons dying in a year, in different stages, or periods, of it, in Mr. Kerseboom's table above-mentioned, and in Dr. Halley's table of probabilities deduced from the observations made at Breslaw, and in several other tables of the same nature.

The decrements of human life in some periods of life become greater, and in others become less, as the age of life advances.

CL. From this observation on the course of the decrements of human life Mr. De Moivre was led to conjecture that, if the said decrements were supposed to be equal in every year, throughout the whole extent of life, as well as during particular periods of it, and the values of life-annuities were computed upon that supposition, they would be very nearly equal to

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the

the true values of them when computed in strict conformity to Dr. Halley's said Bressaw table, or any other table of the probabilities of the duration of human life that was grounded on actual observations. For, as, by such a reduction of the irregular variation of the decrements of life to an absolute equality of the said decrements throughout life, the values of some of the future payments of an annuity for any given age, in some particular years of life, would be made greater than they ought to be, it is evident, the values of some others of the future payments of it, in other years of human life, would be made less than they ought to be, and these contrary variations in the values of different future payments of the annuity in different parts of human life, Mr. De Moivre supposed, would pretty nearly counter-balance each other, and make the sum of all the said future payments, or the value of the whole annuity throughout the whole extent of life, pretty nearly the same as it would have been if it had been regularly computed in strict conformity to Dr. Halley's, or some other, table of the probabilities of human life derived from actual observations, in the manner above described and exemplified in Art. 94. And he tells us that, upon trial, he found this conjecture to be well-founded: for that, having computed the value of an annuity for a life of a certain age upon the ground of this hypothesis, and compared it with the value of the same annuity for the same life, as computed by Dr. Halley in strict conformity to the Bressaw table of probabilities, he found these values to be so very little different from each other that, for all useful and practical purposes, they might well be considered as the same, and consequently that, in the business of computing the values of life-annuities, he might safely neglect the tables of probabilities deduced from observations, and proceed upon the ground of his own Hypothesis.

CLII. Mr. De Moivre does not inform us what was the age of the life for which he calculated an annuity upon the ground of his Hypothesis, in order to compare it with Dr. Halley's value of the same annuity computed strictly from the Bressaw table of probabilities. But it seems not unlikely to have been an annuity for a life of the age of 10 years, that being one of those which Dr. Halley had computed and set down in his tract above-mentioned, and being in some degree siter than the others there set down for the purpose of the said comparison. The several annuities there set down as having been accurately computed from the Bressaw table of probabilities, are for lives of the ages of one year, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70, years. They were computed upon a supposition that the interest of money was 6 per cent. as at that time (in the year, 1692,) it generally was. This little table of the values of life-annuities is as follows.

Of Dr. Hal-
ley's table of
the values of
life-annuities
for lives of
certain ages,
computed
from the Bressaw
table of
probabilities.

TABLE

LIFE-ANNUITIES.

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T A B L E VI.

containing the values of an annuity of one pound a year for the lives of persons of the ages of 1 year, 5 years, 10 years, 15 years, 20 years, and every following age of human life that exceeds the next preceding age by 5 years, as far as the age of 70 years; computed by Dr. Halley from the Breslaw table of the probabilities of the duration of human life, upon a supposition that the interest of money is 6 per cent.

Years of Age.	Values of an annuity of one pound.	Years of Age.	Values of an annuity of one pound.
1	10.28	40	10.57
5	13.40	45	9.91
10	13.44	50	9.21
15	13.33	55	8.51
20	12.78	60	7.60
25	12.27	65	6.54
30	11.72	70	5.32
35	11.12		

This was, I believe, the first table of the values of life-annuities that was published.

CLIII. That

Of the manner
of comparing
the conjectural
probabilities
of the duration
of human life,
relating from
Mr. De Moivre's
Hypothesis, with the
real probabilities
of the same
as exhibited in
Monsieur de
Parcieux's
table.

CLIII. That we may be the better able to judge of the degree in which Mr. De Moivre's Hypothesis above-mentioned, concerning the decrements of human life, approaches to, or varies from, the real decrements of life, as represented in Monsieur de Parcieux's table of probabilities, it will be proper to form a new table of the numbers of persons that would be living at the ends of the several years of human life after the age of 3 years, (which is the youngest age in Monsieur de Parcieux's table,) out of the same original number of persons living at the said age of 3 years as are represented in Monsieur de Parcieux's table to be living at that age, that is, out of 1000 persons of the age of 3 years, if the whole number were to die in the course of the same number of years in which they are represented to be all dead in Monsieur de Parcieux's table, that is, in the course of 92 years, but were to die in equal numbers in every year of that period, instead of dying in the numbers represented as dying in the several years of the same period in Monsieur de Parcieux's table. Now for this purpose it is necessary, in the first place, to divide the whole original number of persons represented in Monsieur de Parcieux's table as living at the age of 3 years, that is, 1000, by 92, or the number of years in which all the said 1000 lives are there represented to become extinct; because, it is evident, the quotient of this division will give us the number of persons who would die in every year of the said period of 92 years, upon Mr. De Moivre's supposition that that number were the same in every year. This quotient is 10.8697, which consists of the whole number 10 and the decimal fraction .8697; which shews that it is not possible that the number of persons dying every year in the said period of 92 years, (from the age of 3 years to the age of 95 years,) out of the said original number of 1000 persons living at the age of 3 years, should be always exactly the same. But, if we somewhat increase this original number, and make it 1012 instead of 1000, we shall thereby render it capable of an exact division by 92, and the quotient will be 11. And this variation, I apprehend, will not be of much importance, notwithstanding we do not suppose the other numbers in Monsieur de Parcieux's table of probabilities to be increased at the same time in the same proportion of 1012 to 1000, but to continue as before. For, if a table of the values of life-annuities were to be calculated from Monsieur de Parcieux's table of probabilities so altered in its first number only, all the said values would be precisely the same as before, except the value of an annuity for a life of the age of 3 years, which would be less than before in the proportion of $\frac{1}{1012}$ to $\frac{1}{1000}$, or 1000 to 1012. This variation, therefore, in the first number of Monsieur de Parcieux's said table may well enough be made on the present occasion for the sake of avoiding the inconvenience and obscurity that might arise from

from representing the number of persons who are supposed to die every year according to Mr. De Moivre's Hypothesis by a number that involves fraction, such as the quotient 10.8697 above found.

CLIV. Let us therefore suppose this variation to be made in the first number of Monsieur de Parcieux's table of probabilities, but that all the other numbers in it continue as before. And from the first number of this table so increased, or from the number 1012, (which contains the number 11 exactly 92 times,) let us continually subtract the number 11, till the said number 1012 be reduced to nothing. The table of numbers thence arising will represent the numbers of persons supposed to be living at the ages of 4 years, 5 years, 6 years, and every following age of life that exceeds the age next preceding by one year, as far as the age of 95 years, according to Mr. De Moivre's Hypothesis. This table, together with Monsieur de Parcieux's table, with its first number increased from 1000 to 1012, as aforesaid, will be as follows.

	1012	1001	5103	3
60	7	1001	072	4
59	8	992	842	3
58	9	983	757	2
57	10	974	672	1
56	11	965	587	0
55	12	956	502	9
54	13	947	417	8
53	14	938	332	7
52	15	929	247	6
51	16	920	162	5
50	17	911	77	4
49	18	902	0	3
48	19	893		2
47	20	884		1
46	21	875		0
45	22	866		9
44	23	857		8
43	24	848		7
42	25	839		6
41	26	830		5
40	27	821		4
39	28	812		3
38	29	803		2
37	30	794		1
36	31	785		0
35	32	776		9
34	33	767		8
33	34	758		7
32	35	749		6
31	36	740		5
30	37	731		4
29	38	722		3
28	39	713		2
27	40	704		1
26	41	695		0
25	42	686		9
24	43	677		8
23	44	668		7
22	45	659		6
21	46	650		5
20	47	641		4
19	48	632		3
18	49	623		2
17	50	614		1
16	51	605		0
15	52	596		9
14	53	587		8
13	54	578		7
12	55	569		6
11	56	560		5
10	57	551		4
9	58	542		3
8	59	533		2
7	60	524		1
6	61	515		0
5	62	506		9
4	63	497		8
3	64	488		7
2	65	479		6
1	66	470		5
0	67	461		4
	68	452		3
	69	443		2
	70	434		1
	71	425		0
	72	416		9
	73	407		8
	74	398		7
	75	389		6
	76	380		5
	77	371		4
	78	362		3
	79	353		2
	80	344		1
	81	335		0
	82	326		9
	83	317		8
	84	308		7
	85	299		6
	86	290		5
	87	281		4
	88	272		3
	89	263		2
	90	254		1
	91	245		0
	92	236		9
	93	227		8
	94	218		7
	95	209		6
	96	200		5
	97	191		4
	98	182		3
	99	173		2
	100	164		1
	101	155		0
	102	146		9
	103	137		8
	104	128		7
	105	119		6
	106	110		5
	107	101		4
	108	92		3
	109	83		2
	110	74		1
	111	65		0
	112	56		9
	113	47		8
	114	38		7
	115	29		6
	116	20		5
	117	11		4
	118	2		3
	119	0		2
	120	0		1
	121	0		0
	122	0		9
	123	0		8
	124	0		7
	125	0		6
	126	0		5
	127	0		4
	128	0		3
	129	0		2
	130	0		1
	131	0		0
	132	0		9
	133	0		8
	134	0		7
	135	0		6
	136	0		5
	137	0		4
	138	0		3
	139	0		2
	140	0		1
	141	0		0
	142	0		9
	143	0		8
	144	0		7
	145	0		6
	146	0		5
	147	0		4
	148	0		3
	149	0		2
	150	0		1
	151	0		0
	152	0		9
	153	0		8
	154	0		7
	155	0		6
	156	0		5
	157	0		4
	158	0		3
	159	0		2
	160	0		1
	161	0		0
	162	0		9
	163	0		8
	164	0		7
	165	0		6
	166	0		5
	167	0		4
	168	0		3
	169	0		2
	170	0		1
	171	0		0
	172	0		9
	173	0		8
	174	0		7
	175	0		6
	176	0		5
	177	0		4
	178	0		3
	179	0		2
	180	0		1
	181	0		0
	182	0		9
	183	0		8
	184	0		7
	185	0		6
	186	0		5
	187	0		4
	188	0		3
	189	0		2
	190	0		1
	191	0		0
	192	0		9
	193	0		8
	194	0		7
	195	0		6
	196	0		5
	197	0		4
	198	0		3
	199	0		2
	200	0		1

T A B L E

T A B L E VII.

Containing Monsieur de Parcieux's table of the probabilities of the duration of human life, from the age of 3 years to the age of 95 years, with the first number of it increased from 1000 to 1012; and likewise an artificial table fitted to the same original number 1012, in which the decrements of human life, or persons dying in a year, are supposed to be always the same throughout the said period of 92 years, or from the age of 3 years to the age of 95 years, agreeably to Mr. De Moivre's Hypothesis.

Years of Age.	Persons living according to Monsieur de Parcieux's table, with an increase of the first number from 1000 to 1012.	Persons living according to Mr. De Moivre's Hypothesis.	Years of Age.	Persons living according to Monsieur de Parcieux's table.	Persons living according to Mr. De Moivre's Hypothesis.
3	1012	1012	26	766	759
4	970	1001	27	758	748
5	948	990	28	750	737
6	930	979	29	742	726
7	915	968	30	734	715
8	902	957	31	726	704
9	890	946	32	718	693
10	880	935	33	710	681
11	872	924	34	702	671
12	866	913	35	694	660
13	860	902	36	686	649
14	854	891	37	678	638
15	848	880	38	671	627
16	842	869	39	664	616
17	835	858	40	657	605
18	828	847	41	650	594
19	821	836	42	643	583
20	814	825	43	636	572
21	806	814	44	629	561
22	798	803	45	622	550
23	790	792	46	615	539
24	782	781	47	607	528
25	774	770	48	599	517

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Age.	Persons living according to Monsieur de Parcieux's table.	Persons living according to Mr. De Moivre's Hypothesis.	Years of Age.	Persons living according to Monsieur de Parcieux's table.	Persons living according to Mr. De Moivre's Hypothesis.
9	590	506	73	251	242
10	581	495	74	231	231
11	571	484	75	211	220
12	560	473	76	192	209
13	549	462	77	173	198
14	538	451	78	154	187
15	526	440	79	136	176
16	514	429	80	118	165
17	502	418	81	101	154
18	489	407	82	85	143
19	476	396	83	71	132
20	463	385	84	59	121
21	450	374	85	48	110
22	437	363	86	38	99
23	423	352	87	29	88
24	409	341	88	22	77
25	395	330	89	16	66
26	383	319	90	11	55
27	364	308	91	7	44
28	347	297	92	4	33
29	329	286	93	2	22
30	310	275	94	1	11
31	291	264	95	0	0
32	271	253			

CLV. It

An observation on the foregoing table of the real and conjectural probabilities of the duration of human life.

CLV. It appears from the foregoing table, that, from the age of 3 years to the age of 24 years, the numbers of persons represented living at the end of every year are greater in the arithmetical progression 1012, 1001, 990, 979, &c. formed upon Mr. De Moivre's Hypothesis than in Monsieur de Parcieux's series of numbers 1012, 970, 948, 915, &c. and that at the age of 24 years they are nearly the same in both serieses of numbers, being 781 and 782; and that, from the age of 24 years to the age of 74 years, the numbers in Mr. De Moivre's series are less than those in Monsieur de Parcieux's; and that at the age of 74 years they are equal in both serieses, being 231 in both serieses; and that then, after the age of 74 years, Mr. De Moivre's numbers become a second time greater than Monsieur de Parcieux's, and continue so to the end of the table.

A conclusion from the foregoing observation concerning the value of an annuity for a life of the age of 74 years.

CLVI. It follows from the foregoing observation that, if the value of an annuity for a life of 74 years were to be computed according to both these serieses of numbers, the value of it derived from Mr. De Moivre's numbers would be greater than the value of it derived from Monsieur de Parcieux's numbers. For the number P , or the number of persons living at the given age, would be the same in both cases, to wit, 231; and the numbers $P^I, P^{II}, P^{III}, P^{IV}, P^V, P^{VI}, P^{VII}, \&c.$ would be greater in the calculation grounded on Mr. De Moivre's series than in that which

was built on Monsieur de Parcieux's series: and consequently $\frac{1}{P} \times$ the series

$$\frac{P^I}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^V}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} + \&c.$$

(which, by Art. 87, is equal to the value of the annuity for the given life,) would be greater in the former case than in the latter.

Conclusions from the same observation concerning the values of annuities for lives of other ages.

CLVII. But, if we were to compute the value of an annuity for 50 years, and no longer, dependent upon a life of the age of 24 years, by means of these two serieses of numbers, the value of such an annuity derived from Mr. De Moivre's numbers would be less than the other value of it derived from Monsieur de Parcieux's numbers. For in this case the value of P in the former calculation would be 781, and in the latter it would be 782, which is almost equal to 781; and the values of $P^I, P^{II}, P^{III}, P^{IV}, P^V, P^{VI}, P^{VII}, \&c.$ in the former calculation would be 770, 759, 748, 737, 726, 715, 704, &c. and in the latter calculation they

they would be 774, 766, 758, 750, 742, 734, 726, &c. which are respectively greater than the other numbers: and consequently the expression

$$x \text{ the series } \frac{P^1}{r} + \frac{P^{12}}{r^2} + \frac{P^{13}}{r^3} + \frac{P^{14}}{r^4} + \frac{P^V}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} + \&c.$$

then derived from Mr. De Moivre's numbers, will be less than the same expression, when derived from Monsieur de Parcieux's numbers, or the value of an annuity for 50 years, dependent upon a life of the age of 24, will be less according to Mr. De Moivre's numbers than according to Monsieur de Parcieux's numbers.

CLVIII. But, if we continue the last annuity for a life of the age of 4 years to the end of life, or to 70 years from its commencement, instead of restraining it to 50 years, the values of the future contingent payments of it to be made in the last 20 years of its duration, or from the age of 74 to the age of 94 years, will be greater according to Mr. De Moivre's numbers than according to Monsieur de Parcieux's. And consequently the value of a complete life-annuity for a person of the age of 24 years, computed from Mr. De Moivre's numbers, will differ less from the value of it, computed from Monsieur de Parcieux's numbers, than the values of a limited annuity for only 50 years, depending upon the same life, computed according to the same two sets of numbers, differ from each other; because the excess of the values of the future payments to be made after the age of 74 years according to Mr. De Moivre's numbers, above the values of those payments according to Monsieur de Parcieux's numbers, will, in some degree, make amends for the deficiencies of the values of the preceding future payments, from the age of 24 years to the age of 74 years, according to Mr. De Moivre's numbers, below the values of the same payments according to Monsieur de Parcieux's numbers. But it will only do this *in some degree*, and not by any means completely; because the values of the 20 last future payments of the annuity from the age of 74 years to the age of 94 years, are so much less (on account of the distance of time at which those payments are to be made,) than the values of the 50 preceding future payments of it from the age of 24 years to the age of 74 years, that it does not greatly signify towards obtaining the true value of the whole annuity for a life of the age of 24 years, by which of the two serieses of numbers they are computed. And therefore, upon the whole, the value of a complete life-annuity for a person of the age of 24 years, computed according to Mr. De Moivre's numbers, will still be considerably less than its value, computed according to Monsieur de Parcieux's numbers, or than its true value. And, in general, it will be found that the value of an annuity for a life of any age under 45 years, computed according to Mr. De Moivre's numbers, will be less than its value computed from Monsieur de

de Parcieux's numbers, or its true value. But, that we may be the better able to judge of this matter, I will now proceed to compute a small table of the values of life-annuities (such as that which was computed exactly from the Bresslaw table of probabilities by Dr. Halley, and which is inserted above in Art. 152,) according to Mr. De Moivre's numbers, upon a supposition that the interest of money is $3\frac{1}{4}$ per cent. which values we may afterwards compare with the values of the same annuities, as computed above (in Art. 108, 109, &c.—114) from the numbers of Monsieur de Parcieux's table.

The numbers of persons living according to Mr. De Moivre's Hypothesis in Table VII. may be reduced to smaller numbers.

CLIX. As the numbers in Mr. De Moivre's series above-mentioned are, all of them, exact multiples of 11, they will all be divisible by 11 without any remainders, and consequently may be reduced to smaller numbers without altering their proportions to one another. Let them all be so divided by 11; and the numbers thence arising will be as follows, to wit, 92, 91, 90, 89, 88, 87, 86, 85, 84, 83, 82, 81, 80, 79, 78, 77, 76, 75, 74, 73, 72, 71, 70, 69, 68, 67, 66, 65, 64, 63, 62, 61, 60, 59, 58, 57, 56, 55, 54, 53, 52, 51, 50, 49, 48, 47, 46, 45, 44, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, which is an arithmetical progression of the simplest nature possible, in which the terms continually decrease by the same common difference 1. Yet, as these numbers are to each other in exactly the same proportions as the former numbers 1012, 1001, 990, 979, 968, 957, 946, &c. (of which they are exact 11th parts,) it is evident that the values of any life-annuities of one pound deduced from the general expression $\frac{1}{p} \times$ the series $\frac{P^i}{r} + \frac{P^{i+1}}{r^2} + \frac{P^{i+2}}{r^3} + \frac{P^{i+3}}{r^4} + \frac{P^{i+4}}{r^5} + \frac{P^{i+5}}{r^6} + \frac{P^{i+6}}{r^7} + \&c.$ by substituting in its terms, instead of $P, P^i, P^{i+1}, P^{i+2}, P^{i+3}, P^{i+4}, P^{i+5}, P^{i+6}, \&c.$ any numbers taken from the series 92, 91, 90, 89, 88, 87, 86, &c. will be the same with the values deduced from the same general expression by substituting in its terms, instead of $P, P^i, P^{i+1}, P^{i+2}, P^{i+3}, P^{i+4}, P^{i+5}, P^{i+6}, \&c.$ the corresponding numbers of the series 1012, 1001, 990, 979, 968, 957, 946, &c.

CLX. If these new numbers are set down in a regular series near the ages to which they respectively belong, and are supposed to begin from the age of one year, instead of beginning from the age of 3 years (as the numbers in the last table do,) they will be as follows.

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T A B L E VII.

Containing an artificial estimate of the probabilities of the duration of human life, similar to that contained in the second column of Table VII. but expressed in the smallest numbers possible; derived from Mr. De Moivre's Hypothesis that the numbers of persons dying every year, out of any given original number, are always the same throughout the whole extent of human life, and formed upon a supposition that the utmost possible duration of human life is less than 95 years.

Years of Age.	Persons living.	Years of Age.	Persons living.	Years of Age.	Persons living.	Years of Age.	Persons living.
1	94	25	70	49	46	73	22
2	93	26	69	50	45	74	21
3	92	27	68	51	44	75	20
4	91	28	67	52	43	76	19
5	90	29	66	53	42	77	18
6	89	30	65	54	41	78	17
7	88	31	64	55	40	79	16
8	87	32	63	56	39	80	15
9	86	33	62	57	38	81	14
10	85	34	61	58	37	82	13
11	84	35	60	59	36	83	12
12	83	36	59	60	35	84	11
13	82	37	58	61	34	85	10
14	81	38	57	62	33	86	9
15	80	39	56	63	32	87	8
16	79	40	55	64	31	88	7
17	78	41	54	65	30	89	6
18	77	42	53	66	29	90	5
19	76	43	52	67	28	91	4
20	75	44	51	68	27	92	3
21	74	45	50	69	26	93	2
22	73	46	49	70	25	94	1
23	72	47	48	71	24	95	0
24	71	48	47	72	23		

Of the comple-
ment of Life.

CLXI. In the foregoing table the number of persons represented living at any given age is always the same with the number of years which are necessary to be added to the number of years in the given age in order to make it equal to 95 years, or the period in the course of which it is supposed that all the persons set down in the table as living at the end of the several foregoing years, will be dead. Thus, the number of persons living at the age of one year is 94; which, being added to one year, the age of the said persons, makes 95: and the number living at the age of 20 years is 85; which being added to 10, the number of years in the age of the said persons, is 95, as before. And the same observation is true throughout the table, to wit, that the number in the second column, which expresses the persons living at any given age, is always the complement of the number of years in the said age to 95. This complement Mr. De Moivre calls *the complement of life*: so that, according to his Hypothesis, the complement of life at any given age is always equal to the number of persons living at the same age.

A computation of the values of an annuity of one pound a year for the lives of persons of several different ages, according to the foregoing artificial table of the probabilities of the duration of human life, grounded on Mr. De Moivre's Hypothesis, and upon a supposition that the interest of money is 3½ per cent.

For the life of
a child of the
age of one
year.

CLXII. Let it now be required to find, by means of the foregoing artificial table of probabilities, the value of an annuity of one pound a year for the life of a child of the age of one year, when the interest of money is 3½ per cent.

By Art. 87, the general expression for the value of an annuity for a life of any given age is $\frac{1}{P} \times$ the series $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{iv}}{r^4} + \frac{P^{v}}{r^5} + \frac{P^{vi}}{r^6} + \frac{P^{vii}}{r^7} + \&c.$ in which P signifies the number of persons living at the given age, P' the number of persons living at the end of the next year, P'' the number living at the end of the second year, P''' the number living at the end of the third year, and so on to the utmost extent of human life.

These numbers $P, P', P'', P''', P^{iv}, \&c.$ are in the present case, and according to Mr. De Moivre's Hypothesis, 94, 93, 92, 91, &c. There-

fore the value of the annuity sought is $= \frac{1}{94} \times$ the series $\frac{93}{r} + \frac{92}{r^2} + \frac{91}{r^3} + \frac{90}{r^4} + \frac{89}{r^5} + \frac{88}{r^6} + \frac{87}{r^7} + \&c.$ continued to 93 terms, or to the term

$\frac{1}{93} = 1 \times \text{the series } \frac{93}{94^r} + \frac{92}{94^{r^2}} + \frac{91}{94^{r^3}} + \frac{90}{94^{r^4}} + \frac{89}{94^{r^5}} + \frac{88}{94^{r^6}} + \frac{87}{94^{r^7}} + \&c. + \frac{1}{94^{r^{93}}} = 1 \times \text{the series } \frac{94-1}{94^r} + \frac{94-2}{94^{r^2}} + \frac{94-3}{94^{r^3}} + \frac{94-4}{94^{r^4}} + \frac{94-5}{94^{r^5}} + \frac{94-6}{94^{r^6}} + \frac{94-7}{94^{r^7}} + \&c. + \frac{94-93}{94^{r^{93}}}$. Let one

term more be added to this series, to wit, the term $\frac{94-94}{94^{r^{94}}}$, in order to

bring the series to the same form as the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3}$

$+ \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} + \&c. + \frac{n-n}{nr^n}$, in Art. 144.

Such addition of the term $\frac{94-94}{94^{r^{94}}}$ will make no alteration in the value of

the series to which it is added, because the said term is $= \left(\frac{0}{94^{r^{94}}} \right)$ or 0.

Therefore the value of the annuity sought will be $= 1 \times \text{the series } \frac{94-1}{94^r}$

$+ \frac{94-2}{94^{r^2}} + \frac{94-3}{94^{r^3}} + \frac{94-4}{94^{r^4}} + \frac{94-5}{94^{r^5}} + \frac{94-6}{94^{r^6}} + \frac{94-7}{94^{r^7}} + \&c.$

$+ \frac{94-94}{94^{r^{94}}}$. But it has been shewn in Art. 144, that, if n be any whole

number whatsoever, the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4}$

$+ \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} + \&c. + \frac{n-n}{nr^n}$ will be equal to the fraction

$\frac{1-rG}{r-1}$, in which G denotes the sum of the terms of the geometrical

progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$

$+ \frac{1}{r^n}$. Therefore the series $\frac{94-1}{94^r} + \frac{94-2}{94^{r^2}} + \frac{94-3}{94^{r^3}} + \frac{94-4}{94^{r^4}}$

$+ \frac{94-5}{94^{r^5}} + \frac{94-6}{94^{r^6}} + \frac{94-7}{94^{r^7}} + \&c. + \frac{94-94}{94^{r^{94}}}$ will be $=$

$\frac{1-rG}{r-1}$

$\frac{94}{r-1}$; that is, (because the interest of money is supposed to be $3\frac{1}{2}$ per

cent.

cent, and consequently r is $= 1.035$;) the said series will be $=$
 $1 - \frac{1.035G}{94}$, in which G will denote the sum of the terms of the geo-

.035

metrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$

$+ \&c.$ $+ \frac{1}{r^{24}}$, or $\frac{1}{1.035} + \frac{1}{1.035^2} + \frac{1}{1.035^3} + \frac{1}{1.035^4} + \frac{1}{1.035^5} + \frac{1}{1.035^6}$

$+ \frac{1}{1.035^7} + \frac{1}{1.035^8} + \frac{1}{1.035^9} + \&c.$ $+ \frac{1}{1.035^{24}}$

or continued to 94 terms. Now it appears from Mr. Smart's fourth table of compound interest, page 82, that G , or the sum of the terms of the said geometrical progression, is 27.445,426. Therefore $\frac{1.035G}{94}$ is $=$

$$\frac{1.035}{94} \times 27.445,426 = \frac{28.406,015,910}{94} = .302,191,658; \text{ and}$$

$$1 - \frac{1.035G}{94} \text{ is } = 1 - .302,191,658 = .697,808,342; \text{ and } 1 - \frac{1.035G}{94}$$

is $= \frac{.697,808,342}{.035} = 19,908,809$. Therefore the value of an annuity

of one pound a year for the life of a child of the age of one year, is

according to Mr. De Moivre's Hypothesis, $= 1 \times 19,908,809, =$
 $19,908,809$, or 19l. 18s. 2d. Q.E.I.

CLXIII. By the like reasonings as those in the preceding article it may be shewn, that, if n be the complement of life to any other given age whatsoever, the value of an annuity of one pound for a life of the said age, according to Mr. De Moivre's Hypothesis, will be $= 1 \times$ the fraction $\frac{1 - r^n}{r - 1}$; as, I presume, the reader must easily perceive. I will

now therefore proceed to calculate, by means of this expression, $1 \times \frac{1 - r^n}{r - 1}$, the values of an annuity of one pound for lives of the

several

veral ages of 3 years, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70 years, according to Mr. De Moivre's hypothesis, to the end that we may be able to compare them with the values of the same annuity for lives of the same ages, as computed above Art. 108, 109, &c.—114, from Monsieur de Parcieux's table.

CLXIV. When the given life is of the age of 3 years, we shall have or the complement of life, $(= 95 - 3) = 92$, and $\frac{1}{92} = \frac{1.035}{92}$, For the life of a child of the age of 3 years.

and $G =$ the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \frac{1}{r^8} + \frac{1}{r^9} + \frac{1}{r^{10}} + \frac{1}{r^{11}} + \frac{1}{r^{12}} + \frac{1}{r^{13}} + \frac{1}{r^{14}} + \frac{1}{r^{15}} + \frac{1}{r^{16}} + \frac{1}{r^{17}} + \frac{1}{r^{18}} + \frac{1}{r^{19}} + \frac{1}{r^{20}} + \frac{1}{r^{21}} + \frac{1}{r^{22}} + \frac{1}{r^{23}} + \frac{1}{r^{24}} + \frac{1}{r^{25}} + \frac{1}{r^{26}} + \frac{1}{r^{27}} + \frac{1}{r^{28}} + \frac{1}{r^{29}} + \frac{1}{r^{30}} + \frac{1}{r^{31}} + \frac{1}{r^{32}} + \frac{1}{r^{33}} + \frac{1}{r^{34}} + \frac{1}{r^{35}} + \frac{1}{r^{36}} + \frac{1}{r^{37}} + \frac{1}{r^{38}} + \frac{1}{r^{39}} + \frac{1}{r^{40}} + \frac{1}{r^{41}} + \frac{1}{r^{42}} + \frac{1}{r^{43}} + \frac{1}{r^{44}} + \frac{1}{r^{45}} + \frac{1}{r^{46}} + \frac{1}{r^{47}} + \frac{1}{r^{48}} + \frac{1}{r^{49}} + \frac{1}{r^{50}} + \frac{1}{r^{51}} + \frac{1}{r^{52}} + \frac{1}{r^{53}} + \frac{1}{r^{54}} + \frac{1}{r^{55}} + \frac{1}{r^{56}} + \frac{1}{r^{57}} + \frac{1}{r^{58}} + \frac{1}{r^{59}} + \frac{1}{r^{60}} + \frac{1}{r^{61}} + \frac{1}{r^{62}} + \frac{1}{r^{63}} + \frac{1}{r^{64}} + \frac{1}{r^{65}} + \frac{1}{r^{66}} + \frac{1}{r^{67}} + \frac{1}{r^{68}} + \frac{1}{r^{69}} + \frac{1}{r^{70}} + \frac{1}{r^{71}} + \frac{1}{r^{72}} + \frac{1}{r^{73}} + \frac{1}{r^{74}} + \frac{1}{r^{75}} + \frac{1}{r^{76}} + \frac{1}{r^{77}} + \frac{1}{r^{78}} + \frac{1}{r^{79}} + \frac{1}{r^{80}} + \frac{1}{r^{81}} + \frac{1}{r^{82}} + \frac{1}{r^{83}} + \frac{1}{r^{84}} + \frac{1}{r^{85}} + \frac{1}{r^{86}} + \frac{1}{r^{87}} + \frac{1}{r^{88}} + \frac{1}{r^{89}} + \frac{1}{r^{90}} + \frac{1}{r^{91}} + \frac{1}{r^{92}}$

continued to $\frac{1}{r^{92}}$, or to 92 terms, $= \frac{1}{1.035} + \frac{1}{1.035^2} + \frac{1}{1.035^3} + \frac{1}{1.035^4} + \frac{1}{1.035^5} + \frac{1}{1.035^6} + \frac{1}{1.035^7} + \frac{1}{1.035^8} + \frac{1}{1.035^9} + \frac{1}{1.035^{10}} + \frac{1}{1.035^{11}} + \frac{1}{1.035^{12}} + \frac{1}{1.035^{13}} + \frac{1}{1.035^{14}} + \frac{1}{1.035^{15}} + \frac{1}{1.035^{16}} + \frac{1}{1.035^{17}} + \frac{1}{1.035^{18}} + \frac{1}{1.035^{19}} + \frac{1}{1.035^{20}} + \frac{1}{1.035^{21}} + \frac{1}{1.035^{22}} + \frac{1}{1.035^{23}} + \frac{1}{1.035^{24}} + \frac{1}{1.035^{25}} + \frac{1}{1.035^{26}} + \frac{1}{1.035^{27}} + \frac{1}{1.035^{28}} + \frac{1}{1.035^{29}} + \frac{1}{1.035^{30}} + \frac{1}{1.035^{31}} + \frac{1}{1.035^{32}} + \frac{1}{1.035^{33}} + \frac{1}{1.035^{34}} + \frac{1}{1.035^{35}} + \frac{1}{1.035^{36}} + \frac{1}{1.035^{37}} + \frac{1}{1.035^{38}} + \frac{1}{1.035^{39}} + \frac{1}{1.035^{40}} + \frac{1}{1.035^{41}} + \frac{1}{1.035^{42}} + \frac{1}{1.035^{43}} + \frac{1}{1.035^{44}} + \frac{1}{1.035^{45}} + \frac{1}{1.035^{46}} + \frac{1}{1.035^{47}} + \frac{1}{1.035^{48}} + \frac{1}{1.035^{49}} + \frac{1}{1.035^{50}} + \frac{1}{1.035^{51}} + \frac{1}{1.035^{52}} + \frac{1}{1.035^{53}} + \frac{1}{1.035^{54}} + \frac{1}{1.035^{55}} + \frac{1}{1.035^{56}} + \frac{1}{1.035^{57}} + \frac{1}{1.035^{58}} + \frac{1}{1.035^{59}} + \frac{1}{1.035^{60}} + \frac{1}{1.035^{61}} + \frac{1}{1.035^{62}} + \frac{1}{1.035^{63}} + \frac{1}{1.035^{64}} + \frac{1}{1.035^{65}} + \frac{1}{1.035^{66}} + \frac{1}{1.035^{67}} + \frac{1}{1.035^{68}} + \frac{1}{1.035^{69}} + \frac{1}{1.035^{70}} + \frac{1}{1.035^{71}} + \frac{1}{1.035^{72}} + \frac{1}{1.035^{73}} + \frac{1}{1.035^{74}} + \frac{1}{1.035^{75}} + \frac{1}{1.035^{76}} + \frac{1}{1.035^{77}} + \frac{1}{1.035^{78}} + \frac{1}{1.035^{79}} + \frac{1}{1.035^{80}} + \frac{1}{1.035^{81}} + \frac{1}{1.035^{82}} + \frac{1}{1.035^{83}} + \frac{1}{1.035^{84}} + \frac{1}{1.035^{85}} + \frac{1}{1.035^{86}} + \frac{1}{1.035^{87}} + \frac{1}{1.035^{88}} + \frac{1}{1.035^{89}} + \frac{1}{1.035^{90}} + \frac{1}{1.035^{91}} + \frac{1}{1.035^{92}}$

$\frac{1}{1.035} + \frac{1}{1.035^2} + \frac{1}{1.035^3} + \frac{1}{1.035^4} + \frac{1}{1.035^5} + \frac{1}{1.035^6} + \frac{1}{1.035^7} + \frac{1}{1.035^8} + \frac{1}{1.035^9} + \frac{1}{1.035^{10}} + \frac{1}{1.035^{11}} + \frac{1}{1.035^{12}} + \frac{1}{1.035^{13}} + \frac{1}{1.035^{14}} + \frac{1}{1.035^{15}} + \frac{1}{1.035^{16}} + \frac{1}{1.035^{17}} + \frac{1}{1.035^{18}} + \frac{1}{1.035^{19}} + \frac{1}{1.035^{20}} + \frac{1}{1.035^{21}} + \frac{1}{1.035^{22}} + \frac{1}{1.035^{23}} + \frac{1}{1.035^{24}} + \frac{1}{1.035^{25}} + \frac{1}{1.035^{26}} + \frac{1}{1.035^{27}} + \frac{1}{1.035^{28}} + \frac{1}{1.035^{29}} + \frac{1}{1.035^{30}} + \frac{1}{1.035^{31}} + \frac{1}{1.035^{32}} + \frac{1}{1.035^{33}} + \frac{1}{1.035^{34}} + \frac{1}{1.035^{35}} + \frac{1}{1.035^{36}} + \frac{1}{1.035^{37}} + \frac{1}{1.035^{38}} + \frac{1}{1.035^{39}} + \frac{1}{1.035^{40}} + \frac{1}{1.035^{41}} + \frac{1}{1.035^{42}} + \frac{1}{1.035^{43}} + \frac{1}{1.035^{44}} + \frac{1}{1.035^{45}} + \frac{1}{1.035^{46}} + \frac{1}{1.035^{47}} + \frac{1}{1.035^{48}} + \frac{1}{1.035^{49}} + \frac{1}{1.035^{50}} + \frac{1}{1.035^{51}} + \frac{1}{1.035^{52}} + \frac{1}{1.035^{53}} + \frac{1}{1.035^{54}} + \frac{1}{1.035^{55}} + \frac{1}{1.035^{56}} + \frac{1}{1.035^{57}} + \frac{1}{1.035^{58}} + \frac{1}{1.035^{59}} + \frac{1}{1.035^{60}} + \frac{1}{1.035^{61}} + \frac{1}{1.035^{62}} + \frac{1}{1.035^{63}} + \frac{1}{1.035^{64}} + \frac{1}{1.035^{65}} + \frac{1}{1.035^{66}} + \frac{1}{1.035^{67}} + \frac{1}{1.035^{68}} + \frac{1}{1.035^{69}} + \frac{1}{1.035^{70}} + \frac{1}{1.035^{71}} + \frac{1}{1.035^{72}} + \frac{1}{1.035^{73}} + \frac{1}{1.035^{74}} + \frac{1}{1.035^{75}} + \frac{1}{1.035^{76}} + \frac{1}{1.035^{77}} + \frac{1}{1.035^{78}} + \frac{1}{1.035^{79}} + \frac{1}{1.035^{80}} + \frac{1}{1.035^{81}} + \frac{1}{1.035^{82}} + \frac{1}{1.035^{83}} + \frac{1}{1.035^{84}} + \frac{1}{1.035^{85}} + \frac{1}{1.035^{86}} + \frac{1}{1.035^{87}} + \frac{1}{1.035^{88}} + \frac{1}{1.035^{89}} + \frac{1}{1.035^{90}} + \frac{1}{1.035^{91}} + \frac{1}{1.035^{92}}$

by Mr. Smar's fourth table of compound interest, page 82.) is = 7365,227. Therefore $\frac{1}{92}G$ will be = $\frac{1.035}{92} \times 27.365,227 =$

$\frac{323,009,945}{92} = 351,098,962$; and $\frac{1}{92}G$ will be = 1,000,000,000

$\cdot 307,858,803 = 692,141,197$; and $\frac{1}{92}G$ will be = $\frac{692,141,197}{92}$

$\cdot 692,141,197 = 19,775,462$. Consequently $1 \times \frac{1}{92}G$, or the

value of an annuity of one pound a year for a life of the age of 3 years, according to Mr. De Moivre's Hypothesis, will be = $1 \times 19,775,462$, or 19,775,462, or 19,775,462, or 19,775,462.

N.B. This value of the said annuity is almost a quarter of a year's purchase less than its true value, as computed above from Monsieur de Parcieux's table of probabilities, that value being £19,987,654, or 19,987,654. See Table III. Art. 118.

For a life of the age of 5 years.

CLXV. When the given life is of the age of 5 years, we shall have n , or the complement of it to 95 years, = 90, and $\frac{r}{n} = \frac{r}{90} = \frac{1.035}{90}$

and G = the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$

+ &c. continued to $\frac{1}{r^{90}}$, or to 90 terms, and consequently (by Mr.

Smart's tables, page 82) = 27.279,315. Therefore $\frac{r}{n}G$ will be = $\frac{1.035}{90}$

$\times 27.279,315 = \frac{28.234,091,025}{90} = .313,712,122$; and $\frac{1-r}{n}G$ will

be = 1.000,000,000 — .313,712,122 = .686,287,878; and $\frac{1-r}{r}$

will be = $\frac{.686,287,878}{r-1} = \frac{.686,287,878}{.035} = 19.608,225$. Therefore

$\frac{r}{1} \times \left| \frac{1-r}{n}G \right|$, or the value of an annuity of one pound for a life of the

age of 5 years, according to Mr. De Meivre's Hypothesis, will be

= $\frac{r}{1} \times 19.608,225 = £19.608,225$, or 19l. 12s. 2d. Q.E.I.

N.B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £20.526,716, or 20l. 10s. 6d. $\frac{1}{2}$; which is greater than the value just now found for it by almost a year's purchase.

For a life of the age of 10 years.

CLXVI. When the given life is of the age of 10 years, we shall have n (= 95 — 10) = 85, and $\frac{r}{n} = \frac{1.035}{85}$, and G = the geometrical

progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \frac{1}{r^8} + \frac{1}{r^9} + \frac{1}{r^{10}}$ or 10

85 terms, which (by Mr. Smart's tables, page 82,) is = 27.036,803

Therefore $\frac{r}{n}G$ is = $\frac{1.035}{85} \times 27.036,803 = \frac{27.983,091,105}{85} =$

329,212,836

29,212,836; and $\frac{1-r}{n}G$ is = 1,000,000,000 — .329,212,836 =

670,787,164; and $\frac{1-r}{n}G$ is = $\frac{.670,787,164}{.035}$ = 19,165,347.

Therefore $1 \times \frac{1-r}{n}G$, or the value of an annuity of one pound a year

for a life of the age of 10 years, according to Mr. De Moivre's Hypothesis, = £19,165,347 = £19.165,347, or 19l. 3s. 3d. $\frac{2}{3}$. Q.E.I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £20,752,981, or 20l. 15s. 7d. $\frac{1}{4}$; which is greater than the value just now found for it by more than a year and a half's purchase.

CLXVII. When the given life is of the age of 15 years, we shall have n (= 95—15) = 80, and $\frac{r}{n} = \frac{1.035}{80}$, and G = the geometrical years.

Progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c.$ continued to $\frac{1}{r^{80}}$,

to 80 terms, which (by Mr. Smart's tables, page 82) is = 26.748,775.

Therefore $\frac{r}{n}G$ is = $\frac{1.035}{80} \times 26.748,775 = \frac{27.684,982,125}{80} =$

46,062,276; and $\frac{1-r}{n}G$ is = 1,000,000,000 — .346,062,276 =

53,937,724; and $\frac{1-r}{n}G$ is = $\frac{.53,937,724}{.035}$ = 18,683,934.

Therefore $1 \times \frac{1-r}{n}G$, or the value of an annuity of one pound a year

for a life of the age of 15 years, according to Mr. De Moivre's Hypothesis, = £18,683,934 = £18,683,934, or 18l. 13s. 8d. Q.E.I.

B b

N. B. The

N. B. The value of this annuity, as computed above from *Monte de Parcieux's* table, is £20.137,194, or 20l. 25. 9d. which is greater than the value just now found for it by almost a year and a half's purchase.

For a life of the age of 20 years.

CLXVIII. When the given life is of the age of 20 years, we shall have $n (= 95 - 20) = 75$; and $\frac{r}{n} = \frac{1.035}{75}$; and $G =$ the geometrical

progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c.$ continued to the term $\frac{1}{r^{75}}$, or to 75 terms, which (by Mr. Smart's tables, page 80,) is =

26.406,688. Therefore $\frac{r}{n}G$ is = $\frac{1.035}{75} \times 26.406,688 = \frac{27.330,922,000}{75}$

= .364,412,294; and $\frac{1-r}{n}G$ is = 1.000,000,000, — .364,412,294 =

.635,587,706; and $\frac{1-r}{n}G$ is = $\frac{.635,587,706}{.035} = 18.159,648$.

Therefore $1 \times \frac{1-r}{n}G$, on the value of an annuity of one pound a year

for a life of the age of 20 years, according to Mr. De Moivre's Hypothesis, is = $1 \times 18.159,648 =$ £18.159,648, or 18l. 35. 2d. $\frac{1}{4}$. Q.E.I.

N. B. The value of this annuity, as computed above from *Monte de Parcieux's* table, is £19.457,903, or 19l. 9s. 1d. $\frac{3}{4}$; which is greater than the value just now found for it by about a year and a quarter's purchase.

For a life of the age of 25 years.

CLXIX. When the given life is of the age of 25 years, we shall have $n (= 95 - 25) = 70$, and $\frac{r}{n} = \frac{1.035}{70}$, and $G =$ the geometrical

progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \&c.$ continued to the term $\frac{1}{r^{70}}$

or to 70 terms, which (by Mr. Smart's table, page 80,) is = 26.000,396

Therefore

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Therefore $\frac{7}{n}G$ is $= \frac{1.035}{70} \times 26.000,396 = \frac{26.910,411,060}{70} =$

384,434,443; and $\frac{1-r}{n}G$ is $= 1.000,000,000 - .384,434,443 =$

615,565,557; and $\frac{1-r}{n}G$ is $= \frac{.615,565,557}{.035} = 17,587,587.$

Therefore $1 \times \frac{1-r}{n}G$, or the value of an annuity of one pound a year

for a life of the age of 25 years, according to Mr. De Moivre's Hypothesis, is $= 1 \times 17,587,587 = \text{£}17,587,587$, or 17l. 11s. 9d. Q.E.I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is $\text{£}18,827,070$, or 18l. 16s. 6d. $\frac{1}{2}$; which is greater than the value just now found for it by about a year and a quarter's purchase.

CLXX. When the given life is of the age of 30 years, we shall have $n (= 95 - 30) = 65$, and $\frac{r}{n} = \frac{1.035}{65}$, and $G =$ the geometrical years.

progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \frac{1}{r^8} + \frac{1}{r^9} + \frac{1}{r^{10}} + \frac{1}{r^{11}} + \frac{1}{r^{12}} + \frac{1}{r^{13}} + \frac{1}{r^{14}} + \frac{1}{r^{15}} + \frac{1}{r^{16}} + \frac{1}{r^{17}} + \frac{1}{r^{18}} + \frac{1}{r^{19}} + \frac{1}{r^{20}} + \frac{1}{r^{21}} + \frac{1}{r^{22}} + \frac{1}{r^{23}} + \frac{1}{r^{24}} + \frac{1}{r^{25}} + \frac{1}{r^{26}} + \frac{1}{r^{27}} + \frac{1}{r^{28}} + \frac{1}{r^{29}} + \frac{1}{r^{30}} + \frac{1}{r^{31}} + \frac{1}{r^{32}} + \frac{1}{r^{33}} + \frac{1}{r^{34}} + \frac{1}{r^{35}} + \frac{1}{r^{36}} + \frac{1}{r^{37}} + \frac{1}{r^{38}} + \frac{1}{r^{39}} + \frac{1}{r^{40}} + \frac{1}{r^{41}} + \frac{1}{r^{42}} + \frac{1}{r^{43}} + \frac{1}{r^{44}} + \frac{1}{r^{45}} + \frac{1}{r^{46}} + \frac{1}{r^{47}} + \frac{1}{r^{48}} + \frac{1}{r^{49}} + \frac{1}{r^{50}} + \frac{1}{r^{51}} + \frac{1}{r^{52}} + \frac{1}{r^{53}} + \frac{1}{r^{54}} + \frac{1}{r^{55}} + \frac{1}{r^{56}} + \frac{1}{r^{57}} + \frac{1}{r^{58}} + \frac{1}{r^{59}} + \frac{1}{r^{60}} + \frac{1}{r^{61}} + \frac{1}{r^{62}} + \frac{1}{r^{63}} + \frac{1}{r^{64}} + \frac{1}{r^{65}}$, or to 65 terms, which (by Mr. Smart's tables, page 80,) is $=$

25,517,849. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{65} \times 25,517,849 = \frac{26,410,974,715}{65} =$

$= .406,322,672$; and $\frac{1-r}{n}G$ is $= 1.000,000,000 - .406,322,672 =$

593,677,328; and $\frac{1-r}{n}G$ is $= \frac{.593,677,328}{.035} = 16,962,209$;

Therefore $1 \times \frac{1-r}{n}G$, or the value of an annuity of one pound a year

for a life of the age of 30 years, according to Mr. De Moivre's Hypothesis, is $= 1 \times 16,962,209 = \text{£}16,962,209$, or 16l. 19s. 3d.

Bb 2

N. B. The

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £18.095,844, or 18*l.* 1*s.* 11*d.* which is greater than the value just now found for it by about a year and half a quarter's purchase.

For a life of the age of 35 years.

CLXXI. When the given life is of the age of 35 years, we shall have n ($= 95 - 35$) $= 60$, and $\frac{r}{n} = \frac{1.035}{60}$, and $G =$ the geometrical

progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{60}} =$ (by Mr.

Smart's tables, page 80,) 24.944,734. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{60} \times$

$$24.944,734 = \frac{25.817,799,690}{60} = .430,296,661; \text{ and } \frac{1-r}{n}G \text{ is } =$$

$$1.000,000,000 - .430,296,661 = .569,703,339; \text{ and } \frac{1-r}{n-1}G \text{ is } =$$

$$\frac{.569,703,339}{.035} = 16.277,238. \text{ Therefore } \frac{r}{1} \times \frac{1-r}{n}G, \text{ or the value}$$

of an annuity of one pound a year for a life of the age of 35 years, according to Mr. De Moivre's Hypothesis, is $= \frac{r}{1} \times 16.277,238 =$
 $=$ £16.277,238, or 16*l.* 7*s.* 6*d.* $\frac{1}{4}$. QEI.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £17.240,588, or 17*l.* 4*s.* 9*d.* $\frac{3}{4}$; which is greater than the value just now found for it by somewhat less than a year's purchase.

For a life of the age of 40 years.

CLXXII. When the given life is of the age of 40 years, we shall have n ($= 95 - 40$) $= 55$, and consequently $\frac{r}{n} = \frac{1.035}{55}$, and $G =$ the

geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{55}}$

$=$ (by Mr. Smart's tables, page 80,) 24.264,053. Therefore $\frac{r}{n}G$ is $=$

$$\frac{1.035}{55}$$

$$\frac{1.035}{55} \times 24.264,053 = \frac{25.113,294,855}{55} = .456,605,361; \text{ and } \frac{1-r}{n}G$$

$$= 1.000,000,000 - .456,605,361 = .543,394,639; \text{ and } \frac{1-r}{n}G$$

$$= \frac{.543,394,639}{.035} = 15.525,561. \text{ Therefore } \frac{\pounds}{1} \times \frac{1-\frac{r}{n}G}{r-1}, \text{ or the}$$

value of an annuity of one pound a year for a life of the age of 40 years, according to Mr. De Moivre's Hypothesis, is = $\pounds 1 \times 15.525,561 = \pounds 15.525,561$, or 15*l.* 10*s.* 6*d.* QEI.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is $\pounds 16.147,167$, or 16*l.* 2*s.* 11*d.* $\frac{1}{4}$; which is greater than the value just now found for it by something more than half a year's purchase.

CLXXIII. When the given life is of the age of 45 years, we shall have $n (= 95 - 45) = 50$, and consequently $\frac{r}{n} = \frac{1.035}{50}$, and $G =$ the age of 45

the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \frac{1}{r^8} + \frac{1}{r^9} + \frac{1}{r^{10}} + \frac{1}{r^{11}} + \frac{1}{r^{12}} + \frac{1}{r^{13}} + \frac{1}{r^{14}} + \frac{1}{r^{15}} + \frac{1}{r^{16}} + \frac{1}{r^{17}} + \frac{1}{r^{18}} + \frac{1}{r^{19}} + \frac{1}{r^{20}} + \frac{1}{r^{21}} + \frac{1}{r^{22}} + \frac{1}{r^{23}} + \frac{1}{r^{24}} + \frac{1}{r^{25}} + \frac{1}{r^{26}} + \frac{1}{r^{27}} + \frac{1}{r^{28}} + \frac{1}{r^{29}} + \frac{1}{r^{30}}$

$+ \frac{1}{r^{31}} =$ (by Mr. Smar's tables, page 78,) 23.455,617. Therefore

$$\frac{r}{n}G \text{ is } = \frac{1.035}{50} \times 23.455,617 = \frac{24.276,563,595}{50} = .485,531,271;$$

and $\frac{1-r}{n}G \text{ is } = 1.000,000,000 - .485,531,271 = .514,468,729;$

$$\text{and } \frac{1-r}{n}G \text{ is } = \frac{.514,468,729}{.035} = 14.699,106. \text{ Therefore } \frac{\pounds}{1} \times \frac{1-\frac{r}{n}G}{r-1},$$

or the value of an annuity of one pound a year for a life of the age of 45 years, according to Mr. De Moivre's Hypothesis, is = $\pounds 1 \times 14.699,106 = \pounds 14.699,106$, or 14*l.* 14*s.* 0*d.* QED.

N. B. The

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £14,716,120, or 14*l.* 14*s.* 3*d.* $\frac{1}{2}$; which is greater than the value just now found for it by only the trifling sum of 3*d.* $\frac{1}{2}$.

For a life of the age of 50 years.

CLXXIV. When the given life is of the age of 50 years, we shall have n ($= 95 - 50$) $= 45$, and consequently $\frac{r}{n} = \frac{1.035}{45}$, and $G =$

the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \dots + \frac{1}{r^{45}}$

$=$ (by Mr. Smart's tables, page 78,) 22.495,450. Therefore $\frac{r}{n}G$ is $=$

$$\frac{1.035}{45} \times 22.493,450 = \frac{23.282,790,750}{45} = 517,395,350; \text{ and } \frac{1-r}{n}G$$

is $= 1.000,000,000, - .517,395,350, = .482,604,650$; and $\frac{1-r}{r-1}G$

$$\text{is } = \frac{.482,604,650}{.035} = 13,788,704. \text{ Therefore } \frac{\frac{1-r}{n}G}{\frac{1-r}{r-1}}, \text{ or the}$$

value of an annuity of one pound a year for a life of the age of 50 years, according to Mr. De Moivre's Hypothesis, is $= \frac{\frac{1-r}{n}G}{\frac{1-r}{r-1}}$ £13,788,704, or 13*l.* 15*s.* 9*d.* $\frac{1}{2}$. Q.E.I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £13,183,083, or 13*l.* 3*s.* 8*d.* which, instead of being greater than the value just now found for it, (as has been the case in the preceding instances,) is less than that value by more than half a year's purchase.

For a life of the age of 55 years.

CLXXV. When the given life is of the age of 55 years, we shall have n ($= 95 - 55$) $= 40$, and consequently $\frac{r}{n} = \frac{1.035}{40}$, and $G =$ the

geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \dots + \frac{1}{r^{40}}$

$=$ (by Mr. Smart's tables, page 78,) 21,355,072. Therefore $\frac{r}{n}G$ is $=$

$$\frac{1.035}{40}$$

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$$\frac{1.035}{40} \times 21.355,072 = \frac{22.102,499}{40} = .552,562,487; \text{ and } \frac{1-r}{n}G$$

$$\text{is} = 1.000,000,000, - .552,562,487, = .447,437,513; \text{ and } \frac{r-1}{n}$$

$$\frac{1-r}{n}G \text{ is } = \frac{.447,437,513}{.035} = 12.783,928. \text{ Therefore } 1 \times \frac{1-r}{n}G,$$

or the value of an annuity of one pound a year for a life of the age of 55 years, is = $1 \times 12.783,928 = \pounds 12.783,928$, or 12*l.* 15*s.* 8*d.* QEI.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is $\pounds 11.691,801$, or 11*l.* 13*s.* 1*d.* which is less than the value just now found for it by more than a year's purchase.

CLXXVI. When the given life is of the age of 60 years, we shall have n ($= 95 - 60$) $= 35$, and consequently $\frac{r}{n} = \frac{1.035}{35}$, and $G =$ the age of 60 the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \text{&c.}$

$+\frac{1}{r^{35}} =$ (by Mr. Smart's tables, page 78,) 20.000,661. Therefore

$$\frac{r}{n}G \text{ is } = \frac{1.035}{35} \times 20,000,661 = \frac{20.700,684,135}{35} = .591,448,118;$$

$$\text{and } 1 - \frac{r}{n}G \text{ is } = 1.000,000,000, - .591,448,118 = 408,551,882;$$

$$\text{and } \frac{1-r}{n}G \text{ is } = \frac{.408,551,882}{.035} = 11.672,910. \text{ Therefore}$$

$$\frac{1-r}{n}G, \text{ or the value of an annuity of one pound a year for}$$

a life of the age of 60 years, according to Mr. De Moivre's Hypothesis, is = $1 \times 11.672,910 = \pounds 11.672,910$, or 11*l.* 13*s.* 5*d.* $\frac{1}{4}$. QEII.

N. B. The

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £10.104,074, or 10*l.* 2*s.* 1*d.* which is less than the value just now found for it by more than a year and a half's purchase.

For a life of the age of 65 years.

CLXXVII. When the given life is of the age of 65 years, we shall have n ($= 95 - 65$) $= 30$, and consequently $\frac{r}{n} = \frac{1.035}{30}$, and $G =$ the

geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c.$

$+ \frac{1}{r^{30}}$, = (by Mr. Smart's tables, page 78,) 18.392,045. Therefore

$\frac{r}{n}G$ is $= \frac{1.035}{30} \times 18.392,045 = \frac{19,035,766,575}{30} = .634,525,551;$

and $1 - \frac{r}{n}G$ is $= 1.000,000,000, - .634,525,552 = .365,474,448;$

and $\frac{1 - \frac{r}{n}G}{r - 1}$ is $= \frac{.365,474,448}{.035} = 10.442,127.$ Therefore

$\frac{\text{£}}{1} \times \frac{1 - \frac{r}{n}G}{r - 1}$, or the value of an annuity of one pound a year for a

life of the age of 65 years, according to Mr. De Moivre's Hypothesis is $= 1 \times 10.442,127 = \text{£}10.442,127$, or 10*l.* 8*s.* 10*d.* Q.EI.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £8.813,625, or 8*l.* 6*s.* 3*d.* $\frac{1}{4}$; which is less than the value just now found for it by more than two years purchase.

For a life of the age of 70 years.

CLXXVIII. When the given life is of the age of 70 years, we shall have n ($= 95 - 70$) $= 25$, and consequently $\frac{r}{n} = \frac{1.035}{25}$, and $G =$

= the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c.$
 $+ \frac{1}{r^{25}}$

$+\frac{1}{r+1} =$ (by Mr. Smar's tables, page 76,) 16.481,514. Therefore

$$\frac{1}{r}G \text{ is } = \frac{1.035}{25} \times 16.481,514 = \frac{17.058,366,990}{25} = .682,334,679;$$

and $\frac{1-r}{r}G$ is = 1.000,000,000 — .682,334,679 = .317,665,321;

$$\text{and } \frac{1-\frac{r}{n}G}{r-1} \text{ is } = \frac{.317,665,321}{.035} = 9.076,152. \quad \text{Therefore}$$

$$\frac{1}{1} \times \frac{1-\frac{r}{n}G}{r-1}, \text{ or the value of an annuity of one pound a year for a life}$$

of the age of 70 years, according to Mr. De Moivre's Hypothesis, is
 $= 1 \times 9.076,152 = \text{£}9.076,152, \text{ or } 9\text{l. } 1\text{s. } 6\text{d. } \frac{1}{2} \text{ QEI.}$

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is $\text{£}6.575,357$, or 6*l.* 1*s.* 6*d.* which is less than the value just now found for it by about two years and a half's purchase.

CLXXIX. If the foregoing values of an annuity of one pound for the lives of persons of the several ages of 1 year, 3 years, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70, years, which are derived from Mr. De Moivre's Hypothesis, are set down in a regular series, and the values of the same annuity for all the same lives, except the first, as computed above from Monsieur de Parcieux's table, are set down in an adjoining column, and the differences between the said values are likewise set down in a third column, they will together form the following table.

C c : T A B L E.

T A B L E IX.

Containing the conjectural values of an annuity of one pound a year for the lives of persons of several different ages, derived from Mr. De Moivre's Hypothesis, "That the decrements of human life, or the numbers of persons dying every year out of any given original number of persons living at the beginning of life, continue the same throughout the whole extent of human life;" and containing likewise the true values of the same annuity of one pound a year for all the same lives, except the first, (which is a life of the age of 1 year) computed according to M^{onsieur} de Parcieux's table of the probabilities of the duration of human life; upon a supposition, in both ways of computing the values of this annuity, that the interest of money is $3\frac{1}{2}$ per cent. and that the utmost extent of human life is less than 95 years; and containing likewise, in an adjoining column, the differences of the said values of the said annuity from each other.

Years of Age.	Values of a life-annuity of one pound a year, according to Mr. De Moivre's Hypothesis.	Values of the same life-annuity according to M ^{onsieur} de Parcieux's table of probabilities.	Differences of the said values.
1	£ 19.908,809	£ 19.987,654	0.212,192
3	19.775,462	20.526,716	0.918,491
5	19.608,225	20.752,981	1.587,634
10	19.165,347	20.137,194	1.453,260
15	18.683,934	19.457,903	1.298,255
20	18.159,648	18.827,070	1.239,483
25	17.587,587	18.095,844	1.133,635
30	16.962,209	17.240,588	0.963,350
35	16.277,238	16.147,167	0.621,606
40	15.525,561	14.716,120	0.017,014
45	14.699,106	13.183,083	0.605,621
50	13.788,704	11.691,801	1.092,127
55	12.783,928	10.104,074	1.568,836
60	11.672,910	8.313,625	2.128,502
65	10.442,127	6.575,357	2.500,795
70	9.076,152		

CLXXX. It appears from the foregoing table, in the 1st place, That, Remarks on from the age of 3 years to the age of 45 years, the values of life-annuities, the foregoing according to Mr. De Moivre's Hypothesis, are less than their true values, table. as computed above from Monsieur de Parcieux's table of probabilities;

And, 2dly, That during the said interval from the age of 3 years to the age of 45 years, the excess of the true value of a life-annuity of one pound, above its conjectural value, derived from Mr. De Moivre's Hypothesis, increases from £0.212,192, or less than a quarter of a year's purchase, to £1.587,634, or more than a year and a half's purchase, (which is its magnitude at the age of 10 years,) and then decreases again to £0.017,014, or less than one week's purchase, or, we may say, to 0;

And, 3dly, That, from the age of 45 years to the age of 70 years, the values of life-annuities, according to Mr. De Moivre's Hypothesis, are greater than their true values, as computed above from Monsieur de Parcieux's table of probabilities;

And, 4thly, That, during the said interval from the age of 45 years to the age of 70 years, the excess of the conjectural value of a life-annuity of one pound, (according to Mr. De Moivre's Hypothesis,) above its true value increases continually, as the age advances, or grows older, from 0.017,014, or 0, to £2.500,795, or more than two years and a half's purchase, which is its magnitude at the age of 70 years.

CLXXXI. Upon the whole matter, the differences of these values seem to me to be so considerable that I cannot but wonder that Mr. De Moivre should have thought fit to disregard them, and to compute his tables of life-annuities from his own Hypothesis, instead of following Dr. Halley's, or Monsieur de Parcieux's, or some other table of the probabilities of life deduced from observations. And this appears the more surprising when we consider, that the labour of deriving the value of a life-annuity for a life of any given age from that of a like annuity for a life only one year older, in strict conformity to a table of the probabilities of life, by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \frac{1}{1+V} | \text{£}$, is scarcely, if

at all, greater than that of computing the same annuity, according to Mr.

De Moivre's Hypothesis, by means of the expression $1 \times \frac{\text{£}}{r-1} | \frac{1-r}{r} G$.

C c 2

I would

I would therefore recommend it to all future calculators of the values of life-annuities, or at least to those who undertake only to calculate annuities for single lives, to lay aside all consideration of Mr. De Moivre's Hypothesis, and to compute those values from Monsieur de Parcieux's table of probabilities, or some other table of the same kind which they may esteem to be of still better authority, by means of the expression $\frac{1}{n} \times \frac{P}{P+d} \times \frac{1}{1+\sqrt[n]{L}}$ in the manner that has been above explained and exemplified in Art. 100, 101, 102, &c.—117.

End of the explanation of Mr. De Moivre's Hypothesis.

An account of Mr. Weyman Lee's method of computing the values of Life-annuities.

CLXXXII. Some gentlemen who have undertaken to write upon the subject of life-annuities, but who appear to have been unacquainted with the Mathematicks and the doctrine of chances, have considered the method of computing the value of a life-annuity above-delivered in Problem 11, Coroll. 2, Art. 31, page 28, and exemplified in many instances in the subsequent part of this tract, and which is that which all mathematical writers, (from Dr. Halley in the year 1692, down to Dr. Price and Mr. Morgan in their late treatises,) have universally adopted; —I say, some gentlemen have considered this method as erroneous. One of these gentlemen was the late Weyman Lee, Esq; a barrister at law and a benchet of the Society of the Inner Temple. This gentleman published a book in one volume octavo in the year 1738, upon the doctrine of life-annuities, intitled, “*An Essay on the method of ascertaining the value of annuities, and of Leases reduced to annuities certain, for one or more lives;*” and in the year 1751 he published a second pamphlet on the same subject, intitled, “*A valuation of annuities and leases certain for a single life.*” This latter pamphlet I have seen. And in it, I observe, he argues against the method of computing the values of life-annuities above-explained, in a style of great confidence, and with a high degree of contempt of his numerous mathematical adversaries, Dr. Halley, a Mr. H. B. Mr. Richards, Mr. Hodgson, Mr. Hayes, and Mr. De Moivre: but at the same time he expresses himself with great obscurity, as might naturally be expected from a person who endeavoured to invalidate so clear and certain a proposition. And, in lieu of the said method of computing the values of life-annuities (which he conceives himself to have shewn to be erroneous,) he gives us another method of estimating them, which is as follows.

CLXXXIII. When

CLXXXIII. When the value of an annuity for the life of a person of any given age is to be determined, we must look into Dr. Halley's, or Monsieur de Parcieux's, or some other, table of the probabilities of the duration of human life, to find what number of persons are therein represented as living at the said age. And then we must divide this number by 2, and look again into the said table, to find at what later age of life the number of persons represented therein as living will be reduced to this latter number, or to one half of the former number. And an annuity of one pound a year for the life of a person of the first, or given, age will be equal in value, says Mr. Lee, to a like annuity of one pound a year for a term of years certain, equal to the difference of the said two ages, or to the space of time in which the number of persons represented in the table as living at the given age will be reduced to half.

Thus, for example, if the value of an annuity for the life of a person of the age of 10 years is to be determined, we shall find in Monsieur de Parcieux's table above-mentioned, that 880 persons are there represented as living at the age of 10 years. The half of this number is 440. We must therefore, in the second place, look out in the same table the age at which the number of persons living is reduced to 440. Now we shall find, upon this inspection of the said table, that at the age of 61 years there will be 440 persons living, and at the age of 62 years 437 persons living, out of the said 880 persons who were living at the age of 10 years. Therefore at some intermediate age between 61 and 62, as, for instance, about the age of 61 years and nine months, we may suppose that the number of persons living, (out of the said original 880 persons, who were living at the age of 10 years,) will be 440; or, in other words, the original 880 persons, living at the age of 10 years, will be reduced to 440, or to one half, in the course of 51 years and 9 months. Therefore (according to Mr. Weyman Lee's method of valuing annuities,) an annuity of one pound a year for the life of a person of the age of 10 years is of the same value with an annuity of one pound a year for the space of 51 years and 9 months certain, or, in round numbers, for the space of 52 years.

CLXXXIV. According to this method of estimating life-annuities, an annuity of one pound a year for the life of a person of the age of 10 years, when the interest of money is $3\frac{1}{4}$ per cent. is £23.795,764, or 23*l.* 15*s.* 11*d.* that being the value of an annuity of one pound a year for a term certain of 52 years, when the interest of money is $3\frac{1}{4}$ per cent. as appears from Mr. Smart's tables, page 80.

CLXXXV. But

Of the difference of the values of life-annuities computed by this method from their true values.

CLXXXV. But we have seen above, in Art. 94, that the true value of an annuity of one pound a year for the life of a person of the age 10 years, when the interest of money is $3\frac{1}{2}$ per cent. is only £20.73914 or 20*l.* 14*s.* 9*d.* $\frac{2}{3}$; and by the other calculation of it, in Art. 114, it appeared to be only £20.752,981, or 20*l.* 15*s.* 0*d.* $\frac{2}{3}$. Therefore the foregoing value of it, obtained by the method of Mr. Weyman Lee, to 23*l.* 15*s.* 11*d.* is greater than its true value by no less than three years purchase. And in like manner it will be found upon trial, that in all other ages of life, except from the age of 74 years to the age of 83 years, the values of life-annuities obtained by this method of Mr. Lee, will be greater, and for the most part *very much* greater, than their true values.

CLXXXVI. But that the difference of these values of life-annuities found by Mr. Lee's method, from their true values, as computed according to the directions of Problem 11, Coroll. 2, may be the more apparent, I shall here present the reader with a table containing both sets of values of an annuity of one pound a year for the lives of persons of all ages, differing from each other by one year, from the age of 1 years to the age of 93 years, inclusively, computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. and containing likewise the differences of the said values, This table is as follows.

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T A B L E X.

containing the values of an annuity of one pound a year for the lives of persons of the ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively, upon a supposition that the probabilities of the duration of human life are such as they are represented to be in Monsieur de Parcieux's table of them, and that the interest of money is $3\frac{1}{2}$ per cent. computed according to two different methods, to wit, Mr. Weyman Lee's method just now mentioned in Art. 183, and the true method that has been adopted by Dr. Halley and all other mathematicians, and which has been explained above in Problem 11 and its corollaries; and containing likewise, in an adjoining column, the differences of the said values.

Years of Age.	Values of an annuity of one pound by Mr. Weyman Lee's method.	Values of the same annuity by Dr. Halley's method.	Differences of the said values.
3	£ 24.113,295	£ 19.987,654	£ 4.125,641
4	24.113,295	20.327,034	3.786,261
5	24.113,295	20.526,716	3.586,579
6	24.113,295	20.656,349	3.456,946
7	24.035,948	20.729,912	3.306,036
8	23.957,260	20.764,685	3.192,575
9	23.877,206	20.781,222	3.095,984
10	23.795,764	20.752,981	3.042,783
11	23.628,616	20.676,395	2.952,221
12	23.455,617	20.548,338	2.907,279
13	23.366,861	20.415,899	2.950,962
14	23.276,564	20.278,914	2.997,650
15	23.091,244	20.137,194	2.954,050
16	22.899,437	19.990,615	2.908,822
17	22.700,918	19.863,739	2.837,179
18	22.599,067	19.732,778	2.866,289
19	22.495,450	19.597,560	2.897,890
20	22.282,791	19.477,903	2.824,888
21	22.173,686	19.338,821	2.834,865
22	21.949,765	19.216,339	2.733,426
23	21.834,882	19.090,318	2.744,564
24	21.599,103	18.960,613	2.638,490

The Principles of the Doctrine of

Years of Age.	Values of an an- nuity of one pound by Mr. Weyman Lee's method.	Values of the same annuity by Dr. Halley's me- thod.	Differences of the said va- lues.
25	£ 21.478,137	£ 18.827,070	2 651,067
26	21.355,072	18.689,528	2.665,544
27	21.102,499	18.547,817	2.554,682
28	20.972,917	18.401,759	2.571,158
29	20.706,969	18.251,167	2.455,802
30	20.570,525	18.095,844	2.474,681
31	20.290,493	17.935,582	2.354,911
32	20.146,823	17.770,162	2.376,661
33	19.851,962	17.599,354	2.252,608
34	19.700,684	17.422,915	2.277,769
35	19.390,208	17.240,588	2.149,620
36	19.068,865	17.052,103	2.016,762
37	18.904,000	16.857,174	2.046,826
38	18.565,640	16.629,189	1.936,451
39	18.392,045	16.392,655	1.999,390
40	18.035,767	16.147,167	1.888,600
41	17.667,018	15.892,297	1.774,721
42	17.285,364	15.580,939	1.704,425
43	17.089,556	15.303,763	1.785,793
44	16.890,352	15.015,669	1.874,683
45	16.481,514	14.716,120	1.765,394
46	16.058,367	14.404,549	1.653,818
47	15.841,272	14.105,200	1.736,072
48	15.395,716	13.793,859	1.601,857
49	15.167,124	13.494,425	1.672,699
50	14.697,974	13.183,083	1.514,891
51	14.212,403	12.883,449	1.328,954
52	13.963,281	12.596,296	1.366,985
53	13.709,837	12.298,386	1.411,451
54	13.189,681	11.989,093	1.200,588
55	12.922,816	11.691,801	1.231,015
56	12.651,320	11.383,427	1.267,893
57	12.094,116	11.063,485	1.030,631
58	11.808,243	10.755,123	1.053,120
59	11.517,410	10.435,566	1.081,844
60	10.920,520	10.104,074	0.816,446
61	10.302,738	9.759,829	0.542,909
62	9.985,785	9.401,925	0.583,860
63	9.663,334	9.053,059	0.610,275
64	9.335,288	8.690,648	0.644,640

LIFE-ANNUITIES.

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Years of Age.	Values of an an- nuity of one pound by Mr. Weyman Lee's method.	Values of the same annuity by Dr. Halley's me- thod.	Differences of the said va- lues.
65	£ 8.662,023	£ 8.313,625	0.348,398
66	8.316,605	7.944,258	0.372,347
67	7.965,194	7.583,727	0.381,467
68	7.607,686	7.233,699	0.373,987
69	7.243,976	6.896,496	0.347,480
70	6.873,955	6.575,357	0.298,598
71	6.497,515	6.249,840	0.247,675
72	6.114,543	5.945,972	0.168,571
73	5.724,928	5.644,448	0.080,480
74	5.328,553	5.347,806	0.019,253
75	4.925,300	5.059,623	0.134,323
76	4.515,052	4.754,921	0.239,809
77	4.515,052	4.461,839	0.053,213
78	4.097,686	4.187,758	0.09,070
79	3.673,079	3.908,000	0.234,921
80	3.673,079	3.661,781	0.011,298
81	3.241,105	3.427,856	0.186,751
82	3.241,105	3.215,658	0.025,447
83	2.801,636	2.984,473	0.182,837
84	2.801,636	2.717,188	0.084,448
85	2.801,636	2.456,774	0.344,862
86	2.354,544	2.211,911	0.142,633
87	2.354,544	1.999,810	0.354,734
88	1.899,694	1.728,378	0.171,316
89	1.899,694	1.459,699	0.439,995
90	1.436,953	1.197,512	0.239,441
91	1.436,953	0.947,669	0.489,284
92	0.966,183	0.716,468	0.249,715
93	0.966,183	0.48309.	0.483,093
94	0.000,000		

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CLXXXVII.

Observations
on the differ-
ences of the
two sets of
values of life-
annuities set
down in the
foregoing
table.

CLXXXVII. From the foregoing table it appears, that at the age of 3 years the value of a life-annuity obtained by Mr. Lee's method is greater than its true value by more than 4 years purchase; and that, from the age of 3 years to the age of 10 years, inclusively, it exceeds the true value by more than three years purchase; and that, from the age of 10 years to the age of 37 years, inclusively, it exceeds the true value by more than two years purchase; and that, from the age of 37 years to the age of 59 years, inclusively, it exceeds the true value by more than one year's purchase; and that from the age of 59 years to the age of 74 years, inclusively, it exceeds the true value by less than one year's purchase, and that in that last interval, from the age of 59 years to the age of 74 years, its excess above the true value becomes gradually less and less, till, at the said latter age of 74 years, it is almost equal to 0. But, after the said age of 74 years, the value of a life-annuity obtained by the said method of Mr. Lee, is, for about 9 years, or till the age of 83 years, nearly equal to, but for the most part somewhat less than, its true value; after which it again becomes greater than the said true value at the age of 84 years, and continues to exceed the said true value throughout the whole remainder of the table, or in every following year of human life.

CLXXXVIII. The differences of the foregoing values of life-annuities, obtained by Mr. Lee's method of computation, from their true values, are so considerable before the ages of 39 or 40 years, that it would be by no means advisable to have recourse to those values in settling the prices of life-annuities that are to be granted for the lives of persons under those ages. And even to the age of 64 years, inclusively, I should think these differences too great to be neglected. But from the age of 65 years to the age of 85, or 86, years, those values might do tolerably well as a guide in the purchase of life-annuities, if a table containing the true values of the annuities for lives of those ages were not at hand. But, when tables of the true values of life-annuities, (computed strictly from a table of the probabilities of human life, and not from Mr. De Moivre's Hypothesis, or any other inaccurate supposition adopted by calculators for the sake of abridging their trouble,) are ready-computed to our hands, it seems to be imprudent and unsatisfactory to have recourse to any other values of them.

CLXXXIX. We have seen in the foregoing table, that Mr. Lee's method of estimating life-annuities is doubly erroneous. For it makes the values of them much greater than they ought to be throughout the greater part of human life; and, for a few years between the ages of 74 years and

Of Mr. Weyman Lee's attempts to prove the justness of his method of valuing life-annuities.

and 84 years, it makes them rather less than they ought to be. Yet Mr. Lee is very confident that it is the only right method of valuing life-annuities, and seems to wonder that any body can doubt of its being so. And, to remove these doubts, and fully establish the justness of his said method, he has given us what he calls two distinct proofs of it. These proofs are expressed in very vague and loose terms, inasmuch that I must confess myself unable to understand them. The words *chance* and *chances* occur in them very frequently, and sometimes the expression of *the chance of, or to, a chance*: but they seem to be used without any distinct meanings annexed to them. However, as this is a matter which relates to the very foundation of the doctrine of life-annuities, I should be glad that my readers would exercise their own judgements concerning it; and therefore, to enable them to do so, (as Mr. Lee's tracts are not now easily to be met with) I shall here present them with a transcript of the passage, in Mr. Lee's second pamphlet above-mentioned, which contains the said two supposed proofs of the justness of his method of valuing life-annuities. In this passage Mr. Lee has introduced some numbers into his argument, by way of example and illustration, which are grounded upon Dr. Halley's Breslaw table of the probabilities of the duration of human life, and on a supposition that the interest of money is 6 per cent. These numbers I have taken the liberty to alter so as to make them suit with Monsieur de Parcieux's table of probabilities and a supposition that the interest of money is $3\frac{1}{2}$ per cent. in order to avoid the unnecessary trouble and perplexity that might have arisen from the consideration of a different table of probabilities and a different rate of interest from those which are become familiar to us by our frequent use of them in the course of the foregoing articles. These alterations in Mr. Lee's numbers, it is evident, cannot in the least affect the force of his reasonings, (if there is any force in them,) and therefore may be made without injuring him. And, with these alterations in the numbers, the passage containing Mr. Lee's two supposed proofs of the justness of his method of valuing life-annuities, in pages 8, 9, and 10 of his second pamphlet above-mentioned, intitled, *A valuation of annuities for lives*, is in the words following.

Extracts from Mr. Lee's second tract, published in the year 1751.

CXC. " Since the author [Mr. H. B.] is so full of, and so learned in, the doctrine of chances, as he calls it, I will take the liberty of offering to his consideration an argument, or two, in behalf of my rule, according to his way of thinking. The value of the chances in this case depends on, or is one and the same thing as, the value of the annuities dependant on those chances. All the chances which an annuitant has on the life of A [a person of the age of 10 years] are agreed to be 880: and of those he has an even chance to enjoy a moiety, that is, 440.

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“ The annuities which attend those chances for the whole life are annuities
 “ for the term of 85 years, it being supposed possible that he may live for
 “ so many years. The total value of them for 85 years, as I compute it,
 “ is 27,036,803; and the value of those in possession for 52 years is
 “ 23,795,764, and of those in reversion 3,241,039. Since then the an-
 “ nuitant on the life of A has an even chance to 440 chances on his life,
 “ and the annuities will attend those chances;—and, since these 440
 “ chances will, and, from the nature of the life of man, of necessity must,
 “ arise in the first part of those 85 years, if they arise at all; (and ‘tis
 “ supposed in the case to be an even chance that so many chances will
 “ arise in his life :) he must then of necessity have the annuities attending
 “ those very chances, that is, he must have the annuities for the first 52
 “ years of his life. This author [Mr. H. B.], or whoever will undertake
 “ to answer this argument, must shew that ‘tis possible for an annuitant on
 “ the life of A to have the benefit of all these 440 chances (to which
 “ number of chances he has without dispute a right, or an even chance to
 “ enjoy,) unless he does enjoy those which arise in the first part of life and
 “ in immediate possession, that is, those of the greatest value. And this,
 “ I am very sure, he cannot shew.

“ Again; these positions are not contested:—That an annuity for a
 “ term of 52 years is in value £23,795,764;—and that an annuitant
 “ for the life of A has an even chance that A lives for 52 years.—‘Tis a
 “ maxim made use of by this author, and is certainly a true one, That,
 “ in estimating the value of annuities for a life, all the possible chances of
 “ life must be computed. From hence I argue thus. To estimate the
 “ value of an annuity for the life of A, we must compute all the possible
 “ chances on the life of A. The chance that A lives for 52 years is an
 “ even chance, and consequently is one of the possible chances on the life
 “ of A. Therefore, to estimate the value of an annuity for his life, we
 “ must compute this even chance. The even chance on this life is to a term
 “ for 52 years. A term for 52 years is in value £23,795,764. Therefore
 “ the value of an annuity for this life is £23,795,764.”

CXCI. These are the words of Mr. Weyman Lee’s proofs of the truth
 of his method of valuing life-annuities. But what meaning is to be found
 in them, or what train of reasoning Mr. Lee might pursue obscurely in his
 own mind, when he used them, I will not pretend to determine. And
 yet I am inclined to conjecture that his meaning (though certainly it is not
 expressed

expressed in the foregoing words,) might be as follows; to wit, "that, if a conjecture were to make 880 different grants, of one pound a year each, to many different grantees, all of the age of 10 years, for their respective lives, half the said grantees would be dead in the space of 52 years, and the other half would live beyond that term; and consequently 440 of the said 880 annuities would become extinct in the course of the said 52 years, and the other 440 of them would continue to be payable after the said 52 years. And hence, I presume, Mr. Lee was inclined to conclude, that therefore it would be the same thing, in point of advantage, to the grantor of these 880 annuities, whether he granted them to the said 880 persons for the life of each of them respectively, or whether he granted them to the said persons and their executors, or other representatives, for a term certain of 52 years; because that, if he had, first, granted them for a term certain of 52 years, and afterwards was, by the consent of all parties, to change those grants into grants of the same annuities of one pound each to the same 880 persons of the age of 10 years for the lives of the several grantees respectively, *the burthen* that would fall on the said grantor in consequence of this change in the nature of the grants, by means of the payments he would thereby be obliged to make after the expiration of the term of 52 years, to the 440 annuitants who would live beyond that term, would be counter-balanced by *the profit*, or saving, that would accrue to him before the end of the said term of 52 years by the extinction of the annuities that had been granted to the other 440 annuitants who would have died in the course of the said term." This is the only argument that can imagine to have been intended to be expressed in the foregoing passage of Mr. Lee's pamphlet. But, if this was the argument Mr. Lee relied on, he was greatly mistaken in thinking it a just one, though it may, at first sight, appear plausible. For the truth is, that the profit accruing to the grantor of the 880 annuities of one pound by the aforesaid change in his bargain, by means of the extinction of the annuities that had been granted to the 440 annuitants who had died before the end of the 52 years, will *much more than counter-balance* the burthen that would fall on him, in consequence of such a change of his first bargain, by means of the payments he would thereby be obliged to make to the other 440 annuitants who would live beyond the said term of 52 years. And one very obvious reason why we should expect that the said *profit* should *more than counter-balance* the said *burthen*, in so young an age as this of 10 years, is, that the payments which constitute the said profit, namely, the payments of the annuities saved by the deaths of the 440 annuitants who die before the end of the said 52 years, are *much less remote*, and consequently *much more valuable*, than the payments which constitute the said burthen, or the payments incurred by the obligation of continuing the annuities, during their respective lives, to the other 440 annuitants who will live beyond the said term of 52 years. In what precise proportion, or degree, the said profit will

Mr. Lee's mistake in the said argument supposed to be contained in the foregoing extracts.

will exceed the said burthen, is a nice question, and can only be determined by observing, in Monsieur de Parcieux's table of the probabilities human life, how many of the 880 persons who are there represented living at the age of 10 years, (and to whom we suppose annuities of a pound to have been granted for their respective lives,) will die off in the individual year of the whole term of 85 years through which it is possible that their lives may be extended, and computing, first, the values of several payments that will be saved in each of the first 52 years of the term of 85 years, by the deaths of those 440 persons who will die in the course of the said 52 years, and, secondly, the values of the payments that will be incurred in each of the remaining 33 years of the said term of years after the 52d year, by the continuance of the lives of the other annuitants, who will live beyond the said 52 years, and then comparing the sum of the values of the former payments with the sum of the values of the latter payments. This would be a nice and tedious inquiry, and would be tantamount to the computation of the value of an annuity of one pound for the life of a person of the age of 10 years in the manner explained above in Problem 11 and its corollaries, and exemplified in Art. 94. But it is sufficient to invalidate the foregoing argument, (which I have supposed to be that which was meant to be advanced by Mr. Lee, that the *exact equality* of the said profit to the said burthen (which in the argument is taken for granted, as a thing self-evident,) is by no means apparent; but that there is, on the contrary, good reason to suppose (without going into the nice inquiry just now mentioned,) that the profit in this case of an annuity for a life of the age of 10 years, is *greater* than the said burthen. For then it will not be the same thing (as is supposed to be in the said argument) in point of advantage, to the grantor of the said 880 annuities of one pound to as many grantees, all of the age of 10 years, whether he grant them the said annuities for the term of 52 years certain, or whether he grant them for the lives of each several grantee; the latter bargain will be less burthenfome to the grantor than the former, and therefore he ought to receive a less price from the said 880 grantees in the latter case than in the former. And consequently, it will follow from Art. 21, that, when the grantor makes only one such grant of an annuity of one pound to a person of the age of 10 years, he ought to receive a less price for it than he ought to receive for the grant of the same annuity for a term certain of 52 years; or, in other words, there is reason to suppose (even without going into the nice inquiry above-mentioned) that the value of an annuity of one pound a year for the life of a person of the age of 10 years is less than the value of the same annuity for a term certain of 52 years; contrary to what Mr. Lee has asserted.

CXCII. The only way of determining truly the value of any proposed annuity is that which is explained above in Art 21, and made the foundation of this whole doctrine, namely, to suppose the grantor of the annuity to make many more such grants to many other grantees, all of the same age as the first, or proposed, grantee, and then to inquire what ought to be paid by each of the said grantees to the grantor of all said annuities, in order to make him, at the close of the whole transaction, or when all the said annuitants shall be dead, be neither a gainer nor a loser by the sum total of all his bargains; upon a supposition that he proves the money he receives from the said grantees, as the price of his annuities, at compound interest according to a certain given rate, and that the grantees of the said annuities die off every year in the proportions represented in some particular table of the probabilities of the duration of human life that is adopted as the ground of the calculation. At price, and no other, is the *true, or fair, value* of such a life-annuity: and, whenever we attempt to estimate a life-annuity without thus supposing a great number of the like annuities to be granted at the same time to several persons of the same age as the proposed annuitant, we shall find ourselves bewildered and confused, and without any solid foundation to build upon. And I have no doubt that it is only to the want of such a *criterion, measure*, whereby to judge of the true value of a life-annuity, or, perhaps I ought to say, of such a *definition* of such true value, that we ought ascribe the obscurity and confusion with which Mr. Weyman Lee treats this subject, both in the passage above-cited from him, in which he deavours to prove the rectitude of his own erroneous method of estimating life-annuities, and in other parts of the same tract, in which he endeavours to prove the falshood of the method which has been used by Dr. Halley, and all other mathematicians, for that purpose, and which has been above explained and illustrated in Prob. 11 and its corollaries, and several of the following articles. All this perplexity would have been avoided, if he had previously settled clearly and distinctly in his own mind what he meant by the *fair price, or value*, of a life-annuity.

CXCIII. I have been induced to enlarge the more on the falshood of his erroneous method of valuing life-annuities, because I have observed, that not only Mr. Weyman Lee in the year 1751, but a later and much older writer on the same subject, Mr. Dale, in his useful book on life-annuities, intituled, *Calculations deduced from first principles*, &c. published in the year 1772, seems likewise to be persuaded of its truth. See the said book of Mr. Dale, article 2d of the Addenda.

End of the examination of Mr. Weyman Lee's erroneous method of valuing Life-annuities.

CXCIV. From these digressions, concerning Mr. De Moivre's Hypothesis for abridging the computations of life-annuities, and Mr. Weyrauch's erroneous method of valuing those annuities, I now return to the only true and accurate method of computing the values of the said annuities, which is that which has been above explained in Problem 11 and its corollaries. From this method, (as we have shewn above in Art. 100,

101, and 102,) the expression $\frac{1}{r} \times \frac{P}{P+d} \times \frac{1}{1+V|K}$ may be derived,

by which the value of an annuity of one pound for any proposed life may be computed from the value of the same annuity for a life that is one year older. Nothing can well be expected more concise and convenient for the purpose of computing a table of life-annuities, than this expression. Nevertheless it is liable to this one inconvenience, that, as the values of life-annuities for different ages are deduced by it one from another, in regular succession from the older ages to the younger, an error that should happen to be made in computing the value of one of these life-annuities would affect the values of all the following life-annuities, or of the annuities for all the ages that were younger than that in which the error was originally made. This inconvenience we have in some measure endeavoured to guard against in Art. 120, by shewing how an uncommonly large increase, or decrease, in the differences of the values of these successive life-annuities will, for the most part, afford a just ground of suspicion that some mistake has been made in the computation of the last value by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \frac{1}{1+V|K}$, and thereby will induce the calculator to revise his last computation; in consequence of which revision such mistake, if any has been made, will be discovered. But Mr. Morgan in his useful and learned tract upon life-annuities, intitled, *The doctrine of annuities and assurances on lives and survivorships*, (pages 59, 60, 61, &c.—68,) has given us a much better and more satisfactory method of removing this inconvenience, and proving the truth of the several operations, as they arise. This method may be explained as follows.

Of Mr. Morgan's method of proving the truth of the computations of the values of Life-annuities, as fast as they are made.

CXCV. Let A denote the number of persons represented in any table of the probabilities of the duration of human life as living at the youngest age that is set down in the said table; which age in Monsieur de Parcieux's table is the age of 3 years. Secondly, Let P be the number of persons represented in the said table as living at any subsequent, or older, age, consisting of N years; and let n be the number of years by which the said

age of N years exceeds the youngest age in the table. And, thirdly, let the value of one pound together with its interest for one year.

Then, it is evident, that the value of a single payment of one pound to be received at the end of n years by a person of the youngest age in the table, if the said person shall so long live, that is, if he shall live to the

age of N years, will be $\frac{1}{r^n} \times \frac{P}{A}$, or $\text{£}1 \times \frac{P}{Ar^n}$. In Monsieur de

Parcieux's table of probabilities the number of persons represented as living at the age of 3 years (which is the youngest age in the table,) is 1000; and consequently A is = 1000, and $\text{£}1 \times \frac{P}{Ar^n}$ is = $\text{£}1 \times \frac{P}{1000r^n}$.

An expression of the value of a single future payment of one pound, to be received by a person of the youngest age in the table, after a given number of years, if he shall be then alive.

CXCVI. Further, if P' denote the number of persons represented in the table of probabilities as living at the age of $N+1$ years, and P'' the number living at the age of $N+2$ years, and P''' the number living at the age of $N+3$ years, and so on throughout the remainder of the table, the number annexed, (in Roman figures,) to the top of the letter P being every where the same with the number of years added to N , and the said letter P , with such number so annexed, denoting the number of persons living at the age denoted by N with the same number added to it; it is evident that the value of a remote annuity of one pound a year for the life of a person of the youngest age in the table, to commence at the end of n years, or whereof the first payment is to be

An expression of the value of a remote life-annuity of one pound a year, for the life of a person of the youngest age in the table, to commence at a given future age.

received at the end of $n+1$ years, will be equal to $\frac{1}{A} \times$ the series

$$\frac{P'}{r^{n+1}} + \frac{P''}{r^{n+2}} + \frac{P'''}{r^{n+3}} + \frac{P^{iv}}{r^{n+4}} + \frac{P^v}{r^{n+5}} + \frac{P^{vi}}{r^{n+6}} + \frac{P^{vii}}{r^{n+7}} + \&c.$$

continued to the end of the table, or to the utmost possible extent of human life, which in Monsieur de Parcieux's table is 95 years; or it will

$$\text{be equal to } \text{£}1 \times \text{the series } \frac{P'}{Ar^{n+1}} + \frac{P''}{Ar^{n+2}} + \frac{P'''}{Ar^{n+3}} + \frac{P^{iv}}{Ar^{n+4}}$$

$$+ \frac{P^v}{Ar^{n+5}} + \frac{P^{vi}}{Ar^{n+6}} + \frac{P^{vii}}{Ar^{n+7}} + \&c. \text{ continued to the end of the}$$

table, or to the utmost possible extent of human life.

The Proposition that is the ground of Mr. Morgan's method above mentioned.

CXCIV. Now this remote annuity of one pound a year for the life of a person of the youngest age in the table is equal to the product that arises by multiplying the value of a like annuity of one pound a year for the life of a person of the age of N years, into $\text{£}1 \times \frac{P}{A_n}$, or the present value of the above-mentioned single payment of one pound to be made to a person of the youngest age in the table at the end of n years, or when he shall have attained the age of N years, if he shall so long live; or, if we put V for the value of an annuity of one pound a year for the life of a person of the age of N years, and R for the value of a remote annuity of the same sum of one pound a year for the life of a person of the youngest age in the table to commence at the distance of n years, or whereof the first payment is to be received at the end of $n+1$ years, we shall have

$$R = V \times 1 \times \frac{P}{A_n}.$$

DEMONSTRATION.

For V , or the value of an annuity of one pound a year for the life of a person of the age of N years, is (by Art. 86 and 87) = $\text{£}1 \times$ the series $\frac{P}{P_r} + \frac{P^{i1}}{P_{r^2}} + \frac{P^{i11}}{P_{r^3}} + \frac{P^{i1v}}{P_{r^4}} + \frac{P^v}{P_{r^5}} + \frac{P^{vi}}{P_{r^6}} + \frac{P^{vii}}{P_{r^7}} + \&c.$ continued to the end of the table of probabilities, or to the utmost extent of human life. Therefore $\text{£}1 \times \frac{P}{A_n} \times V$ is = $\text{£}1 \times \frac{P}{A_n} \times \text{£}1 \times$ the series $\frac{P}{P_r} + \frac{P^{i1}}{P_{r^2}} + \frac{P^{i11}}{P_{r^3}} + \frac{P^{i1v}}{P_{r^4}} + \frac{P^v}{P_{r^5}} + \frac{P^{vi}}{P_{r^6}} + \frac{P^{vii}}{P_{r^7}} + \&c.$ continued to the end of the table of probabilities, and consequently is = $\text{£}1 \times$ the series $\frac{P^i}{A_{n+1}} + \frac{P^{i1}}{A_{n+2}} + \frac{P^{i11}}{A_{n+3}} + \frac{P^{i1v}}{A_{n+4}} + \frac{P^v}{A_{n+5}} + \frac{P^{vi}}{A_{n+6}} + \frac{P^{vii}}{A_{n+7}} + \&c.$ continued to the end of the table of probabilities. But, by Art. 196, R , or the value of a remote annuity

annuity of one pound a year for the life of a person of the youngest age in the table of probabilities, to commence at the end of n years, or whereof the first payment is to be received at the end of $n+1$ years, is $= \text{£}1 \times$ the series $\frac{P_i}{A^{n+1}} + \frac{P_{i+1}}{A^{n+2}} + \frac{P_{i+2}}{A^{n+3}} + \frac{P_v}{A^{n+4}} + \frac{P_v}{A^{n+5}} + \frac{P_i}{A^{n+6}} + \frac{P_{vii}}{A^{n+7}} + \&c.$ continued to the end of the table of probabilities. Therefore $\text{£} \frac{P}{A^n} \times V$, or $V \times \text{£} \frac{P}{A^n}$, is $= R$. QED.

CXCVIII. From the foregoing proposition Mr. Morgan's rule for proving the truth of a computation of a life-annuity for a life of any given age is an easy consequence. This rule is as follows. Let V be put for the value of an annuity of one pound a year for the life of a person of the given age of N years, which number of years exceeds the number of years in the youngest age set down in the table of probabilities by n years.

Then, in order to try whether V has been rightly computed, compute the quantity $\text{£}1 \times \frac{P}{A^n}$, or the value of a single future payment of one pound to be made to a person of the youngest age set down in the table of probabilities at the end of n years, if such person shall then be living; and likewise compute the several terms of the series $\frac{P_i}{A^{n+1}} + \frac{P_{i+1}}{A^{n+2}} + \frac{P_{i+2}}{A^{n+3}} + \frac{P_v}{A^{n+4}} + \frac{P_v}{A^{n+5}} + \frac{P_{vii}}{A^{n+6}} + \frac{P_{vii}}{A^{n+7}} + \&c.$ continued to the end

of the table of probabilities, or to the utmost extent of human life; and then add the terms of this series up together into one sum total, which call S : and, lastly, multiply V into $1 \times \frac{P}{A^n}$. And, if the product of this multiplication is equal to $\text{£}1 \times S$, or if $V \times \frac{P}{A^n}$ is equal to S , we may conclude that V , or the value of the said annuity of one pound a year for a life of the age of N years, has been rightly computed.

Proof of the
said rule.

For it is evident from Art. 196 that $1 \times S$ is the value of a remaining annuity of one pound a year for the life of a person of the younger age in the table, to commence at the distance of n years, so that the first payment thereof shall be received at the distance of $n+1$ years, when

value is called R in Art. 197; that is, $1 \times S$ is $= R$. Therefore when $\frac{R}{V \times 1} \times \frac{P}{A^n}$ is equal to $1 \times S$, it will be equal likewise to R , and

consequently (by Art. 197,) the value denoted by V must have been rightly computed. QED.

An account of
the construction
of the
table of num-
bers next en-
suing, or
Table XI.

CXCIX. To make the manner of applying this rule of Mr. Morgan more apparent, it will be proper to set down again in regular order, in a new table, the values of the life-annuities which we computed above in Art. 108, ——— 115, from Monsieur de Parcieux's table by means of the expression $\frac{1}{1.035} \times \frac{P}{P-1} \times \frac{1}{1+V} \times \text{£}$. And, as, in obtaining the said

values one from another by means of that expression, we proceeded upwards, or from the older lives to the younger, it will be convenient to set down the said values in the same order in the following table in a column adjoining to another which contains the numbers of years in the corresponding ages. After these two columns of the ages of the lives and the values of the annuities, I shall set down, in a third column, the present values of a single payment of one pound to be received by a person of the age of 3 years (which is the youngest age in Monsieur de Parcieux's table of probabilities,) at the ends of 91 years, 90 years, 89 years, 88 years, and every following number of years respectively (reckoning in this backward order) to the end of one year, or at the ages of 94 years, 93 years, 92 years, 91 years, and every following younger age (proceeding by difference of one year,) to the age of 4 years; which series, it is evident

will contain in it all the terms of the series $\frac{P}{A^{n+1}} + \frac{P}{A^{n+2}} + \frac{P}{A^{n+3}}$

$+ \frac{P}{A^{n+4}} + \frac{P}{A^{n+5}} + \frac{P}{A^{n+6}} + \frac{P}{A^{n+7}} + \text{\&c.}$ in whatever part of

the table of probabilities the number P be taken, or whatever be the number of years denoted by $N+1$, or by N . And then, in a fourth column, I shall set down the sums of the terms in the foregoing series of values contained in the third column, as they arise; so that every term in the

T A B L E XI.

Consisting of five columns of numbers; in the first of which the number of years in the several ages of human life, that differ from each other by a year, from the age of 94 years to the age of 3 years, inclusively, are set down in regular order; and in the second column are set down the several values of an annuity of one pound a year for the lives of persons of the several ages set down in the first column, respectively; computed from Monsieur de Parcien's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{4}$ per cent. and in the third column are set down the present values of a single payment of one pound to be received by a child of the age of 3 years at the ends of 91 years, 90 years, 89 years, 88 years, and of every following lesser number of years down to one year, respectively, if the child shall live to the ends of the said years; and in the fourth column are set down the numbers that arise by the continual addition of the numbers set down in the third column, so that each number in the said fourth column is equal to the sum of all the numbers in the third column that are placed above it, or that correspond to the preceding, or older, ages; and in the fifth and last column are set down the products that arise by multiplying the terms of the second column (or the values of the life-annuities for the several ages set down in the first column,) by the corresponding terms of the third column, respectively.

Years of Age.	Values of a life-annuity of one pound a year.	Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.	Sums of the values in the third column.	Products of the multiplication of the numbers in the second and third columns.
94	£ 0.000,000	£ 0.000,043	£ 0.000,043	£ 0.000,043
93	0.483,091	0.000,090	0.000,043	0.000,133
92	0.716,468	0.000,187	0.000,133	0.000,321
91	0.947,669	0.000,339	0.000,320	0.000,659
90	1.197,512	0.000,551	0.000,659	0.001,211
89	1.459,699	0.000,830	0.001,210	

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Years of age.	Value of a life- annuity of one pound a year.	Present value of a single payment of one pound, to be received by a child of the age of 3 years, at the end of 91, 90, 89, 88, &c. years, if he shall be living at the end of those years respectively.	Sum of the va- lues in the third column.	Products of the multiplication of the numbers in the second and third columns.
88	£ 1,728,378	£ 0.001,181	£ 0.002,1040	£ 0.002,041
87	1,999,810	0.001,612	0.003,221	0.003,223
86	2,211,911	0.002,186	0.004,1833	0.004,835
85	2,456,774	0.002,858	0.007,019	0.007,021
84	2,717,188	0.003,636	0.009,877	0.009,879
83	2,984,473	0.004,529	0.013,513	0.013,516
82	3,215,658	0.005,612	0.018,042	0.018,046
81	3,427,856	0.006,901	0.023,654	0.023,655
80	3,661,781	0.008,345	0.030,555	0.030,557
79	3,908,000	0.009,955	0.038,900	0.038,904
78	4,187,758	0.011,667	0.048,855	0.048,858
77	4,461,839	0.013,566	0.060,522	0.060,529
76	4,754,921	0.015,583	0.074,088	0.074,095
75	5,059,623	0.017,724	0.089,671	0.089,676
74	5,347,806	0.020,083	0.107,395	0.107,399
73	5,644,448	0.022,586	0.127,478	0.127,485
72	5,945,972	0.025,239	0.150,064	0.150,070
71	6,249,840	0.028,050	0.175,303	0.175,308
70	6,575,357	0.030,928	0.203,353	0.203,362
69	6,896,496	0.033,372	0.234,281	0.234,287
68	7,233,699	0.037,085	0.268,253	0.268,261
67	7,583,727	0.040,263	0.305,338	0.305,343
66	7,944,258	0.043,505	0.345,601	0.345,614
65	8,313,625	0.046,805	0.389,106	0.389,119
64	8,690,648	0.050,160	0.435,911	0.435,922
63	9,053,059	0.053,693	0.486,071	0.486,085
62	9,401,925	0.057,411	0.539,764	0.539,773
61	9,759,829	0.061,188	0.597,175	0.597,184
60	10,104,074	0.065,159	0.658,363	0.658,371
59	10,435,566	0.069,834	0.723,522	0.723,539
58	10,755,123	0.073,720	0.792,856	0.792,867
57	11,063,485	0.078,329	0.866,576	0.866,591
56	11,383,427	0.083,008	0.944,905	0.944,915
55	11,691,801	0.087,919	1.027,913	1.027,931
54	11,989,093	0.093,072	1.115,832	1.115,848

Years of Age.	Values of a life-annuity of one pound a year.	Present value of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.	Sum of the values in the third column.	Products of the multiplication of the numbers in the second and third columns.
53	£ 12.298,386	£ 0.098,300	£ 1.208,904	£ 1.208,931
52	12.596,296	0.103,779	1.307,204	1.307,231
51	12.883,449	0.109,521	1.410,983	1.411,008
50	13.183,083	0.115,339	1.520,504	1.520,533
49	13.494,425	0.121,225	1.635,843	1.635,861
48	13.793,859	0.127,382	1.757,068	1.757,089
47	14.105,200	0.133,601	1.884,450	1.884,468
46	14.404,549	0.140,100	2.018,051	2.018,077
45	14.716,120	0.146,654	2.158,151	2.158,177
44	15.015,669	0.153,495	2.304,805	2.304,830
43	15.303,763	0.160,635	2.458,300	2.458,320
42	15.580,939	0.168,087	2.618,935	2.618,953
41	15.847,705	0.175,864	2.787,022	2.787,040
40	16.104,542	0.183,980	2.962,886	2.962,904
39	16.351,906	0.192,448	3.146,866	3.146,891
38	16.590,228	0.201,283	3.339,314	3.339,330
37	16.819,920	0.210,502	3.540,597	3.540,626
36	17.016,528	0.220,440	3.751,099	3.751,123
35	17.206,612	0.230,816	3.971,539	3.971,561
34	17.390,462	0.241,649	4.202,355	4.202,387
33	17.568,352	0.252,957	4.444,004	4.444,037
32	17.740,543	0.264,761	4.696,961	4.697,003
31	17.907,280	0.277,080	4.961,722	4.961,748
30	18.068,798	0.289,938	5.238,802	5.238,831
29	18.225,318	0.303,357	5.528,740	5.528,777
28	18.377,051	0.317,360	5.832,097	5.832,140
27	18.524,196	0.331,971	6.149,457	6.149,495
26	18.666,944	0.347,216	6.481,428	6.481,461
25	18.805,476	0.363,122	6.828,644	6.828,682
24	18.939,963	0.379,715	7.191,766	7.191,788
23	19.070,569	0.397,026	7.571,481	7.571,511
22	19.197,449	0.415,083	7.968,507	7.968,534
21	19.320,751	0.433,918	8.383,590	8.383,621
20	19.440,616	0.453,563	8.817,508	8.817,544

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Years of Age.	Values of a life- annuity of one pound a year.	Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the end of those years respectively.	Sums of the va- lues in the third column.	Products of the multiplication of the numbers in the second and third columns.
19	£ 19.581,000	£ 0.473,474	£ 9.271,071	£ 9.271,094
18	19.716,914	0.494,224	9.744,545	9.744,572
17	19.848,510	0.515,847	10.238,769	10.238,809
16	19.976,052	0.538,378	10.754,616	10.754,666
15	20.123,320	0.561,191	11.292,994	11.293,026
14	20.265,616	0.584,943	11.854,185	11.854,230
13	20.403,141	0.609,669	12.439,128	12.439,162
12	20.536,088	0.635,410	13.048,797	13.048,835
11	20.664,641	0.662,206	13.684,207	13.684,249
10	20.741,729	0.691,671	14.346,413	14.346,451
9	20.770,473	0.724,015	15.038,084	15.038,134
8	20.754,438	0.759,459	15.762,099	15.762,144
7	20.720,153	0.797,369	16.521,558	16.521,607
6	20.647,176	0.838,806	17.318,927	17.318,975
5	20.518,022	0.884,967	18.157,733	18.157,772
4	20.318,825	0.937,197	19.042,700	19.042,741
3	19.979,961		19.979,897	

In this table the numbers in the fifth column agree with those in the fourth column in either the five or the six first places of figures; and those of the last year, or that correspond to the age of 4 years, to wit, £19.042,741 and £19.042,700, agree with each other in the six first places of figures, their difference being only £0.000,041, or 41 millionth parts of a pound. And therefore we may safely conclude that all the values of an annuity of one pound a year set down in this table have been rightly computed to the same degree of exactness, or to a least five places of figures.

Of the degree of exactness of the values of life-annuities given in the foregoing table.

Ff

CC. The

An account of an error in the computation of Table XI, which makes the values of an annuity of one pound for lives under the age of 41 years a little too great.

CC. The reader will observe that the numbers set down in the foregoing table as the values of annuities of one pound a year for the lives of persons under the age of 42 years differ a little from the values of them exhibited in Table III, page 131, et seq. This is owing to an error in the calculation of the value of an annuity for a life of 41 years of age in that table, which error runs through all the rest of the table, or all the values of it that are derived from that value, that is, all the values of it for lives of younger ages than that of 41 years. The value of an annuity of one pound a year for a life of 41 years is equal to $\frac{1}{1.035} \times \frac{64}{69}$ $\times$ £16.580,939, which in page 123 (where the values afterwards set down in Table III, are computed,) is made to be equal to $\frac{1}{1.035} \times$ £16.448,528, or £15.892,297, whereas it is in truth equal to $\frac{1}{1.035} \times$ £16.402,375, or £15.847,705. But the difference between

these two values, £15.892,297 and £15.847,705, is only £0.044,592, or about 10d. $\frac{2}{3}$, which is but a trifling difference upon a sum of near 15l. 17s. And the differences between the subsequent values in the two tables are still less than 10d. $\frac{2}{3}$. Thus, for example, the difference between £17.770,162, the value of an annuity of one pound a year for a life of the age of 32 years in Table III, page 132, and £17.740,543, the value of the same annuity given in the last table, is £0.029,619; which is less than 7d. $\frac{2}{3}$; and the difference between £20.752,981, the value of an annuity of one pound a year for a life of the age of 10 years, in Table III, page 131, and £20.741,729, the value of the same annuity given in the last table, is £0.011,252; which is less than 2d. $\frac{2}{3}$; and the difference between £19.987,654, the value of an annuity of one pound a year given in Table III, page 131, for a life of only 3 years of age (which is the youngest age in the table,) and £19.979,961, the value of the same annuity given in the last table, is £0.007,693; which is less than 2d. We may therefore consider the values given in Table III as being exact to all purposes of practice, or as co-inciding with those in the last table, which has been computed without any mistake. And therefore I did not think it worth while to cause the errors of Table III, and of some articles that follow it and refer to it, to be corrected, when I had discovered them, which was not till after the sheets in which that table and those subsequent articles are contained had been printed off. I discovered these errors by means of Mr. Morgan's method, above-mentioned, of proving the truth of these computations as we go on. For when, in computing the fifth column in the last table (which consists of the products of the numbers in

second and third columns,) I came to the product belonging to the age of 41 years, I found that the said product was greater than it ought to be, exceeded by a greater difference than it ought to have done the corresponding number in the fourth column; and thereupon I computed anew the value of an annuity of one pound a year for a life of the age of 41 years from that of the like annuity for a life of the age of 42 years, by

$$\text{means of the expression } \frac{1}{1.035} \times \frac{643}{650} \times £16.580,939, \text{ and found that}$$

the said expression was not equal to $\frac{1}{1.035} \times £16.448,528$, or $£15.892,297$;

as it is supposed to be in page 123,) but to $\frac{1}{1.035} \times £16.402,375$, or

£15,847,705. And, having thus found out and corrected this error

belonging to the age of 41 years, I computed the values of an annuity of one pound a year for lives of all ages younger than 41 years over again

by means of the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \frac{1}{1+V|k}$, and proved them,

as I went on, by Mr. Morgan's method above-mentioned, and therefore I am confident that this last table is correct.

CCI. Having now gone through every thing I proposed to offer concerning the manner of computing the values of annuities for single lives, I shall proceed to exhibit several different tables of those values suited to the several following rates of the interest of money, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. All computed fairly and strictly from Monsieur de Parcieux's table of the probabilities of the duration of human life above-mentioned by

means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \frac{1}{1+V|k}$. One of them, to wit,

the fourth, or that which relates to the interest of $3\frac{1}{2}$ per cent. is the same with the first and second columns of Table XI, which is given above in Art. 199. I have inserted this part of the said table a second time in its proper place among the other tables, because I thought it would be more convenient for the readers who shall want to consult and make use of it, to look for it there than in the former parts of this tract. But I have there set down only the said two first columns of Table XI, or the numbers of years in the several ages of human life and

the numbers that exhibit the values of the annuities that correspond to them, without the additional numbers that are contained in the third, fourth, and fifth columns of Table XI. and which are only useful as proofs of the exactness of the former numbers. This fourth of the following set of tables, or table relating to the interest of $3\frac{1}{2}$ per cent. I have computed myself. The others have been computed by Mr. Denham, an able arithmetician, whom Dr. Price recommended for that purpose. And they were proved by him, as he proceeded in the work, by Mr. Morgan's method above-mentioned, though I have not thought it necessary to cause all those proof-numbers to be printed. But I have looked over those proof-numbers myself, and found that they confirm the truth of the computations.

And, as these tables are not inserted in this place to illustrate the manner in which they have been computed, (as was the case with Table XI,) but are intended to be resorted to and made use of, as exhibiting the true values of life-annuities for the different ages of human life, according to the different rates of the interest of money, without any reference to the method of computing them, I have set down the values of the annuities in each table in their natural order, beginning with the youngest age, which is that of 3 years. These tables are as follows.

TABLE

Contd	per	fol	Mo	bu	Year	of	Age.
33							3
32							4
31							5
30							6
29							7
28							8
27							9
26							10
25							11
24							12
23							13
22							14
21							15
20							16
19							17
18							18
17							19
16							20
15							21
14							22
13							23
12							24
11							25
10							26
9							27
8							28
7							29
6							30
5							31
4							32
3							33

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T A B L E XII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 2 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 27.382,830	34	£ 21.782,266	65	£ 9.238,319
4	27.794,317	35	21.474,025	66	8.795,049
5	28.008,120	36	21.158,940	67	8.365,278
6	28.121,217	37	20.836,776	68	7.950,607
7	28.153,865	38	20.475,233	69	7.553,307
8	28.130,823	39	20.104,909	70	7.176,577
9	28.080,318	40	19.725,499	71	6.798,054
10	27.967,401	41	19.336,686	72	6.445,751
11	27.788,463	42	18.938,139	73	6.098,544
12	27.540,613	43	18.529,510	74	5.759,088
13	27.287,412	44	18.110,436	75	5.431,073
14	27.028,710	45	17.680,537	76	5.087,894
15	26.764,350	46	17.239,415	77	4.759,614
16	26.494,172	47	16.815,956	78	4.453,776
17	26.250,605	48	16.381,354	79	4.144,112
18	26.001,981	49	15.963,864	80	3.871,790
19	25.748,153	50	15.553,376	81	3.613,948
20	25.488,966	51	15.123,598	82	3.380,105
21	25.256,792	52	14.729,083	83	3.127,536
22	25.020,193	53	14.324,685	84	2.838,919
23	24.779,034	54	13.909,922	85	2.559,295
24	24.533,179	55	13.511,804	86	2.297,450
25	24.282,487	56	13.103,799	87	2.070,661
26	24.026,812	57	12.685,379	88	1.784,099
27	23.766,002	58	12.283,071	89	1.502,199
28	23.499,897	59	11.870,902	90	1.228,718
29	23.228,331	60	11.448,295	91	0.969,461
30	22.951,131	61	11.014,604	92	0.730,488
31	22.668,117	62	10.569,115	93	0.490,196
32	22.379,101	63	10.137,299	94	0.000,000
33	22.083,886	64	9.693,983		

T A B L E XII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $2\frac{1}{2}$ per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 24.462,676	34	£ 20.136,887	65	£ 8.911,789
4	24.849,735	35	19.878,238	66	8.495,160
5	25.062,078	36	19.612,791	67	8.090,289
6	25.185,830	37	19.340,316	68	7.698,809
7	25.238,680	38	19.030,630	69	7.323,022
8	25.242,492	39	18.712,036	70	6.966,149
9	25.222,411	40	18.384,189	71	6.606,509
10	25.146,756	41	18.046,727	72	6.271,426
11	25.011,897	42	17.699,272	73	5.940,421
12	24.814,820	43	17.341,428	74	5.616,112
13	24.612,646	44	16.972,778	75	5.302,157
14	24.405,208	45	16.592,885	76	4.972,521
15	24.192,333	46	16.201,291	77	4.656,602
16	23.973,844	47	15.825,188	78	4.361,896
17	23.779,193	48	15.437,457	79	4.062,687
18	23.579,731	49	15.064,768	80	3.799,480
19	23.375,296	50	14.680,583	81	3.549,972
20	23.165,719	51	14.311,129	82	3.323,657
21	22.980,544	52	13.957,047	83	3.078,502
22	22.791,199	53	13.592,614	84	2.797,255
23	22.597,546	54	13.217,294	85	2.524,251
24	22.399,442	55	12.856,800	86	2.268,242
25	22.196,735	56	12.485,883	87	2.046,481
26	21.989,269	57	12.103,960	88	1.765,079
27	21.776,880	58	11.736,386	89	1.487,784
28	21.559,396	59	11.358,341	90	1.218,152
29	21.336,639	60	10.969,190	91	0.962,095
30	21.108,421	61	10.568,230	92	0.725,758
31	20.874,547	62	10.154,683	93	0.487,824
32	20.634,811	63	9.753,041	94	0.000,000
33	20.388,999	64	9.339,034		

T A B L E XIV.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 3 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 22.027,85	34	£ 18.681,86	65	£ 8.603,99
4	22.390,40	35	18.464,13	66	8.211,93
5	22.597,31	36	18.239,84	67	7.830,08
6	22.725,72	37	18.008,72	68	7.460,10
7	22.791,23	38	17.742,48	69	7.104,30
8	22.813,30	39	17.467,42	70	6.765,92
9	22.814,53	40	17.183,14	71	6.423,92
10	22.766,00	41	16.889,24	72	6.104,99
11	22.664,17	42	16.585,30	73	5.789,19
12	22.505,84	43	16.270,90	74	5.479,13
13	22.342,75	44	15.945,54	75	5.178,44
14	22.174,71	45	15.608,74	76	4.861,62
15	22.001,56	46	15.260,00	77	4.557,43
16	21.823,10	47	14.924,96	78	4.273,29
17	21.666,23	48	14.578,02	79	3.984,05
18	21.504,89	49	14.244,41	80	3.729,54
19	21.338,90	50	13.899,02	81	3.488,01
20	21.168,08	51	13.566,71	82	3.268,91
21	21.019,54	52	13.248,20	83	3.030,89
22	20.867,17	53	12.919,06	84	2.756,77
23	20.710,84	54	12.578,70	85	2.490,19
24	20.550,40	55	12.251,64	86	2.239,87
25	20.385,70	56	11.913,80	87	2.023,06
26	20.210,57	57	11.564,55	88	1.746,59
27	20.042,84	58	11.228,16	89	1.473,61
28	19.864,33	59	10.880,86	90	1.207,74
29	19.680,86	60	10.521,96	91	0.954,82
30	19.492,23	61	10.150,71	92	0.721,08
31	19.298,24	62	9.766,26	93	0.485,43
32	19.098,66	63	9.392,18	94	0.000,00
33	18.893,29	64	9.005,09		

T A B L E XV.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{4}$ per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 19.979,961	34	£ 17.390,462	65	£ 8.313,625
4	20.318,825	35	17.206,612	66	7.944,258
5	20.518,022	36	17.016,528	67	7.583,727
6	20.647,176	37	16.819,920	68	7.233,699
7	20.720,153	38	16.593,228	69	6.896,496
8	20.754,438	39	16.351,906	70	6.575,357
9	20.770,473	40	16.104,542	71	6.249,840
10	20.741,729	41	15.847,705	72	5.945,972
11	20.664,641	42	15.580,939	73	5.644,448
12	20.536,088	43	15.303,763	74	5.341,806
13	20.403,141	44	15.015,663	75	5.059,623
14	20.265,616	45	14.716,120	76	4.754,921
15	20.123,320	46	14.404,549	77	4.461,839
16	19.976,052	47	14.105,200	78	4.187,758
17	19.848,540	48	13.793,859	79	3.908,000
18	19.716,914	49	13.494,425	80	3.661,781
19	19.581,000	50	13.183,083	81	3.427,856
20	19.440,616	51	12.883,449	82	3.215,658
21	19.320,751	52	12.596,296	83	2.984,473
22	19.197,449	53	12.298,386	84	2.717,188
23	19.070,569	54	11.989,093	85	2.456,774
24	18.939,963	55	11.691,801	86	2.211,911
25	18.805,476	56	11.383,427	87	1.999,810
26	18.666,944	57	11.063,485	88	1.728,378
27	18.524,196	58	10.755,123	89	1.459,099
28	18.377,051	59	10.435,566	90	1.197,512
29	18.225,318	60	10.104,074	91	0.947,669
30	18.068,798	61	9.759,829	92	0.716,468
31	17.907,280	62	9.401,925	93	0.483,091
32	17.740,543	63	9.053,059	94	0.000,000
33	17.568,352	64	8.690,648		

T A B L E XVI.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 18.242,464	34	£ 16.239,735	65	£ 8.039,349
4	18.558,931	35	16.084,014	66	7.690,960
5	18.749,209	36	15.922,447	67	7.350,186
6	18.876,581	37	15.754,736	68	7.018,692
7	18.953,475	38	15.555,857	69	6.698,802
8	18.995,706	39	15.348,644	70	6.393,749
9	19.021,902	40	15.132,664	71	6.083,658
10	19.007,582	41	14.907,457	72	5.793,941
11	18.949,243	42	14.672,537	73	5.505,835
12	18.843,753	43	14.427,388	74	5.221,832
13	18.734,230	44	14.171,466	75	4.945,465
14	18.620,487	45	13.904,190	76	4.652,254
15	18.502,326	46	13.624,948	77	4.369,724
16	18.379,539	47	13.356,700	78	4.105,200
17	18.274,964	48	13.076,491	79	3.834,477
18	18.166,641	49	12.807,002	80	3.596,174
19	18.054,395	50	12.525,605	81	3.369,530
20	17.938,041	51	12.254,766	82	3.163,947
21	17.840,730	52	11.995,305	83	2.939,337
22	17.740,368	53	11.725,075	84	2.678,656
23	17.635,818	54	11.443,400	85	2.424,216
24	17.529,936	55	11.172,645	86	2.181,654
25	17.419,509	56	10.890,825	87	1.977,157
26	17.305,557	57	10.597,210	88	1.710,503
27	17.187,730	58	10.314,093	89	1.446,020
28	17.065,909	59	10.019,612	90	1.187,434
29	16.939,905	60	9.712,978	91	0.940,608
30	16.809,518	61	9.393,319	92	0.711,907
31	16.674,537	62	9.059,665	93	0.480,769
32	16.534,739	63	8.733,893	94	0.000,000
33	16.389,888	64	8.394,166		

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T A B L E XVII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $4\frac{1}{2}$ per cent.

Years of Age.	Value of an annuity of one pound a year.	Years of Age.	Value of an annuity of one pound a year.	Years of Age.	Value of an annuity of one pound a year.
3	£ 16.756,318	34	£ 15.210,67	65	£ 7.779,99
4	17.051,91	35	15.078,39	66	7.451,01
5	17.232,78	36	14.940,68	67	7.128,57
6	17.356,81	37	14.797,23	68	6.814,32
7	17.435,21	38	14.624,42	69	6.510,57
8	17.482,39	39	14.443,64	70	6.220,54
9	17.515,42	40	14.254,42	71	5.924,89
10	17.511,62	41	14.056,29	72	5.648,45
11	17.467,53	42	13.848,74	73	5.372,97
12	17.380,04	43	13.631,21	74	5.100,89
13	17.288,86	44	13.403,15	75	4.835,69
14	17.193,80	45	13.163,92	76	4.553,37
15	17.094,65	46	12.912,88	77	4.280,86
16	16.991,21	47	12.671,81	78	4.025,43
17	16.904,67	48	12.418,90	79	3.763,33
18	16.814,73	49	12.175,72	80	3.532,59
19	16.721,21	50	11.920,73	81	3.312,91
20	16.623,93	51	11.675,33	82	3.113,67
21	16.544,44	52	11.440,38	83	2.895,38
22	16.462,26	53	11.194,74	84	2.641,07
23	16.377,28	54	10.937,70	85	2.392,40
24	16.289,34	55	10.690,66	86	2.157,98
25	16.198,31	56	10.432,56	87	1.954,95
26	16.104,02	57	10.162,64	88	1.692,95
27	16.006,32	58	9.902,29	89	1.432,57
28	15.905,03	59	9.630,51	90	1.177,51
29	15.799,96	60	9.346,46	91	0.933,64
30	15.690,92	61	9.049,21	92	0.707,40
31	15.577,70	62	8.737,74	93	0.478,46
32	15.460,07	63	8.433,15	94	0.000,00
33	15.337,82	64	8.114,30		

T A B L E XVIII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 5 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 15.475,38	34	£ 14.287,24	65	£ 7.534,57
4	15.751,70	35	14.174,54	66	7.223,59
5	15.923,11	36	14.056,84	67	6.918,17
6	16.042,87	37	13.933,83	68	6.619,96
7	16.121,17	38	13.783,16	69	6.331,26
8	16.171,20	39	13.624,89	70	6.055,28
9	16.208,71	40	13.458,56	71	5.773,18
10	16.212,55	41	13.283,68	72	5.505,20
11	16.179,35	42	13.099,71	73	5.245,60
12	16.106,03	43	12.906,09	74	4.984,76
13	16.029,32	44	12.702,21	75	4.730,12
14	15.949,04	45	12.487,42	76	4.458,12
15	15.864,99	46	12.261,04	77	4.195,13
16	15.776,95	47	12.043,77	78	3.948,35
17	15.704,68	48	11.814,85	79	3.694,48
18	15.629,33	49	11.594,84	80	3.470,95
19	15.550,71	50	11.363,18	81	3.257,93
20	15.468,67	51	11.140,30	82	3.064,75
21	15.403,32	52	10.927,09	83	2.852,53
22	15.335,63	53	10.703,34	84	2.604,35
23	15.265,47	54	10.468,30	85	2.361,25
24	15.192,73	55	10.242,48	86	2.131,77
25	15.117,25	56	10.005,69	87	1.933,03
26	15.038,89	57	9.757,12	88	1.675,04
27	14.957,50	58	9.517,33	89	1.418,35
28	14.872,90	59	9.266,13	90	1.166,21
29	14.784,92	60	9.002,61	91	0.924,25
30	14.693,37	61	8.725,83	92	0.702,94
31	14.598,05	62	8.434,68	93	0.476,19
32	14.498,74	63	8.149,54	94	0.000,00
33	14.395,22	64	7.849,93		

T A B L E XIX.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 6 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 13.390,78	34	£ 12.704,49	65	£ 7.081,91
4	13.633,23	35	12.622,00	66	6.803,15
5	13.786,59	36	12.535,35	67	6.528,33
6	13.896,64	37	12.444,26	68	6.259,06
7	13.971,93	38	12.328,53	69	5.997,22
8	14.023,69	39	12.206,01	70	5.746,68
9	14.065,55	40	12.076,23	71	5.489,21
10	14.078,91	41	11.938,66	72	5.247,98
11	14.060,56	42	11.792,75	73	5.006,12
12	14.007,46	43	11.637,89	74	4.765,93
13	13.951,50	44	11.473,46	75	4.530,73
14	13.892,50	45	11.298,74	76	4.277,83
15	13.830,25	46	11.112,99	77	4.032,52
16	13.764,54	47	10.935,03	78	3.801,85
17	13.712,73	48	10.745,94	79	3.563,34
18	13.658,38	49	10.564,46	80	3.353,32
19	13.601,33	50	10.371,80	81	3.152,81
20	13.541,40	51	10.186,65	82	2.971,06
21	13.496,36	52	10.009,95	83	2.770,32
22	13.449,57	53	9.823,15	84	2.533,81
23	13.400,92	54	9.625,44	85	2.301,35
24	13.350,30	55	9.435,74	86	2.081,39
25	13.297,59	56	9.235,40	87	1.890,99
26	13.242,66	57	9.023,54	88	1.642,24
27	13.185,37	58	8.819,24	89	1.393,57
28	13.125,58	59	8.603,71	90	1.148,64
29	13.063,13	60	8.376,00	91	0.913,30
30	12.997,84	61	8.135,06	92	0.694,19
31	12.929,54	62	7.879,69	93	0.471,69
32	12.858,02	63	7.628,91	94	0.000,00
33	12.783,08	64	7.363,46		

L I F E - A N N U I T I E S .

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T A B L E XX.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 7 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 11.777,44	34	£ 11.404,51	65	£ 6.673,74
4	11.991,62	35	11.343,49	66	6.422,78
5	12.128,80	36	11.279,09	67	6.174,46
6	12.229,00	37	11.211,03	68	5.930,35
7	12.299,56	38	11.120,95	69	5.692,65
8	12.350,20	39	11.024,87	70	5.464,47
9	12.392,90	40	10.912,30	71	5.228,74
10	12.411,09	41	10.812,72	72	5.007,66
11	12.401,71	42	10.695,57	73	4.785,15
12	12.361,77	43	10.570,22	74	4.563,42
13	12.319,38	44	10.436,01	75	4.345,69
14	12.274,34	45	10.292,20	76	4.110,04
15	12.226,48	46	10.138,01	77	3.880,74
16	12.175,56	47	9.990,64	78	3.664,71
17	12.137,07	48	9.832,76	79	3.440,23
18	12.096,46	49	9.681,55	80	3.242,57
19	12.053,56	50	9.519,73	81	3.053,54
20	12.008,23	51	9.364,51	82	2.882,31
21	11.976,34	52	9.216,85	83	2.692,20
22	11.943,15	53	9.059,63	84	2.466,56
23	11.908,59	54	8.892,00	85	2.244,04
24	11.872,55	55	8.731,51	86	2.033,01
25	11.834,93	56	8.560,84	87	1.850,43
26	11.795,64	57	8.379,07	88	1.609,95
27	11.754,55	58	8.203,96	89	1.368,64
28	11.711,53	59	8.017,98	90	1.130,11
29	11.666,45	60	7.820,13	91	0.900,21
30	11.619,16	61	7.609,27	92	0.685,649
31	11.569,50	62	7.384,13	93	0.467,289
32	11.517,30	63	7.162,51	94	0.000,000
33	11.462,37	64	6.926,23		

T A B L E XXI.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 8 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 10.498,90	34	£ 10.323,80	65	£ 6.305,04
4	10.689,50	35	10.278,24	66	6.078,24
5	10.812,58	36	10.229,96	67	5.853,05
6	10.903,61	37	10.178,73	68	5.630,99
7	10.968,95	38	10.107,71	69	5.414,20
8	11.017,21	39	10.031,41	70	5.205,73
9	11.059,02	40	9.949,36	71	4.989,28
10	11.079,47	41	9.861,03	72	4.786,10
11	11.075,61	42	9.765,86	73	4.580,86
12	11.044,54	43	9.663,21	74	4.375,67
13	11.011,33	44	9.552,42	75	4.173,66
14	10.975,79	45	9.432,72	76	3.953,62
15	10.937,73	46	9.303,30	77	3.738,86
16	10.896,93	47	9.179,99	78	3.536,16
17	10.867,35	48	9.046,81	79	3.324,52
18	10.835,97	49	8.919,59	80	3.138,19
19	10.802,63	50	8.782,39	81	2.959,71
20	10.767,17	51	8.651,10	82	2.798,19
21	10.743,97	52	8.526,72	83	2.617,95
22	10.719,82	53	8.393,38	84	2.402,45
23	10.694,65	54	8.250,20	85	2.189,26
24	10.668,38	55	8.113,50	86	1.986,62
25	10.640,95	56	7.967,16	87	1.811,41
26	10.612,25	57	7.810,22	88	1.578,80
27	10.582,20	58	7.659,28	89	1.344,52
28	10.550,69	59	7.497,95	90	1.112,13
29	10.517,60	60	7.325,16	91	0.887,458
30	10.482,82	61	7.139,72	92	0.677,297
31	10.446,20	62	6.940,29	93	0.462,962,9
32	10.407,60	63	6.743,60	94	0.000,0.0
33	10.366,85	64	6.532,39		

TABLE XXII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 9 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 9.464,49	34	£ 9.415,24	65	£ 5.970,66
4	9.635,36	35	9.380,92	66	5.764,92
5	9.746,28	36	9.344,45	67	5.559,98
6	9.829,07	37	9.305,64	68	5.357,29
7	9.889,33	38	9.248,97	69	5.158,93
8	9.934,73	39	9.187,66	70	4.967,89
9	9.974,87	40	9.121,25	71	4.768,56
10	9.996,17	41	9.049,24	72	4.581,32
11	9.995,79	42	8.971,06	73	4.391,54
12	9.970,90	43	8.886,08	74	4.201,23
13	9.944,11	44	8.793,62	75	4.013,41
14	9.915,24	45	8.692,92	76	3.807,52
15	9.884,08	46	8.583,14	77	3.606,01
16	9.850,43	47	8.478,93	78	3.415,49
17	9.826,98	48	8.365,47	79	3.215,61
18	9.801,97	49	8.257,46	80	3.039,69
19	9.775,24	50	8.140,06	81	2.870,94
20	9.746,65	51	8.028,06	82	2.718,38
21	9.729,30	52	7.922,48	83	2.547,30
22	9.711,26	53	7.808,53	84	2.341,29
23	9.692,47	54	7.685,33	85	2.136,85
24	9.672,88	55	7.568,13	86	1.942,11
25	9.652,42	56	7.441,86	87	1.773,87
26	9.631,02	57	7.305,54	88	1.548,74
27	9.608,60	58	7.174,74	89	1.321,18
28	9.585,10	59	7.034,06	90	1.094,68
29	9.560,41	60	6.882,41	91	0.875,038
30	9.534,43	61	6.718,55	92	0.669,135
31	9.507,05	62	6.541,08	93	0.458,715
32	9.478,15	63	6.365,76	94	0.000,000
33	9.447,60	64	6.176,19		

T A B L E XXIII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 10 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
3	£ 8.612,41	34	£ 8.643,42	65	£ 5.666,42
4	8.766,66	35	8.617,37	66	5.479,11
5	8.867,12	36	8.589,66	67	5.291,94
6	8.942,62	37	8.560,12	68	5.106,33
7	8.998,15	38	8.514,37	69	4.924,28
8	9.040,62	39	8.464,55	70	4.748,71
9	9.078,77	40	8.410,21	71	4.564,65
10	9.100,13	41	8.350,86	72	4.391,68
11	9.101,99	42	8.285,95	73	4.215,77
12	9.081,56	43	8.214,86	74	4.038,86
13	9.059,42	44	8.136,92	75	3.863,86
14	9.035,38	45	8.051,35	76	3.670,85
15	9.009,25	46	7.957,30	77	3.481,41
16	8.980,80	47	7.868,40	78	3.302,03
17	8.961,70	48	7.770,84	79	3.112,98
18	8.941,21	49	7.678,32	80	2.946,03
19	8.919,19	50	7.576,99	81	2.786,85
20	8.895,48	51	7.480,66	82	2.642,58
21	8.882,16	52	7.390,37	83	2.480,03
22	8.868,33	53	7.292,30	84	2.282,89
23	8.853,95	54	7.185,54	85	2.086,67
24	8.838,99	55	7.084,42	86	1.899,38
25	8.823,39	56	6.974,90	87	1.737,73
26	8.807,10	57	6.855,69	88	1.519,72
27	8.790,06	58	6.741,74	89	1.298,58
28	8.772,21	59	6.618,46	90	1.077,73
29	8.753,47	60	6.484,73	91	0.802,938
30	8.733,77	61	6.339,28	92	0.661,157
31	8.713,02	62	6.180,65	93	0.454,545
32	8.691,12	63	6.023,74	94	0.000,000
33	8.667,96	64	5.852,93		

cerning life-annuities that are payable every half-year or every quarter of a year.

CC. The foregoing tables exhibit the values of life-annuities, according to the above rates of interest, upon a supposition that those annuities are to be paid only once a year at the ends of several years of the annuitant's life that shall follow the time of granting them. But it is often stipulated between the grantors and grantees of life-annuities that they shall be paid every half-year, and sometimes that they shall be paid every quarter of a year, during the life of the annuitant. In these cases they will be somewhat more valuable than if they are to be paid only once a year; and consequently a small addition ought to be made to the values them set down in the foregoing tables. Mr. Simpson (in his doctrine of annuities, pages 78, 79, 80, 81,) says that this addition ought, in the case of half-yearly payments, to be about a quarter of a year's annuity, and in the case of quarterly payments, to be about three eighth parts of a year's annuity. But these are only approximations to the values of these annuities: and it is impossible to determine them with perfect precision without a table of the probabilities of the duration of human life that would exhibit the numbers of persons who die off in every half-year and quarter of a year, as well as in every whole year, of the whole possible extent of human life, out of a given original number of persons of the same age all living at the same time: and no such table has yet been published. But we may determine the value of the first of these additions, that which is to be made in the case of half-yearly payments, to a sufficient degree of exactness for all useful purposes in the manner following.

Observations on the value of a life-annuity that is payable every half-year.

CCL. It has been shown above in Art. 86, 87, that the value of an annuity of one pound a year for the life of a person of the age of N years is $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4} + \frac{P^{(4)}}{r^5} + \frac{P^{(5)}}{r^6} + \dots$ continued to $E - N$ terms, or, (according to Monsieur

Parcieux's table of probabilities,) to $94 - N$ terms, or to the utmost possible extent of human life.

Now let us suppose this grant of an annuity of one pound a year for the life of the person aged N years to be changed into a grant of an annuity, or rather *pension*, of one pound a year for the life of the same person, to be paid by half-yearly payments of 10 shillings each during the said person's life. Then it is evident that the said person will gain by this change the three following advantages. In the first place, he will have a chance of receiving one half-year's payment after he has attained the age of 94 years; because it is supposed, in Monsieur de Parcieux's table of the probabilities

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probabilities of the duration of human life, that one person, out of the original 1000 persons who are living at the age of 3 years, will live beyond the age of 94 years and before he attains the age of 95 years; and therefore, if the annuitant should be that person, and should live to the age of 94 years and a half, he may, upon this second supposition of the annuity's being paid by half-yearly payments, receive a payment of 10 shillings in that remote period of his life, which it is absolutely impossible he should receive upon the former supposition of its being paid only at the end of every year. This advantage is, however, so exceeding small, that it is hardly worth attending to. And, in the second place, the annuitant will, upon this second supposition of the annuity's being paid every half year, receive one half of all the rest of the money which will be paid him by the grantor of the annuity, half a year sooner than on the former supposition, namely, all those payments of 10 shillings each which he will receive at the ends of half a year, a year and a half, two years and a half, three years and a half, and every following intermediate half-year during his life; from which circumstance those half-yearly payments will be more valuable to him, and more burthensome to the grantor of the annuity, than they were before, when they were to be made only at the ends of a whole year, two years, three years, four years, and every following whole number of years during the same person's life. And, in the last place, the annuitant's chances of receiving those payments on the second supposition of their being made at the ends of half a year, a year and a half, two years and a half, three years and a half, &c. will be somewhat better than they were before, when he was to receive them half a year later, or at the ends of a whole year, two years, three years, four years, &c. because fewer persons out of the whole number *P* of persons represented in Monsieur de Parcieux's table as living at the age of the annuitant, at the time of granting the annuity, will be dead at the ends of half a year, a year and a half, two years and a half, three years and a half, &c. than at the ends of a whole year, two years, three years, four years, &c. And for these reasons, and chiefly for the two latter, (as the first is not worth attending to,) the value of an annuity of one pound a year payable every half-year by payments of 10 shillings each, during the life of a person of a given age, will be somewhat greater than that of a like annuity of one pound a year that is payable only at the end of every year.

CCII. In order to determine the value of one of these half-yearly annuities with exactness it would be necessary to have a table of the probabilities of the duration of human life that should set forth the number of persons who may be supposed to die in every half-year, as well as years of human life, out of a given original number of very young persons of the same age all living at the same time. But no such tables have yet

been published. We must therefore content ourselves with determining the values of these annuities by an approximation; which may be done in a considerable and sufficient degree of exactness in the following manner.

An investigation of the said value by approximation.

CCIII. Let V denote (as in the foregoing articles,) the value of an annuity of one pound a year payable at the end of every year, during the life of a person of the age of N years; and W the value of a like annuity of one pound a year payable at the end of every year, during the life of a person one year younger than the former, or of the age of $N-1$ years. And let P be the number of persons represented in Monsieur de Parcieux's table of probabilities as living at the age of N years, and $P+d$ be the number of persons represented therein as living at the age of $N-1$ years. And let r be the value of one pound together with its interest for one year, as in the foregoing articles.

Then will $\frac{r-1}{2} \times \text{£}1$ be the interest of one pound for a year, and consequently $\frac{r-1}{2} \times \text{£}1$ will be the interest of one pound for half a year: or, when money is lent for less than a whole year at any particular rate of interest denominated from the whole year, the parties to such short loans generally mean that the interest of the sum lent for the time the loan is to last shall bear the same proportion to its interest for a whole year as the time for which it is lent bears to a whole year. Thus, if the interest of money is 4 per cent. per annum, and 100 pounds are lent for half a year, it is generally understood that the borrower, when he repays the 100 pounds at the end of the half-year, is to pay with it an additional sum of 40 shillings by way of interest for it. And, in like manner in general, if the interest upon one pound for a whole year is $r-1 \times \text{£}1$, its interest for half a year will be $\frac{r-1}{2} \times \text{£}1$, and its interest for the m^{th} part of a year will be $\frac{r-1}{m} \times \text{£}1$.

Therefore the value of one pound together with its interest for half year will be $\text{£}1 + \frac{r-1}{2} \times \text{£}1$, (or $\frac{2}{2} \times \text{£}1 + \frac{r-1}{2} \times \text{£}1$, or $\frac{2+r-1}{2} \times \text{£}1$) or $\frac{1+r}{2} \times \text{£}1$. Let this value be denoted by b .

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$$+\frac{P^{11}}{r^2} + \frac{\Pi^{111}}{r^3} + \frac{P^{111}}{r^3} + \frac{\Pi^{11V}}{r^4} + \frac{P^{11V}}{r^4} + \frac{\Pi^{11V}}{r^5} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$$

or than $\frac{1}{\bar{p}}$ x the series $\frac{\Pi^I}{r^{\frac{1}{2}}} + \frac{\Pi^{II}}{r^{\frac{3}{2}}} + \frac{\Pi^{III}}{r^{\frac{5}{2}}} + \frac{\Pi^{IV}}{r^{\frac{7}{2}}} + \frac{\Pi^V}{r^{\frac{9}{2}}} + \&c.$

together with $\frac{1}{\bar{p}}$ x the series $\frac{P^I}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^V}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$

Therefore the present value of a set of half-yearly payments of one shilling, or half a pound, and

for the life of the same person will be less than $\frac{1}{2\bar{p}}$ x the series $\frac{\Pi^I}{r^{\frac{1}{2}}} + \frac{\Pi^{II}}{r^{\frac{3}{2}}} + \frac{\Pi^{III}}{r^{\frac{5}{2}}} + \frac{\Pi^{IV}}{r^{\frac{7}{2}}} + \frac{\Pi^V}{r^{\frac{9}{2}}} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ together with $\frac{1}{2\bar{p}}$ x the series $\frac{P^I}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^V}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$

CCIX. Now $\frac{1}{\bar{p}}$ x the series $\frac{P^I}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^V}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$ is equal to $\bar{V}$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a person of the age of N years. Therefore $\frac{1}{2\bar{p}}$ x the series $\frac{P^I}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^V}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$ is = $\frac{\bar{V}}{2}$.

Therefore the present value of a set of half-yearly payments of 10 shillings each for the life of a

person of the age of N years is less than $\frac{1}{2\bar{p}}$ x the series $\frac{\Pi^I}{r^{\frac{1}{2}}} + \frac{\Pi^{II}}{r^{\frac{3}{2}}} + \frac{\Pi^{III}}{r^{\frac{5}{2}}} + \frac{\Pi^{IV}}{r^{\frac{7}{2}}} + \frac{\Pi^V}{r^{\frac{9}{2}}} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ together with $\frac{\bar{V}}{2}$

Therefore

Therefore it is less than $r^{\frac{1}{2}} \times \frac{1}{2} \times \frac{1}{2} \times$ the series $\frac{\Pi^I}{r} + \frac{\Pi^{II}}{r^2} + \frac{\Pi^{III}}{r^3}$ together with $\frac{1}{2}$.

CCX. Now it is probable that the number of persons who die in the first half of every year of human life is somewhat less than the number of those who die in the second half of the same year; and consequently the number of persons living at the end of the first half-year will be somewhat greater than an arithmetical mean proportional between the numbers of persons living at the beginning of the first half-year and at the end of the second half-year. Therefore Π^I will be somewhat greater than an arithmetical mean between P^I and P^{II} , and Π^{II} than an arithmetical mean between P^{II} and P^{III} , and Π^{III} than an arithmetical mean between P^{III} and P^{IV} , and between P^{IV} and P^V , &c. Consequently $\Pi^I, \Pi^{II}, \Pi^{III}, \Pi^{IV}$, &c. will be somewhat greater than $\frac{P^I + P^{II}}{2}, \frac{P^{II} + P^{III}}{2}, \frac{P^{III} + P^{IV}}{2}$, and $\frac{P^{IV} + P^V}{2}$, &c.

But the difference will probably be so extremely small that we may very well consider them as equal. And upon this supposition the series

$\frac{\Pi^I}{r} + \frac{\Pi^{II}}{r^2} + \frac{\Pi^{III}}{r^3} + \frac{\Pi^{IV}}{r^4} + \frac{\Pi^V}{r^5} + \frac{\Pi^E - N + 1}{r^6}$ will be equal to the series $\frac{P + P^I}{r} + \frac{P^I + P^{II}}{2r^2} + \frac{P^{II} + P^{III}}{2r^3} + \frac{P^{III} + P^{IV}}{2r^4}$

$+ \frac{P^{IV} + P^V}{2r^5} + \frac{\Pi^E - N + 1}{r^6}$ and consequently to one half of

the series $\frac{P + P^I}{r} + \frac{P^I + P^{II}}{r^2} + \frac{P^{II} + P^{III}}{r^3} + \frac{P^{III} + P^{IV}}{r^4} + \frac{P^{IV} + P^V}{r^5} + \frac{\Pi^E - N + 1}{r^6}$ together with the term $\frac{\Pi^E - N + 1}{r^6}$, and consequently to one half of the series $\frac{P}{r} + \frac{P^I}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \frac{P^V}{r^6} + \frac{\Pi^E - N + 1}{r^6}$

+ &c. together with one half of the series $\frac{P}{r} + \frac{P^I}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \frac{P^V}{r^6} + \frac{\Pi^E - N + 1}{r^6}$

+ &c. and with the term $\frac{\Pi^E - N + 1}{r^6}$.

Therefore

Therefore $r^{\frac{1}{2}} \times \frac{1}{2P} \times$ the series $\frac{\Pi^I}{r} + \frac{\Pi^{II}}{r^2} + \frac{\Pi^{III}}{r^3} + \frac{\Pi^{IV}}{r^4} + \&c.$ is equal $r^{\frac{1}{2}} \times \frac{1}{2P} \times \frac{1}{2} \times$ the series $\frac{P}{r} + \frac{P^I}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \&c.$ together with $r^{\frac{1}{2}} \times \frac{1}{2P} \times$ the series $\frac{P}{r} + \frac{P^I}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \&c.$ on account of its extreme smallness,)

$$+ \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \&c. + \frac{r^{\frac{1}{2}} \times \frac{1}{2P}}{4} \times \frac{1}{P} \times \text{the series } \frac{P}{r} + \frac{P^I}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \&c.$$

(if we neglect this last quantity $\frac{r^{\frac{1}{2}}}{4} \times \frac{1}{P} \times \frac{\Pi E - N + 1}{r E - N + 1}$ on account of its extreme smallness,)

$$+ \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \&c. + \frac{r^{\frac{1}{2}} \times \frac{1}{2P}}{4} \times \frac{1}{P} \times \text{the series } \frac{P}{r} + \frac{P^I}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \&c.$$

CCXI. Therefore $r^{\frac{1}{2}} \times \frac{1}{2P} \times$ the series $\frac{\Pi^I}{r} + \frac{\Pi^{II}}{r^2} + \frac{\Pi^{III}}{r^3} + \frac{\Pi^{IV}}{r^4} + \frac{\Pi^{V}}{r^5} + \&c.$ together with $\frac{r^{\frac{1}{2}}}{2} \times \frac{1}{P} \times$ the series $\frac{P}{r} + \frac{P^I}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} + \frac{P^{V}}{r^6} + \&c.$

$$\frac{L}{P} \times \frac{1}{P} \times \text{the series } \frac{P}{r} + \frac{P^1}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{111}}{r^4} + \frac{P^{1111}}{r^5} + \&c.$$

$$+ \frac{r^{\frac{1}{2}} \times \frac{L}{V}}{4} + \frac{L}{2} = \frac{r^{\frac{1}{2}}}{4} \times \frac{1}{P} \times \text{the series } \frac{P}{r} + \frac{P^1}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{111}}{r^4} + \frac{P^{1111}}{r^5} + \&c.$$

$$+ \frac{P^{11111}}{r^6} + \&c. + \frac{r^{\frac{1}{2}} + 2}{4} \times \frac{L}{V}.$$

CCXII. But it has been shewn, in Art. 209, that the present value of a set of half-yearly payments of 10 shillings each, during the life of a person of the age of N years, is somewhat less than $r^{\frac{1}{2}} \times \frac{1}{2P} \times \text{the series } \frac{P}{r} + \frac{P^1}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{111}}{r^4} + \frac{P^{1111}}{r^5} + \frac{P^{11111}}{r^6} + \&c.$

$$+ \frac{P^{111111}}{r^7} + \frac{P^{1111111}}{r^8} + \frac{P^{11111111}}{r^9} + \frac{P^{111111111}}{r^{10}} + \&c. + \frac{P^{E-N+1}}{r^{E-N+1}} \text{ together with } \frac{L}{2}.$$

Therefore the said value of the said half-yearly payments is somewhat less than $\frac{r^{\frac{1}{2}}}{4} \times \frac{1}{P} \times \text{the series } \frac{P}{r} + \frac{P^1}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{111}}{r^4} + \frac{P^{1111}}{r^5} + \frac{P^{11111}}{r^6} + \frac{P^{111111}}{r^7} + \frac{P^{1111111}}{r^8} + \frac{P^{11111111}}{r^9} + \frac{P^{111111111}}{r^{10}} + \&c.$

CCXIII. Now $\frac{L}{W}$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a person of the age of $N-1$ years, is $= \frac{1}{P+d} \times \text{the series } \frac{P}{r} + \frac{P^1}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{111}}{r^4} + \frac{P^{1111}}{r^5} + \frac{P^{11111}}{r^6} + \frac{P^{111111}}{r^7} + \frac{P^{1111111}}{r^8} + \frac{P^{11111111}}{r^9} + \frac{P^{111111111}}{r^{10}} + \&c.$

$$\text{Therefore } \frac{P+d}{P} \times \frac{L}{W} \text{ is } = \frac{1}{P} \times \text{the series } \frac{P}{r} + \frac{P^1}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{111}}{r^4} + \frac{P^{1111}}{r^5} + \frac{P^{11111}}{r^6} + \frac{P^{111111}}{r^7} + \frac{P^{1111111}}{r^8} + \frac{P^{11111111}}{r^9} + \frac{P^{111111111}}{r^{10}} + \&c.$$

$$+ \frac{P^{1111111111}}{r^{11}} + \frac{P^{11111111111}}{r^{12}} + \&c. \text{ and } \frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \frac{L}{W} \text{ is } = \frac{r^{\frac{1}{2}}}{4} \times \frac{1}{P} \times \text{the series } \frac{P}{r} + \frac{P^1}{r^2} + \frac{P^{11}}{r^3} + \frac{P^{111}}{r^4} + \frac{P^{1111}}{r^5} + \frac{P^{11111}}{r^6} + \frac{P^{111111}}{r^7} + \frac{P^{1111111}}{r^8} + \frac{P^{11111111}}{r^9} + \frac{P^{111111111}}{r^{10}} + \frac{P^{1111111111}}{r^{11}} + \frac{P^{11111111111}}{r^{12}} + \&c.$$

Therefore

CCXV. Since 1 is to $r\frac{1}{2}$ as $r\frac{1}{2}$ is to r , $1+r$ will be greater than $r\frac{1}{2}$, and consequently $1+r + 2r\frac{1}{2}$ will be greater than $2r\frac{1}{2} + 2r\frac{1}{2}$, or $1 + 2r\frac{1}{2} + r$ will be greater than $4r\frac{1}{2}$, and therefore $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}}$

The excess of the quantity $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}}$

or $1 + 2r\frac{1}{2} + r$ will be greater than $4r\frac{1}{2}$, and therefore $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}}$ will be greater than $\frac{1}{V}$. But the difference between them will be extremely small. Thus, for example, if the interest of money is 4 per cent. we shall have $r = 1.04$, and $r\frac{1}{2} = \sqrt{1.04} = 1.0198$, and consequently $2r\frac{1}{2} = 2 \times 1.0198 = 2.0396$, and $1 + 2r\frac{1}{2} + r = 1 + 2.0396 + 1.04 = 4.0796$, and $4r\frac{1}{2} = 4.0792$, and $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}}$

$\times \frac{1}{V}$, or the second member of the foregoing near value of an annuity of one pound a year, paid half-yearly, above $\frac{1}{V}$, or the value of the same annuity, when paid at the end of every year, is always extremely small.

$\frac{1}{V} = \frac{4.0796}{4.0792} \times \frac{1}{V} = 1.000,09 \times \frac{1}{V}$, or less than $\frac{1}{V} + .0001 \times \frac{1}{V}$, or $\frac{1}{V} + \frac{1}{10000}$ part of $\frac{1}{V}$. And, when the interest of money is 10 per cent. (which is the highest interest supposed in the foregoing tables,) and r is consequently equal to 1.1, we shall have $r\frac{1}{2} = 1.0488$, and $2r\frac{1}{2} = 2.0976$, and $1 + 2r\frac{1}{2} + r = 1 + 2.0976 + 1.1 = 4.1976$, and $4r\frac{1}{2} = 4 \times 1.0488 = 4.1952$, and consequently $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}}$

$\frac{1}{V} = \frac{4.1976}{4.1952} \times \frac{1}{V} = 1.00057 \times \frac{1}{V}$, or $\frac{1}{V} + \frac{57}{100,000} \times \frac{1}{V}$, or $\frac{1}{V} + \frac{1}{1754}$ part of $\frac{1}{V}$. Therefore, if the interest of money be either 10 per cent. or any lower interest, the excess of $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}}$

$\times \frac{1}{V}$ above $\frac{1}{V}$ will be either $\frac{1}{1754}$ part, or some lesser part, of $\frac{1}{V}$; and consequently the excess of $\frac{1}{4r\frac{1}{2}} + \frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \frac{1}{V}$ above $\frac{1}{4r\frac{1}{2}} + \frac{1}{V}$ will be only the 1754^{th} , or some lesser, part of $\frac{1}{V}$. And

therefore, if we suppose the said quantity $\frac{1}{4r\frac{1}{2}} + \frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \frac{1}{V}$, which

A conjectural which has been shewn to be always somewhat, and but a very little conclusion concerning the magnitude of the true value of the said half-yearly annuity.

greater than the true value of an annuity of one pound a year paid half yearly for the life of a person of the age of N years, to be greater than the said true value by the same excess by which it exceeds the quantity $\frac{1}{4r^{\frac{1}{2}}} + V$, the said true value will be equal to $\frac{1}{4r^{\frac{1}{2}}} + V$. Q.E.I.

Agreement of the said conjectural conclusion with the value of the said half-yearly annuity assigned by Mr. Thomas Simpson.

CCXVI. If this last supposition be a true one, the value of an annuity of one pound a year for the life of a person of the age of N years, when paid half-yearly, will be obtained by adding to V , or the value of the like annuity, when paid at the end of every year, (which is given in the foregoing tables,) the sum of $\frac{1}{4r^{\frac{1}{2}}}$, or somewhat less than $\frac{1}{4}$, or a quarter of a year's annuity; which agrees with the direction given by Mr. Thomas Simpson for this purpose in his *Doctrine of Annuities*, &c. page 81.

A computation of the several different values of the expression $\frac{1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times V$, (found in Art. 203—214,) according to the twelve different values of r , or the twelve different rates of interest, above-mentioned.

CCXVII. But, if this last supposition should not be true, or if the truth of it should not be thought sufficiently evident or probable, it will

be necessary to have recourse to the full expression $\frac{1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times V$, which has been shewn in the foregoing articles to be always somewhat greater than, but very nearly equal to, the true value of an annuity of one pound a year that is paid half-yearly, during the life of a person of

age of N years. Now, if we compute the different values of this expression according to the twelve different rates of interest above-mentioned, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. per cent. and 10 per cent. those values will be as follows.

In the first place, when the interest of money is 2 per cent. we shall
 $r = 1.02$, and $r^{\frac{1}{2}} = \sqrt{1.02} = 1.009,950$, and $2r^{\frac{1}{2}} = 2.019,900$,
 and $4r^{\frac{1}{2}} = 4.039,800$, and $1 + 2r^{\frac{1}{2}} + r (= 1 + 2.019,900 + 1.02)$

$$4.039,900; \text{ and consequently } \frac{1}{4r^{\frac{1}{2}}} = \frac{1}{4.039,800} = .2475\text{£, and}$$

$$\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \text{£} = \frac{4.039,900}{4.039,800} \times \text{£} = 1.000,024 \times \text{£}. \text{ Therefore,}$$

when the interest of money is 2 per cent. the value of an annuity of one pound a year, for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,
 $475\text{£} + 1.000,024 \times \text{£}.$

Secondly, when the interest of money is $2\frac{1}{2}$ per cent. we shall have
 $= 1.025$, and $r^{\frac{1}{2}} = \sqrt{1.025} = 1.012,422$, and $2r^{\frac{1}{2}} = 2.024,844$,
 and $4r^{\frac{1}{2}} = 4.049,688$, and $1 + 2r^{\frac{1}{2}} + r (= 1 + 2.024,844 + 1.025)$

$$= 4.049,844; \text{ and consequently } \frac{1}{4r^{\frac{1}{2}}} = \frac{1}{4.049,688} = .2469\text{£, and}$$

$$\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \text{£} = \frac{4.049,844}{4.049,688} \times \text{£} = 1.000,038 \times \text{£}. \text{ Therefore,}$$

when the interest of money is $2\frac{1}{2}$ per cent. the value of an annuity of one pound a year, for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$2469\text{£} + 1.000,038 \times \text{£}.$$

Thirdly, when the interest of money is 3 per cent. we shall have
 $= 1.03$, and $r^{\frac{1}{2}} = \sqrt{1.03} = 1.014,889$, and $2r^{\frac{1}{2}} = 2.029,778$,
 and $4r^{\frac{1}{2}} = 4.059,556$, and $1 + 2r^{\frac{1}{2}} + r (= 1 + 2.029,778 + 1.03)$
 $= 4.059,778$

$$= 4.059,778; \text{ and consequently } \frac{\text{£}}{4r\frac{1}{2}} = \frac{\text{£}}{4.059,556} = .2463\text{£}, \text{ and}$$

$$\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£}V = \frac{4.059,778}{4.059,556} \times \text{£}V = 1.000,054 \times \text{£}V. \text{ Therefore,}$$

when the interest of money is 3 per cent. the value of an annuity of one pound a year, for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$.2463\text{£} + 1.000,054 \times \text{£}V.$$

Fourthly, when the interest of money is $3\frac{1}{2}$ per cent. we shall have $r = 1.035$, and $r\frac{1}{2} = \sqrt{1.035} = 1.017,349$, and $2r\frac{1}{2} = 2.034,698$, and $4r\frac{1}{2} = 4.069,396$, and $1 + 2r\frac{1}{2} + r (= 1 + 2.034,698 + 1.035)$

$$= 4.069,698; \text{ and consequently } \frac{\text{£}}{4r\frac{1}{2}} = \frac{\text{£}}{4.069,396} = .2457\text{£}, \text{ and}$$

$$\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£}V = \frac{4.069,698}{4.069,396} \times \text{£}V = 1.000,074 \times \text{£}V. \text{ Therefore,}$$

when the interest of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$.2457\text{£} + 1.000,074 \times \text{£}V.$$

Fifthly, when the interest of money is 4 per cent. we shall have $r = 1.04$, and $r\frac{1}{2} = \sqrt{1.04} = 1.019,804$, and $2r\frac{1}{2} = 2.039,608$, and $4r\frac{1}{2} = 4.079,216$, and $1 + 2r\frac{1}{2} + r (= 1 + 2.039,608 + 1.04)$

$$= 4.079,608; \text{ and consequently } \frac{\text{£}}{4r\frac{1}{2}} = \frac{\text{£}}{4.079,216} = .2451\text{£}, \text{ and}$$

$$\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£}V = \frac{4.079,608}{4.079,216} \times \text{£}V = 1.000,096 \times \text{£}V. \text{ Therefore,}$$

when the interest of money is 4 per cent. the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$.2451\text{£} + 1.000,096 \times \text{£}V.$$

Sixthly, when the interest of money is $4\frac{1}{2}$ per cent. we shall have
 $= 1.045$, and $r\frac{1}{2} = \sqrt{1.045} = 1.022,252$, and $2r\frac{1}{2} = 2.044,504$,
 and $4r\frac{1}{2} = 4.089,008$, and $1 + 2r\frac{1}{2} + r (= 1 + 2.044,504 + 1.045)$

$$= 4.089,504; \text{ and consequently } \frac{1}{4r\frac{1}{2}} = \frac{1}{4.089,008} = .2445 \text{ £, and}$$

$$\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£} = \frac{4.089,504}{4.089,008} \times \text{£} = 1.000,121 \times \text{£}. \text{ Therefore,}$$

when the interest of money is $4\frac{1}{2}$ per cent. the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$2445 \text{ £} + 1.000,121 \times \text{£}.$$

Seventhly, when the interest of money is 5 per cent. we shall have
 $= 1.05$, and $r\frac{1}{2} = \sqrt{1.05} = 1.024,695$, and $2r\frac{1}{2} = 2.049,390$,
 and $4r\frac{1}{2} = 4.098,780$, and $1 + 2r\frac{1}{2} + r (= 1 + 2.049,390 + 1.05)$

$$= 4.099,390; \text{ and consequently } \frac{1}{4r\frac{1}{2}} = \frac{1}{4.098,780} = .2439 \text{ £, and}$$

$$\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£} = \frac{4.099,390}{4.098,780} \times \text{£} = 1.000,148 \times \text{£}. \text{ Therefore,}$$

when the interest of money is 5 per cent. the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$2439 \text{ £} + 1.000,148 \times \text{£}.$$

Eighthly, when the interest of money is 6 per cent. we shall have
 $= 1.06$, and $r\frac{1}{2} = \sqrt{1.06} = 1.029,563$, and $2r\frac{1}{2} = 2.059,126$,
 and $4r\frac{1}{2} = 4.118,252$, and $1 + 2r\frac{1}{2} + r (= 1 + 2.059,126 + 1.06)$

$$= 4.119,126; \text{ and consequently } \frac{1}{4r\frac{1}{2}} = \frac{1}{4.118,252} = .2428 \text{ £, and}$$

$$\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£} = \frac{4.119,126}{4.118,252} \times \text{£} = 1.000,212 \times \text{£}. \text{ Therefore,}$$

when

when the interest of money is 6 per cent. the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$.2428\text{£} + 1.000,212 \times \text{£}V.$$

Ninthly, when the interest of money is 7 per cent. we shall have $r = 1.07$, and $r^{\frac{1}{2}} = \sqrt{1.07} = 1.034,408$, and $2r^{\frac{1}{2}} = 2.068,816$, and $4r^{\frac{1}{2}} = 4.137,632$, and $1 + 2r^{\frac{1}{2}} + r = (1 + 2.068,816 + 1.07)$

$$= 4.138,816; \text{ and consequently } \frac{1}{4r^{\frac{1}{2}}} = \frac{1}{4.137,632} = .2416\text{£}, \text{ and}$$

$$\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \text{£}V = \frac{4.138,816}{4.137,632} \times \text{£}V = 1.000,286 \times \text{£}V. \text{ Therefore,}$$

when the interest of money is 7 per cent. the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$.2416\text{£} + 1.000,286 \times \text{£}V.$$

Tenthly, when the interest of money is 8 per cent. we shall have $r = 1.08$, and $r^{\frac{1}{2}} = \sqrt{1.08} = 1.039,230$, and $2r^{\frac{1}{2}} = 2.078,460$, and $4r^{\frac{1}{2}} = 4.156,920$, and $1 + 2r^{\frac{1}{2}} + r = (1 + 2.078,460 + 1.08)$

$$= 4.158,460; \text{ and consequently } \frac{1}{4r^{\frac{1}{2}}} = \frac{1}{4.156,920} = .2405\text{£}, \text{ and}$$

$$\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \text{£}V = \frac{4.158,460}{4.156,920} \times \text{£}V = 1.000,370 \times \text{£}V. \text{ Therefore,}$$

when the interest of money is 8 per cent. the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid half-yearly, is somewhat less than, but very nearly equal to,

$$.2405\text{£} + 1.000,370 \times \text{£}V.$$

Eleventhly, when the interest of money is 9 per cent. we shall have
 $= 1.09$, and $r\frac{1}{2} = \sqrt{1.09} = 1.044,031$, and $2r\frac{1}{2} = 2.088,062$,
 and $4r\frac{1}{2} = 4.176,124$, and $1 + 2r\frac{1}{2} + r (= 1 + 2.088,062 + 1.09)$
 $= 4.178,062$; and consequently $\frac{1}{4r\frac{1}{2}} = \frac{1}{4.176,124} = .2394\text{£}$, and
 $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£} = \frac{4.178,062}{4.176,124} \times \text{£} = 1.000,464 \times \text{£}$. Therefore,
 when the interest of money is 9 per cent. the value of an annuity of
 one pound a year for the life of a person of the age of N years, to
 be paid half-yearly, is somewhat less than, but very nearly equal to,
 $2394\text{£} + 1.000,464 \times \text{£}$.

And, lastly, when the interest of money is 10 per cent. we shall
 have $r = 1.1$, and $r\frac{1}{2} = \sqrt{1.1} = 1.048,809$, and $2r\frac{1}{2} = 2.097,618$,
 and $4r\frac{1}{2} = 4.195,236$, and $1 + 2r\frac{1}{2} + r (= 1 + 2.097,618 + 1.1)$
 $= 4.197,618$; and consequently $\frac{1}{4r\frac{1}{2}} = \frac{1}{4.195,236} = .2383\text{£}$, and
 $\frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \text{£} = \frac{4.197,618}{4.195,236} \times \text{£} = 1.000,567 \times \text{£}$. Therefore,
 when the interest of money is 10 per cent. the value of an annuity of
 one pound a year for the life of a person of the age of N years, to
 be paid half-yearly, is somewhat less than, but very nearly equal to,
 $2383\text{£} + 1.000,567 \times \text{£}$.

CCXVIII. The substance of the conclusions obtained in the last article may be expressed more concisely in the manner following.

The conclusions obtained in the last article, expressed more concisely.

Let $\text{£}V$ denote, as before, the value of an annuity of one pound a year for the life of a person of the age of N years, to be paid at the end of every year; and let $\text{£}H$ denote the value of the same annuity for the same

K

same life, when it is to be paid at the end of every half-year by payments of 10 shillings each. Then, if the interest of money be

$$2 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2475\mathcal{L} + 1.000,024 \times V^{\mathcal{L}};$$

$$\text{If } 2\frac{1}{2} \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2469\mathcal{L} + 1.000,038 \times V^{\mathcal{L}};$$

$$\text{If } 3 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2463\mathcal{L} + 1.000,054 \times V^{\mathcal{L}};$$

$$\text{If } 3\frac{1}{2} \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2457\mathcal{L} + 1.000,074 \times V^{\mathcal{L}};$$

$$\text{If } 4 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2451\mathcal{L} + 1.000,096 \times V^{\mathcal{L}};$$

$$\text{If } 4\frac{1}{2} \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2445\mathcal{L} + 1.000,121 \times V^{\mathcal{L}};$$

$$\text{If } 5 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2439\mathcal{L} + 1.000,148 \times V^{\mathcal{L}};$$

$$\text{If } 6 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2428\mathcal{L} + 1.000,212 \times V^{\mathcal{L}};$$

$$\text{If } 7 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2416\mathcal{L} + 1.000,286 \times V^{\mathcal{L}};$$

$$\text{If } 8 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2405\mathcal{L} + 1.000,370 \times V^{\mathcal{L}};$$

$$\text{If } 9 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2394\mathcal{L} + 1.000,464 \times V^{\mathcal{L}};$$

$$\text{And, if } 10 \text{ per cent. } H^{\mathcal{L}} \text{ will be (nearly) } = .2383\mathcal{L} + 1.000,567 \times V^{\mathcal{L}}.$$

Expressions of . CCXIX. Therefore the values of $H^{\mathcal{L}} - V^{\mathcal{L}}$, or the differences of the differences values of the said yearly and half-yearly annuities of one pound a year for the life of a person of the age of N years, according to the aforesaid different rates of the interest of money, will be as follows.

If the interest of money is

$$2 \text{ per cent. } H^{\mathcal{L}} - V^{\mathcal{L}} \text{ will be (nearly) } = .2475\mathcal{L} + 0.000,024 \times V^{\mathcal{L}};$$

$$\text{If } 2\frac{1}{2} \text{ per cent. } H^{\mathcal{L}} - V^{\mathcal{L}} \text{ will be (nearly) } = .2469\mathcal{L} + 0.000,038 \times V^{\mathcal{L}};$$

$$\text{If } 3 \text{ per cent. } H^{\mathcal{L}} - V^{\mathcal{L}} \text{ will be (nearly) } = .2463\mathcal{L} + 0.000,054 \times V^{\mathcal{L}};$$

If $3\frac{1}{2}$ per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2457£ + 0.000,074 $\times \overset{f}{V}$;

If 4 per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2451£ + 0.000,096 $\times \overset{f}{V}$;

If $4\frac{1}{2}$ per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2445£ + 0.000,121 $\times \overset{f}{V}$;

If 5 per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2439£ + 0.000,148 $\times \overset{f}{V}$;

If 6 per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2428£ + 0.000,212 $\times \overset{f}{V}$;

If 7 per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2416£ + 0.000,286 $\times \overset{f}{V}$;

If 8 per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2405£ + 0.000,370 $\times \overset{f}{V}$;

If 9 per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2394£ + 0.000,464 $\times \overset{f}{V}$;

And, if 10 per cent. $\overset{f}{H}-\overset{f}{V}$ will be (nearly) = .2383£ + 0.000,567 $\times \overset{f}{V}$.

CCXX. The second terms of the foregoing values of $\overset{f}{H}-\overset{f}{V}$, to wit, Of the smallness of the second terms of the foregoing expressions in comparison of the first terms, .2475£, .2469£, .2463£, &c. as will be evident from the following computation of their magnitudes when they are greatest, or when the values of $\overset{f}{V}$ are greatest. terms.

CCXXI. When the interest of money is 2 per cent. it appears from The magnitudes of the foregoing tables, (Table XII, page 221,) that $\overset{f}{V}$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a person of the age of N years, is greatest when N is = 7, or the greatest annuitant is 7 years of age; and then the said annuity is worth 28.153,865.

Therefore the greatest magnitude of .000,024 $\times \overset{f}{V}$, or of the second term of the expression .2475£ + 0.000,024 $\times \overset{f}{V}$, (which, at this rate of

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of interest, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is = $0.000,024 \times \overset{\pounds}{28.153,865} = \overset{\pounds}{0.000,675}$; which is less than the 366th part of $\overset{\pounds}{.2475\pounds}$, or the first term of the said expression.

When the interest of money is $2\frac{1}{2}$ per cent. it appears from the foregoing tables (Table XIII, page 222,) that $\overset{\pounds}{V}$, or the value of a life-annuity of one pound a year, paid yearly, is greatest when the annuitant is of the age of 8 years, and that it is then = $\overset{\pounds}{25.242,492}$. Therefore the greatest magnitude of $0.000,038 \times \overset{\pounds}{V}$, or of the second term of the expression $\overset{\pounds}{.2469\pounds} + 0.000,038 \times \overset{\pounds}{V}$, (which, at this rate of interest, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is = $0.000,038 \times \overset{\pounds}{25.242,492} = \overset{\pounds}{0.000,959}$; which is less than 0.001 , or $\frac{1}{1000}$ part of a pound, and consequently than the 246th part of $\overset{\pounds}{.2469\pounds}$, or of the first term of the said expression.

When the interest of money is 3 per cent. it appears from Table XIV, page 223, that the greatest magnitude of $\overset{\pounds}{V}$ is at the age of 9 years, and that it then is = $\overset{\pounds}{22.814,53}$. Therefore the greatest magnitude of $0.000,054 \times \overset{\pounds}{V}$, or of the second term of the expression $\overset{\pounds}{.2463\pounds} + 0.000,054 \times \overset{\pounds}{V}$, (which, at this rate of interest, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is = $0.000,054 \times \overset{\pounds}{22.814,53} = \overset{\pounds}{0.001,221}$; which is less than the 201st part of $\overset{\pounds}{2463\pounds}$, or of the first term of the said expression.

When the interest of money is $3\frac{1}{2}$ per cent. it appears from Table XV, page 224, that the greatest magnitude of $\overset{\pounds}{V}$ is at the age of 9 years, and that it is then = $\overset{\pounds}{20.770,473}$. Therefore the greatest magnitude of

of $0.000,074 \times \overset{\text{£}}{V}$, or of the second term of the expression $.2457\text{£}$
 $+ 0.000,074 \times \overset{\text{£}}{V}$ (which, at this rate of interest, is equal to $\overset{\text{£}}{H} - \overset{\text{£}}{V}$,)
 $s = 0.000,074 \times 20.770,473 = 0.001,537$; which is less than the
 159^{th} part of $.2457\text{£}$, or of the first term of the said expression.

When the interest of money is 4 per cent. it appears from Table XVI,
 page 225, that the greatest magnitude of $\overset{\text{£}}{V}$ is at the age of 9 years,
 and that it is then $= 19.021,902$. Therefore the greatest magnitude
 of $0.000,096 \times \overset{\text{£}}{V}$, or of the second term of the expression $.2451\text{£}$
 $+ 0.000,096 \times \overset{\text{£}}{V}$, (which, at this rate of interest, is equal to $\overset{\text{£}}{H} - \overset{\text{£}}{V}$,)
 $s = 0.000,096 \times 19.021,902 = 0.001,826$; which is less than the
 134^{th} part of $.2451\text{£}$, or of the first term of the said expression.

When the interest of money is $4\frac{1}{2}$ per cent. it appears from Table XVII,
 page 226, that the greatest magnitude of $\overset{\text{£}}{V}$ is at the age of 9 years,
 and that it is then $= 17.51542$. Therefore the greatest magnitude
 of $0.000,121 \times \overset{\text{£}}{V}$, or of the second term of the expression $.2445\text{£}$
 $+ 0.000,121 \times \overset{\text{£}}{V}$, (which, at this rate of interest, is $= \overset{\text{£}}{H} - \overset{\text{£}}{V}$,) is
 $= 0.000,121 \times 17.51542 = 0.002,129$; which is less than the 114^{th}
 part of $.2445\text{£}$, or of the first term of the said expression.

When the interest of money is 5 per cent. it appears from Table XVIII,
 page 227, that the greatest magnitude of $\overset{\text{£}}{V}$ is at the age of 10 years,
 and that it is then $= 16.212,55$. Therefore the greatest magnitude
 of $0.000,148 \times \overset{\text{£}}{V}$, or of the second term of the expression $.2439\text{£}$
 $+ 0.000,148 \times \overset{\text{£}}{V}$,

$+ 0.000,148 \times \overset{f}{V}$, (which, at this rate of interest, is $= \overset{f}{H} - \overset{f}{V}$),
 $= 0.000,148 \times 16.212,55 = 0.002,499$; which is less than the 9th
 part of .2439*f*, or of the first term of the said expression.

When the interest of money is 6 per cent. it appears from Table XIX
 page 228, that the greatest magnitude of $\overset{f}{V}$ is at the age of 10 years
 and that it is then $= 14.078,91$. Therefore the greatest magnitud
 of $0.000,212 \times \overset{f}{V}$, or of the second term of the expression .2428
 $+ 0.000,212 \times \overset{f}{V}$, (which, at this rate of interest, is $= \overset{f}{H} - \overset{f}{V}$), i
 $= 0.000,212 \times 14.07891 = 0.002,984$; which is less than the 8th
 part of .2428*f*, or of the first term of the said expression.

When the interest of money is 7 per cent. it appears from Table XX
 page 229, that the greatest magnitude of $\overset{f}{V}$ is at the age of 10 years
 and that it is then $= 12.411,09$. Therefore the greatest magnitud
 of $0.000,286 \times \overset{f}{V}$, or of the second term of the expression .2416
 $+ 0.000,286 \times \overset{f}{V}$, (which, at this rate of interest, is $= \overset{f}{H} - \overset{f}{V}$), i
 $= 0.000,286 \times 12.411,09 = 0.003,549$; which is less than the 6th
 part of .2416*f*, or of the first term of the said expression.

When the interest of money is 8 per cent. it appears from Table XXI
 page 230, that the greatest magnitude of $\overset{f}{V}$ is at the age of 10 years
 and that it is then $= 11.079,47$. Therefore the greatest magnitud
 of $0.000,370 \times \overset{f}{V}$, or of the second term of the expression .2405
 $+ 0.000,370 \times \overset{f}{V}$, (which, at this rate of interest, is $= \overset{f}{H} - \overset{f}{V}$), i
 $= 0.000,370 \times 11.079,47 = 0.004,099$; which is less than the 5th
 part of .2405*f*, or of the first term of the said expression.

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When the interest of money is 9 per cent. it appears from Table XXII, page 231, that the greatest magnitude of $\dot{V}$ is at the age of 10 years, and that it is then $= 9.996,17$. Therefore the greatest magnitude of $0.000,464 \times \dot{V}$, or of the second term of the expression $.2394\text{£} + 0.000,464 \times \dot{V}$, (which, at this rate of interest, is $= \dot{H} - \dot{V}$,) is $= 0.000,464 \times 9.996,17 = 0.004,638$; which is less than the 51st part of $.2394\text{£}$, or of the first term of the said expression.

And, when the interest of money is 10 per cent. it appears from Table XXIII, page 232, that the greatest magnitude of $\dot{V}$ is at the age of 11 years, and that it is then $= 9.101,99$. Therefore the greatest magnitude of $0.000,567 \times \dot{V}$, or of the second term of the expression $.2383\text{£} + 0.000,567 \times \dot{V}$, (which, at this rate of interest, is $= \dot{H} - \dot{V}$,) is $= 0.000,567 \times 9.101,99 = 0.005,160$; which is less than the 46th part of $.2383\text{£}$, or of the first term of the said expression.

CCXXII. It follows from the foregoing article that the greatest possible magnitudes of the several expressions $.2475\text{£} + 0.000,024 \times \dot{V}$, $.2469\text{£} + 0.000,038 \times \dot{V}$, $.2463\text{£} + 0.000,054 \times \dot{V}$, $.2457\text{£} + 0.000,074 \times \dot{V}$, $.2451\text{£} + 0.000,096 \times \dot{V}$, $.2445\text{£} + 0.000,121 \times \dot{V}$, $.2439\text{£} + 0.000,148 \times \dot{V}$, $.2428\text{£} + 0.000,212 \times \dot{V}$, $.2416\text{£} + 0.000,286 \times \dot{V}$, $.2405\text{£} + 0.000,370 \times \dot{V}$, $.2394\text{£} + 0.000,464 \times \dot{V}$, and $.2383\text{£} + 0.000,567 \times \dot{V}$, (to which the general expression

$$\frac{\text{£}}{47\frac{1}{2}}$$

$\frac{£}{4r\frac{1}{2}} + \frac{1 + 2r\frac{1}{2} + r}{4r\frac{1}{2}} \times \frac{£}{V}$ becomes equal when r is equal to 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, and 1.10, or when the interest of money is 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. 3½ per cent. 4 per cent. 4½ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent.) are as follows,

When the interest of money is 2 per cent. the greatest possible magnitude of the expression $.2475£ + 0.000,024 \times \frac{£}{V}$ is ($= .2475£ + 0.000,675£$) = .248,175£.

When the interest of money is $2\frac{1}{2}$ per cent. the greatest possible magnitude of the expression $.2469£ + 0.000,038 \times \frac{£}{V}$ is ($= .2469£ + 0.000,959£$) = .247,859£.

When the interest of money is 3 per cent. the greatest possible magnitude of the expression $.2463£ + 0.000,054 \times \frac{£}{V}$ is ($= .2463£ + 0.001,221£$) = .247,521£.

When the interest of money is $3\frac{1}{2}$ per cent. the greatest possible magnitude of the expression $.2457£ + 0.000,074 \times \frac{£}{V}$ is ($= .2457£ + 0.001,537£$) = .247,237£.

When the interest of money is 4 per cent. the greatest possible magnitude of the expression $.2451£ + 0.000,096 \times \frac{£}{V}$ is ($= .2451£ + 0.001,826£$) = .246,926£.

When the interest of money is $4\frac{1}{2}$ per cent. the greatest possible magnitude of the expression $.2445£ + 0.000,121 \times \frac{£}{V}$ is ($= .2445£ + 0.002,129£$) = .246,629£.

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When the interest of money is 5 per cent. the greatest possible

magnitude of the expression $.2439\text{£} + 0.000,148 \times \overset{\text{£}}{V}$ is ($= .2439\text{£}$
 $+ 0.002,499\text{£}) = .246,399\text{£}.$

When the interest of money is 6. per cent. the greatest possible

magnitude of the expression $.2428\text{£} + 0.000,212 \times \overset{\text{£}}{V}$ is ($= .2428\text{£}$
 $+ 0.002,984\text{£}) = .245,784\text{£}.$

When the interest of money is 7 per cent. the greatest possible

magnitude of the expression $.2416\text{£} + 0.000,286 \times \overset{\text{£}}{V}$ is ($= .2416\text{£}$
 $+ 0.003,549\text{£}) = .245,149\text{£}.$

When the interest of money is 8 per cent. the greatest possible

magnitude of the expression $.2405\text{£} + 0.000,370 \times \overset{\text{£}}{V}$ is ($= .2405\text{£}$
 $+ 0.004,099\text{£}) = .244,599\text{£}.$

When the interest of money is 9 per cent. the greatest possible

magnitude of the expression $.2394\text{£} + 0.000,464 \times \overset{\text{£}}{V}$ is ($= .2394\text{£}$
 $+ 0.004,638\text{£}) = .244,038\text{£}.$

And, when the interest of money is 10 per cent. the greatest possible

magnitude of the expression $.2383\text{£} + 0.000,567 \times \overset{\text{£}}{V}$ is ($= .2383\text{£}$
 $+ 0.005,160\text{£}) = .243,460\text{£}.$

The said greatest magnitudes of the whole expressions mentioned in Art. 217 are, all of them, less than a quarter of a Year's annuity.

CCXXIII. These several quantities, .248,175*£*, .247,859*£*, .247,521*£*, .247,237*£*, .246,926*£*, .246,629*£*, .246,399*£*, .245,784*£*, .245,149*£*, .244,599*£*, .244,038*£*, and .243,460*£*, or (neglecting the two last figures, as inconsiderable,) .2482*£*, .2478*£*, .2475*£*, .2472*£*, .2469*£*, .2466*£*, .2464*£*, .2458*£*, .2451*£*, .2446*£*, .2440*£*, and .2434*£*, (which are the greatest possible magnitudes of the several values of the expression $\frac{1}{4r^{\frac{1}{2}}} + \frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{1}{V}$, or

$\frac{1}{4r^{\frac{1}{2}}}$, according to the twelve different rates of interest above-mentioned,) are all of them less than .2500*£*, or $\frac{1}{4}$ of a pound, or a quarter of a year's annuity. Therefore the greatest possible magnitude of the excess of the value of an annuity of one pound a year, paid half-yearly by payments of 10 shillings each, above the value of a like annuity of one pound a year, paid at the end of every year, when the interest of money is at any of the rates before-mentioned, is less than $\frac{1}{4}$ part of a pound, or a quarter of a year's annuity.

CCXXIV. Since the several values of $\frac{1}{4r^{\frac{1}{2}}}$, or of the first term of

The second terms of the said expressions, mentioned in Art. 217, are so small in comparison of the first terms, that they may justly be neglected.

the expression $\frac{1}{4r^{\frac{1}{2}}} + \frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{1}{V}$, at the several rates of interest before-mentioned, have been shewn, in Art. 217, to be .2475*£*, and .2469*£*, and .2463*£*, and .2457*£*, and .2451*£*, and .2445*£*, and .2439*£*, and .2428*£*, and .2416*£*, and .2405*£*, and .2394*£*, and .2383*£*; and (by Art. 221) the greatest possible magnitudes of the second term, $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{1}{V}$, of the same expression, at the same rates of interest, are only equal to 0.000,675*£*, and 0.000,959*£*, and 0.001,221*£*, and 0.001,537*£*, and 0.001,826*£*, and 0.002,129*£*, and 0.002,499*£*, and 0.002,984*£*, and 0.003,549*£*, and 0.004,099*£*, and 0.004,638*£*, and 0.005,160*£*; of which latter quantities the last and greatest, to wit, 0.005,160*£*, is only about 5 thousandth parts, or one two hundredth part, of a pound, or one two hundredth part of a year's

year's annuity, or less than two days annuity; we may safely neglect the

$$\text{second term, } \frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V}, \text{ of the expression } \frac{\text{£}}{4r^{\frac{1}{2}}} + \frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}}$$

$\times \frac{\text{£}}{V}$, as being inconsiderable in comparison of the first term, $\frac{\text{£}}{4r^{\frac{1}{2}}}$,

and may therefore esteem the first term $\frac{\text{£}}{4r^{\frac{1}{2}}}$ to be equal to the whole

expression, and consequently to $\frac{\text{£}}{H} - \frac{\text{£}}{V}$, or to the difference of the values of an annuity of one pound a year paid yearly and paid half-yearly. And, upon this supposition, we shall have $\frac{\text{£}}{H} - \frac{\text{£}}{V}$,

The values of the said expressions, when their second terms are neglected.

When the interest of money is 2 per cent. = .2475 £;
 And, when it is $2\frac{1}{2}$ per cent. = .2469 £;
 And, when it is 3 per cent. = .2463 £;
 And, when it is $3\frac{1}{2}$ per cent. = .2457 £;
 And, when it is 4 per cent. = .2451 £;
 And, when it is $4\frac{1}{2}$ per cent. = .2445 £;
 And, when it is 5 per cent. = .2439 £;
 And, when it is 6 per cent. = .2428 £;
 And, when it is 7 per cent. = .2416 £;
 And, when it is 8 per cent. = .2405 £;
 And, when it is 9 per cent. = .2394 £;
 And, when it is 10 per cent. = .2383 £.

CCXXV. And, if these last-mentioned quantities, .2475 £, .2469 £, .2463 £, &c. (which, it is evident, differ very little from each other,) be considered as equal to each other, and also as equal to .25 £, (from which they differ by only about the 100th part of a pound, or about four

Agreement of these values with Mr. Thomas Simpson's determination,

days annuity,) we shall then have $\frac{\text{£}}{H} - \frac{\text{£}}{V}$, or the difference of the values of an annuity of one pound a year, paid half-yearly, and paid yearly, in all the different rates of interest above-mentioned, equal to .25 £, or $\frac{1}{4}$ part of a pound, or a quarter of a year's annuity; agreeably to Mr. Thomas Simpson's determination above-mentioned.

Of the value of a life-annuity that is paid at the end of every quarter of a year.

CCXXVI. When the annuity is to be paid at the end of every quarter of a year, it will be worth somewhat more than when it is to be paid at the end of every half-year; and Mr. Simpson says that it will be worth an eighth part of a year's annuity more than in the other case, or $\frac{1}{8}$ of a year's annuity more than when it is to be paid at the end of every year. And this, I think, seems very probable. But after the foregoing very tedious investigation of the difference of the values of a yearly and a half-yearly annuity, (which yet I knew not how to make shorter or easier,) I shall not trouble either myself or my reader with any attempt to determine it exactly. Indeed I thought the difference of the values of a yearly and a half-yearly annuity hardly worth attending to, and should not therefore have gone into the foregoing investigation of it, (especially when I found into what length it led me) if I had not observed that several other writers upon this subject, and particularly Mr. De Moivre, Mr. Simpson, and Dr. Price, have thought fit to mention it as an object of some importance. But, having now found that the said difference is worth only about $.24\frac{1}{2}$, or less than a quarter of a year's annuity, we may be the better satisfied that the further addition that ought to be made to V , or to the value of the annuity when paid yearly, in order to make it equal to the value of the same annuity paid quarterly, can be but a very trifling quantity. And therefore we may safely decline any further consideration of it.

[End of the consideration of the values of life-annuities paid half-yearly and paid quarterly.]

Of the limit of the increase of a sum of money at compound interest.

CCXXVII. Nevertheless, as it is evident that the advantage arising from a given sum of money put out at compound interest continually increases in *some* degree when the term for which the money is lent is diminished, it will, I doubt not, be entertaining to the reader to see determination of *the limit* of this increase, or the sum to which the given sum of money may, (by increasing the number, and diminishing the lengths, of the terms for which the money is lent,) be made to approach as near as we please, in the space of a year, or in any other given time, but which it can never exceed, or even absolutely become equal to, in whatever number of parts the year be divided. Now this limit may be determined without much difficulty by the solution of the following Problem.

A Problem

A Problem in the Doctrine of Compound Interest.

CCXXVIII. Let a be any given sum of money, and b its interest for one year. Let m be any whole number whatsoever; and let it be supposed that the money a is lent out for the m^{th} part of a year at an interest that bears the same proportion to the interest b , (which would be due for a whole year,) as the term for which it is lent, to wit, the m^{th} part of a year, bears to a whole year; and that, when the interest and principal are both repaid at the end of the m^{th} part of a year, they are added together and immediately lent out again at the same rate of interest for another m^{th} part of a year; and so on continually, at the end of every m^{th} part of a year, till the whole year is exhausted. It is required to find the sum to which the original sum a will have increased, by these repeated loans of principal and interest, at the end of the whole year.

S O L U T I O N.

Since b is the interest of the sum a for a whole year, it is evident that $\frac{b}{a}$ will be the interest of the same sum a for the m^{th} part of a year. Therefore the sum to be lent at the end of the first, and the beginning of the second, m^{th} part of a year will be $\frac{a+b}{m}$. This sum is to be lent for the m^{th} part of a year, as the sum a was at first; and it is to be lent at the same interest. Therefore the said sum $\frac{a+b}{m}$ together with its interest for the m^{th} part of a year will bear the same proportion to the said sum $\frac{a+b}{m}$ alone as the said sum $\frac{a+b}{m}$ (which is equal to the original sum a together with its interest for the m^{th} part of a year,) bears to the original sum a . Therefore the said sum $\frac{a+b}{m}$, together with its interest for the m^{th} part of a year, is equal to $\frac{a+b}{m}$; or the quantity to which the original sum a will have increased by these loans at the end of the

second

second m^{th} part of a year will be $\frac{a + \frac{b}{m}}{a}$. In the same manner

may be shewn that the quantity to which the original sum a will have increased in the next, or third, m^{th} part of the year will be $\frac{a + \frac{b}{m}}{a} \times \frac{a + \frac{b}{m}}{a}$, or $\frac{a + \frac{b}{m}}{aa}$; and the quantity to which it

will have increased in the fourth m^{th} part of the year will be $\frac{a + \frac{b}{m}}{aa} \times \frac{a + \frac{b}{m}}{a}$, or $\frac{a + \frac{b}{m}}{a^3}$; and consequently that the quantity to which it will have increased in the last m^{th} part of the year

or in the course of the whole year, will be $= \frac{a + \frac{b}{m}}{a^{m-1}}$. QEI.

Examples of the foregoing Solution.

Case of interest paid every half-year.

CCXXIX. COROLL. 1. Let m be $= 2$; or let the sum a be lent for only half a year, and then paid in with the interest due upon it and immediately lent out again, together with the interest, at the same

rate of interest, for the other half-year. Then will $\frac{a + \frac{b}{m}}{a^{m-1}}$ be

$$\frac{a + \frac{b}{2}}{a^{2-1}} = \frac{a + \frac{b}{2}}{a} = \frac{aa + 2a \times \frac{b}{2} + \frac{bb}{4}}{a^2} = \frac{a + b + \frac{bb}{4a}}{a}.$$

There

fore by lending the sum a in this manner for two successive half-years instead of lending it at once for a whole year, the said sum will, at the end of the year, be increased to $\frac{a + b + \frac{bb}{4a}}{a}$, instead of being increased only to $a + b$.

Thus, for example, if we suppose the sum a to be £100, and b , the interest of it for a year, to be 5 pounds, we shall have $bb = 25$, $\frac{b}{a} = \frac{5}{100} = \frac{1}{20} = \frac{5}{400}$, and $\frac{bb}{4a} = \frac{25}{400} = \frac{1}{16}$. Therefore, if a man were to lend a hundred pounds at the interest of 5 per cent. per annum, for two successive half-years, so that the principal together with its interest for the first half-year, should be made a new principal and lent out at the same interest for the second half-year, he would thereby increase it in the course of the whole year to only 15. 3d. more than it would have been increased to in the same time by lending it once for a whole year at the same interest.

CCXXX. COROLL. 2. If m is = 4, we shall have $\frac{a + \frac{b}{m}}{m}$ Case of interest paid every quarter of a year.

$$\frac{a + \frac{b}{4}}{4} \left(a^4 + \frac{4a^3b}{4} + \frac{6aabb}{16} + \frac{4a \times b^3}{64} + \frac{b^4}{256} \right) = a^4 + a^3b \text{ year.}$$

$$+ \frac{3aabb}{8} + \frac{ab^3}{16} + \frac{b^4}{256}, \text{ and } am-1 = a4-1 = a^3; \text{ and conse-}$$

$$\frac{a + \frac{b}{m}}{m} = \frac{a + b + \frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}}{m-1}. \text{ Therefore,}$$

if the sum a be lent out for only a quarter of a year, or three months, the interest of b per annum, or $\frac{b}{4}$ for the said three months, and it be then paid in with the interest due upon it; and then the said principal and interest be added together, so as to make a new principal, and the said new principal be lent out again immediately, at the same rate of interest as before, for three months more; and so on for the two remaining quarters of the year; the quantity to which the original sum a will be increased at the end of the year by means of these four successive loans, will be $\frac{a + b + \frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}}{4}$, which is greater than $\frac{a + b}{4}$, (or the quantity to which the said sum a would have increased in the same time by means of a single loan at the same interest for the whole year,) by $\frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$.

If

If a is 100 pounds, and b is 5 pounds, we shall have $\frac{3bb}{8a} (= \frac{3 \times 25}{8 \times 100})$

$$= \frac{3 \times 1}{8 \times 4} = \frac{3 \times 1}{32} = \frac{3 \times 20}{32} = \frac{60}{32} = \frac{720}{32} = 22.5 = 1, 10\frac{1}{2}; \text{ and}$$

$$\frac{b^3}{16aa} (= \frac{125}{16 \times 10,000}) = \frac{25}{16 \times 2000} = \frac{5}{16 \times 400} = \frac{1}{16 \times 80} =$$

$$\frac{1}{1280} = \frac{240}{1280} = \frac{24}{128} = \frac{3}{16} = 0.1875; \text{ and}$$

$$(\frac{625}{256 \times 1000,000} = \frac{125}{256 \times 200,000} = \frac{25}{256 \times 40,000} = \frac{5}{256 \times 8000})$$

$$= \frac{1}{256 \times 1600} = \frac{240}{256 \times 1600} = \frac{24}{256 \times 160} = \frac{3}{256 \times 20} = \frac{3}{5120}$$

$$= 0.000581. \text{ Therefore } \frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^2} \text{ will be } = 1, 10\frac{1}{2}$$

+ 0.18750 + 0.00058 = 1, 10. 68808, or less than 1, 10 $\frac{1}{2}$; that is the excess of the increase of the original sum of a hundred pounds in the space of a year by means of these four quarterly loans above the increase of it in the same time by means of a single loan of it for the

whole year at the interest of 5 pounds, is a little more than 1, 10 $\frac{1}{2}$ but less than 1, 10 $\frac{3}{4}$.

Case of interest paid every month.

CCXXXI. COROLL. 3. If m is = 12, or the sum a is lent out for only the 12th part of a year, or a calendar month, and then paid in, and the principal and interest added together so as to form a new principal, and the said new principal lent out again immediately for another calendar month at the same interest as before; and the loans of the principal with its monthly interest, be repeated for the

remaining ten months of the year, we shall have $\frac{a + \frac{b}{12}}{m} = \frac{a + \frac{b}{12}}{12}$

$$\text{and } am - 1 = a'', \text{ and consequently } \frac{a + \frac{b}{m}}{am - 1} = \frac{a + \frac{b}{12}}{a''}$$

If a is 100, and b is 5, we shall have $bb = 25$, and $\frac{11b}{24}$

$$\left(= \frac{11 \times 25}{24 \times 100} = \frac{11 \times 25}{24 \times 4 \times 25} = \frac{11 \times 1}{24 \times 4} = \frac{11}{96} = \frac{11 \times 240}{96} = \frac{2640}{96} \right)$$

$$= 27.5) = 2, 3.5, \text{ or } 2, 3 \frac{1}{2}; \text{ and } \frac{55b^3}{432aa} \left(= \frac{55 \times 125}{432 \times 100 \times 100} = \right)$$

$$\frac{55 \times 25 \times 5}{432 \times 4 \times 25 \times 5 \times 20} = \frac{55}{432 \times 4 \times 20} = \frac{55}{432 \times 4 \times 4 \times 5} = \frac{11}{432 \times 4 \times 4}$$

$$= \frac{11}{432 \times 16} = \frac{11}{6912} = \frac{11 \times 240}{6912} = \frac{2640}{6912} = 0.38, \text{ or less than one half-penny.}$$

Therefore $\frac{11bb}{24a} + \frac{55b^3}{432aa}$ will be equal to 2, 3.5 + 0.38,

or 2, 3.88, or less than 2, 4. So that the excess of the interest of 100 pounds for a year, arising from these 12 monthly loans, at this rate of interest, above 5 pounds, or the interest of the same sum of money for the same time, arising from a single loan of it for the whole year, is only 2, 3.88, or less than 2, 4.

Case of interest paid every week.

CCXXXII. COROLL. 4. If m is = 52, or the sum a is lent out for only the 52d part of a year, or for one week, and then paid in, and, with its interest, immediately lent out again for another week, and so on for every week throughout the whole year, we shall have

$$\overline{a+b}^m = \overline{a+b}^{52} \quad \text{and } a^{m-1} = a^{51}, \text{ and consequently } \frac{a+b}{a^{51}}$$

$$= \frac{\overline{a+b}^{52}}{a^{51}}. \quad \text{Now, by Sir Isaac Newton's binomial theorem,}$$

$$\overline{a+b}^{52} \text{ is } \left(= a^{52} + \frac{52}{1} \times a^{51} \times \frac{b}{52} + \frac{52}{1} \times \frac{52-1}{2} \times \frac{a^{50} \times b^2}{52 \times 52} + \frac{52}{1} \right)$$

$$\begin{aligned}
& + \frac{52}{1} \times \frac{52-1}{2} \times \frac{52-2}{3} \times \frac{a^{49} \times b^3}{52 \times 52 \times 52} + \&c. = a^{52} + a^{51}b \\
& + \frac{51a^{50}bb}{104} + \frac{51}{2} \times \frac{50}{3} \times \frac{a^{49}b^3}{52 \times 52} + \&c. = a^{52} + a^{51}b + \frac{51a^{50}bb}{104} \\
& + \frac{17 \times 25 \times a^{49}b^3}{52 \times 52} + \&c.) = a^{52} + a^{51}b + \frac{51a^{50}bb}{104} + \frac{425a^{49}b^3}{2704} \\
& + \&c. \text{ Therefore } \frac{a+b}{a^{51}} \text{ is } = a + b + \frac{51bb}{104a} + \frac{425b^3}{2704aa} + \&c.
\end{aligned}$$

Therefore the sum a will, in the course of the whole year, have increased, by means of these 52 repeated weekly loans, to $a + b + \frac{51bb}{104a} + \frac{425b^3}{2704aa} + \&c.$ which exceeds $a + b$ (or the quantity to which the sum a would have increased in the same time by a single loan for the interest b ;) by the difference $\frac{51bb}{104a} + \frac{425b^3}{2704aa} + \&c.$ in which difference all the terms after the second term, $\frac{425b^3}{2704aa}$, are omitted on account of their extreme smallness in comparison of $\frac{51bb}{104a}$ and $\frac{425b^3}{2704aa}$.

Let a be = 100, as before, and $b = 5$. Then will bb be = 25,

$$\begin{aligned}
& \text{and } \frac{51bb}{104a} \text{ will be } (= \frac{51 \times 25}{104 \times 100} = \frac{51 \times 25}{104 \times 4 \times 25} = \frac{51 \times 1}{104 \times 4} = \frac{51 \times 240}{104 \times 4} \\
& \frac{51 \times 60}{104} = \frac{51 \times 15}{26} = \frac{766}{26}) = 29.46, \text{ or } 2, 5.46; \text{ and } \frac{425b^3}{2704aa}
\end{aligned}$$

$$\begin{aligned}
& \text{will be } (= \frac{425 \times 5 \times 5 \times 5}{2704 \times 100 \times 100} = \frac{425 \times 5 \times 5 \times 5}{2704 \times 5 \times 5 \times 4 \times 5 \times 20} = \frac{425 \times 1}{2704 \times 4 \times 20} \\
& \frac{425 \times 1}{2704 \times 4 \times 4 \times 5} = \frac{85 \times 1}{2704 \times 4 \times 4} = \frac{85 \times 240}{2704 \times 4 \times 4} = \frac{85 \times 60}{2704 \times 4} \\
& \text{Mm 2} = \frac{85 \times 15}{2704}
\end{aligned}$$

$$= \frac{85 \times 15}{2704} = \frac{1275}{2704} = 0.47; \text{ and consequently } \frac{51bb}{104a} + \frac{425b}{2704a}$$
 will be $= 2, 5.46 + 0.47 = 2, 5.93$, or less than 2, 6. Consequently the excess of the compound interest of 100 pounds for a year arising from these 52 weekly loans of it, at this rate of interest, above 5 pounds, or the simple interest of the same sum of money for the same time, or the interest arising from a single loan of it for the whole year, is only $2, 5.93$, or less than 2, 6.

Recapitulation of the conclusions obtained in the four preceding articles.

CCXXXIII. Thus it appears that, when the interest of money is 5 per cent. per annum, the excess of the compound interest of 100 pounds for a year, arising from two successive half-yearly loans of it, that interest, above the simple interest of it, is only 1, 3; and the excess of the compound interest above the simple interest when the loans are made for a quarter of a year at a time, is about 1, 10 $\frac{1}{4}$; and when the said loans are made for a month at a time, it is 2, 3 $\frac{1}{2}$; and when they are made for only a week at a time, it is about 2, 6. So that very little advantage arises to the lender of a sum of money by lending it for terms less than a year, or by receiving the interest of it at the end of every half-year, or quarter of year, or month, or week, instead of lending it for a whole year at once, or receiving his interest only at the end of the year.

Case of interest paid at the end of every minute, or second, of time, or rather, of every infinitely small portion of time,

CCXXXIV. COROLL. 5. The limit of the magnitude to which the sum a will increase in the course of a year by supposing the term for which it is lent to be continually diminished, and the number of the terms to be increased in the same proportion, may be found by supposing the number m (which denotes the number of those terms contained in the whole year,) to become infinite.

Now, when m is infinite, the several fractions $\frac{m-1}{2}, \frac{m-2}{3}, \frac{m-3}{4}, \frac{m-4}{5}, \frac{m-5}{6}, \&c.$ will become equal to $\frac{m}{2}, \frac{m}{3}, \frac{m}{4}, \frac{m}{5}, \&c.$

cause the subtracted numbers 1, 2, 3, 4, 5, &c. in the several numerators of those fractions will be infinitely smaller than the number m , from which they are subtracted. Therefore $\frac{a + \frac{b}{m}}{m}$, (which, by Sir Isaac

Newton's binomial theorem, is in all cases $= a^m + \frac{m}{1} a^{m-1} \frac{b}{m} + \frac{m-1}{2} \times a^{m-2} \times \frac{b^2}{mm} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{b^3}{m^3} + \frac{m-1}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{b^4}{m^4} + \frac{m-3}{4} \times \frac{b^5}{m^5} + \&c.$ continued to the

$m+1^{\text{th}}$ term,) will in this case be $= a^m + \frac{m}{1} \times a^{m-1} \times \frac{b}{m} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{b^2}{mm} + \frac{m}{mm} \times \frac{m-1}{1} \times \frac{m-2}{3} \times \frac{b^3}{m^3} + \frac{m}{mm} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{b^4}{m^4} + \frac{m}{mm} \times \frac{m-1}{1} \times \frac{m-2}{2} \times \frac{m-3}{3} \times \frac{m-4}{5} \times \frac{b^5}{m^5} + \&c.$

$ad\ infinitum = a^m + a^{m-1}b + \frac{a^{m-2}bb}{1.2} + \frac{a^{m-3}b^3}{1.2.3} + \frac{a^{m-4}b^4}{1.2.3.4} + \frac{a^{m-5}b^5}{1.2.3.4.5} + \&c.$ $ad\ infinitum$; and consequently $\frac{a + \frac{b}{m}}{m}$ will in

this case be $= a + b + \frac{b^2}{1.2.a} + \frac{b^3}{1.2.3.aa} + \frac{b^4}{1.2.3.4.a^3} + \frac{b^5}{1.2.3.4.5.a^4} + \&c.$ $ad\ infinitum$. Therefore the limit of the

quantity to which the original sum a may be made to increase in the space of a year by continually diminishing the magnitude, and increasing the number, of the parts into which the year is divided, or the periods

for which the money is to be lent, is the said series $a + b + \frac{bb}{1.2.a}$

$+ \frac{b^3}{1.2.3.aa} + \frac{b^4}{1.2.3.4.a^3} + \frac{b^5}{1.2.3.4.5.a^4} + \&c.$ $ad\ infinitum$; or

$a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c.$ $ad\ infinitum$;

and

and the excess of the said limit above $a + b$, or the quantity to which the said sum a would have increased in the same time by a single loan of it at the interest b for the whole year, is = the series

$$\frac{bb}{2a} + \frac{b^2}{2.3.aa} + \frac{b^3}{2.3.4.a^2} + \frac{b^4}{2.3.4.5.a^3} + \&c. \text{ ad infinitum. } QEL$$

Now let a (as before) be = 100, and b be = 5. Then will

$$\frac{bb}{2a} \text{ be } \left(= \frac{b \times b}{2a} = \frac{\text{£ } 5 \times \frac{5}{2 \times 100}} = \frac{\text{£ } 1}{20} \times \frac{1}{20} = \frac{\text{£ } 1}{400} = \frac{100}{400} \right. \\ \left. = \frac{1200}{40} \right) = 30 = 2, 6; \text{ and } \frac{b^3}{2.3.aa} \text{ will be } \left(= \frac{bb}{2a} \times \frac{b}{3a} = \frac{bb}{2a} \right.$$

$$\times \frac{1}{3 \times 20} = \frac{bb}{2a} \times \frac{1}{60} = 30 \times \frac{1}{60} \left. \right) = \frac{1}{2}; \text{ and } \frac{b^4}{2.3.4.a^3} \text{ will be}$$

$$\left(= \frac{b^3}{2.3.aa} \times \frac{b}{4a} = \frac{b^3}{2.3.aa} \times \frac{1}{4 \times 20} = \frac{b^3}{2.3.aa} \times \frac{1}{80} = \frac{1}{2} \times \frac{1}{80} \right) = \frac{1}{160};$$

$$\text{and } \frac{b^5}{2.3.4.5.a^4} \text{ will be } \left(= \frac{b^4}{2.3.4.a^3} \times \frac{b}{5a} = \frac{b^4}{2.3.4.a^3} \times \frac{1}{5 \times 20} \right.$$

$$\left. = \frac{b^4}{2.3.4.a^3} \times \frac{1}{100} = \frac{1}{160} \times \frac{1}{100} \right) = \frac{1}{16000}; \text{ and consequently the}$$

$$\text{series } \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^2} + \frac{b^5}{2.3.4.5.a^3} + \&c. \text{ will be}$$

$$\left(= 30 + \frac{1}{2} + \frac{1}{160} + \frac{1}{16000} + \&c. = 30 + 0.500,000 \right.$$

$$\left. + 0.006,250 + 0.000,062 + \&c. = 30.506,312 \right) = 2, 6.506,312,$$

$$\text{or } 2, 6\frac{1}{2} + 0.006,312, \text{ or } 2, 6\frac{1}{2} \text{ together with } \frac{6312}{1000,000} \text{ parts, or one}$$

158th part, of a Penny. Therefore the limit of the excess of the sum to which it is possible to increase the sum of 100 pounds in the space of a year by lending it at the interest of 5 per cent. per annum for very small portions of a year, above 105 pounds, or the sum to which it will increase

in the same time by a single loan of it for the whole year at the same rate of interest, is only two shillings, and sixpence, half-penny, together with the 158th part of a penny. QEI.

From this and the preceding articles we may conclude that the advantages that may be made by lending money for very small portions of a year, instead of lending it for a year, or half a year, at a time, are very small as not to be worth attending to.

A remarkable analogy between the foregoing limit of the increase of a sum of money at compound interest, in the course of a year, and a certain ordinate of a logarithmick curve.

CCXXXV. Let the sum a be represented by the subtangent of any logarithmick curve; and let an ordinate to the axis of the said curve be drawn that shall be equal to the subtangent of it; which, from the nature of the said curve, (which admits of ordinates of all possible magnitudes,) is always possible: and from the bottom of the said ordinate, (or the point in which it touches the axis of the curve,) and on at side of the curve on which the ordinates increase, let a portion of the said axis be taken that shall bear the same proportion to the said ordinate, (or to the subtangent of the curve,) as b , (the interest of the sum a for a year, when lent at once for the whole year,) bears to the said sum a itself; and at the extremity of the said portion, or absciss, draw the axis, let another ordinate be erected; which, it is evident, will be greater than the former ordinate, or than the subtangent. Then will the said second, or greater, ordinate represent, or be proportional to, the series $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c.$ *infinitum*; or the said first, or lesser ordinate, (which is equal to the subtangent,) the said absciss of the axis intercepted between the two ordinates, and the said greater ordinate, will bear to each other the same proportions, respectively, as the original sum a , the sum b (which is the interest of a for one year, when lent out at once for the whole year,) and the series $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c.$ *ad infinitum*, or the limit of the quantity to which the sum a may be made to increase in the same space of one year by lending it at compound interest for very short terms.

For

For it is shewn in my Elements of Plane Trigonometry, Art. 354, page 470, that, if a be the subtangent of a logarithmick curve, and a and $a+x$ be two ordinates to the axis of the said curve, and z the absciss of the axis intercepted between the said ordinates, the difference

x , of the said ordinates will be equal to the series $z + \frac{zx}{2a} + \frac{z^2}{2.3.4a}$

$+ \frac{z^3}{2.3.4.a^3} + \frac{z^4}{2.3.4.5.a^4} + \&c.$ *ad infinitum.* Consequently $a+x$,

or the greater of the said two ordinates, will be equal to the series

$a + z + \frac{zx}{2a} + \frac{z^2}{2.3.aa} + \frac{z^3}{2.3.4.a^3} + \frac{z^4}{2.3.4.5.a^4} + \&c.$ *ad infinitum.*

Therefore, if we denote the said absciss of the axis by the letter b instead

of the letter z , we shall have $a+x =$ the series $a + b + \frac{bb}{2a} + \frac{b^2}{2.3.aa}$

$+ \frac{b^3}{2.3.4.a^3} + \frac{b^4}{2.3.4.5.a^4} + \&c.$ *ad infinitum.* QED.

N. B. This analogy between the foregoing limit of the increase of the sum a in the course of a year, by being lent out for short periods at compound interest, and the ordinate of a logarithmick curve, has something in it that is pleasing and amusing to the imagination: and therefore I thought the reader might be glad to see it stated in this place, though I do not know of any practical use in the doctrine of annuities to which it can be applied. See upon this subject Dr. Keil's treatise on Logarithms, at the end of Commandine's edition of Euclid's Elements, printed at Oxford in the year 1747, page 71.

Of the foregoing infinite series, obtained for the limit of the increase of the sum a at compound interest, when b denotes the simple interest of it for a greater, or lesser, term than one year.

CCXXXVI. In Art. 227 and all the following articles we have supposed b to be the interest of the sum a for a single year, because that is the most common way of estimating, or denominating, the interest of money. But the reasonings used in those articles will be equally just, and the conclusions obtained in them will be equally true, if b be made to denote the interest of a for any other given time, either greater or less than a year; as, for example, if b be made to denote the interest of a

for fifteen or twenty years. And therefore it will always be true, (whatsoever be the length of the term for which the money is lent, or whatever be the magnitude of the interest b ;) that the limit of the quantity to be the original sum a may be made to increase, (in the same time in which the interest b becomes due,) by lending it at compound interest for a great number of very short terms, is equal to the foregoing series to the original sum a .

$$a + b + \frac{bb}{2a} + \frac{b^2}{2.3.aa} + \frac{b^3}{2.3.4.a^2} + \frac{b^4}{2.3.4.5.a^3} + \text{&c. ad infinitum.}$$

One of these cases seems more curious than the rest, namely, that in which the interest b , which is to be paid for the original sum a at the end of the term for which it is lent, is equal to the said sum a itself; as, for example, if the sum a were to be lent for a term of 20 years, and were then to be repaid with an interest equal to itself; which, it is evident, would be by no means a hard or unreasonable agreement. Now in this

CASE of the equality of b to a .

$$\text{the foregoing series } a + b + \frac{bb}{2a} + \frac{b^2}{2.3.aa} + \frac{b^3}{2.3.4.a^2} + \frac{b^4}{2.3.4.5.a^3} + \text{&c.}$$

$$\text{ad infinitum will become equal to } a + a + \frac{aa}{2a} + \frac{a^2}{2.3.aa} + \text{&c.}$$

$$+ \frac{a^4}{2.3.4.a^3} + \frac{a^5}{2.3.4.5.a^4} + \text{&c. ad infinitum} = a + a + \frac{a}{2} + \frac{a}{2.3} + \frac{a}{2.3.4} + \frac{a}{2.3.4.5} + \text{&c. ad infinitum} = a \times \text{the series } 1 + 1 + \frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5} + \text{&c. ad infinitum} = a \times 2.718, 281, 828,$$

459, 036, &c. Therefore, if the interest due for the sum a for any given time were equal to the said sum itself, instead of being only a 20th, or 25th, part of it, (as is most commonly the case, when money is lent only for a year,) the limit of the quantity to which the said sum might be made to increase in the same space of time by improving it at compound interest by means of very frequent payments of the interest and immediate renewals of the loan of both principal and interest, would be equal to 2.718, 281, 828, 459, 036, &c. times, or less than three times, the original sum a .

CCXXXVII. The computation of the terms of the series A computed: on of the value of the said series in that case to fifteen places of figures.

Let

Let

Let the capital letters *A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, Q, R, S* be put for the several terms of the said series, as they arise; so the

A shall be = 1, and *B* shall be = 1, and *C* shall be = $\frac{1}{2}$, and

D shall be = $\frac{1}{2.3}$, and *E* shall be = $\frac{1}{2.3.4}$, and *F* shall be =

$\frac{1}{2.3.4.5}$, and so on of the following terms. Then, from the law of the

continuation of the terms of this series, it is evident that *D* will be =

$\frac{C}{3}$, and *E* will be = $\frac{D}{4}$, and *F* will be = $\frac{E}{5}$, and *G* will be = $\frac{F}{6}$

and *H* will be = $\frac{G}{7}$, and *I* will be = $\frac{H}{8}$, and *K* will be = $\frac{I}{9}$,

and *L* will be = $\frac{K}{10}$, and *M* will be = $\frac{L}{11}$, and *N* will be = $\frac{M}{12}$,

and *O* will be = $\frac{N}{13}$, and *P* will be = $\frac{O}{14}$, and *Q* will be = $\frac{P}{15}$,

and *R* will be = $\frac{Q}{16}$, and *S* will be = $\frac{R}{17}$. We shall therefore have

$$A = 1 = 1.000,000,000,000,000;$$

$$\text{and } B = 1 = 1.000,000,000,000,000;$$

$$\text{and } C = \frac{1}{2} = .500,000,000,000,000;$$

$$\text{and } D = \frac{C}{3} = .166,666,666,666,666;$$

$$\text{and } E = \frac{D}{4} = .041,666,666,666,666;$$

$$\text{and } F = \frac{E}{5} = .008,333,333,333,333;$$

$$\text{and } G = \frac{F}{6} = .001,388,888,888,888;$$

$$\text{and } H = \frac{G}{7} = .000,198,412,698,412;$$

and

$$\text{and } I = \frac{H}{8} = .000,024,801,587,301;$$

$$\text{and } K = \frac{I}{9} = .000,002,755,731,922;$$

$$\text{and } L = \frac{K}{10} = .000,000,275,573,192;$$

$$\text{and } M = \frac{L}{11} = .000,000,025,052,108;$$

$$\text{and } N = \frac{M}{12} = .000,000,002,087,075;$$

$$\text{and } O = \frac{N}{13} = .000,000,000,160,590;$$

$$\text{and } P = \frac{O}{14} = .000,000,000,011,470;$$

$$\text{and } Q = \frac{P}{15} = .000,000,000,000,764;$$

$$\text{and } R = \frac{Q}{16} = .000,000,000,000,047;$$

$$\text{and } S = \frac{R}{17} = .000,000,000,000,002.$$

$$2.718,281,828,459,036.$$

N. B. The proportion of this number, 2.718,281,828,459,036, &c. 1, is what Mr. Coates calls the *ratio modularis*, being the ratio, or proportion, of two ordinates to the axis of a logarithmick curve drawn through the extremities of an absciss of the axis equal to the *subtangent* of the curve, or that ratio of which the subtangent of a logarithmick curve, and, in general, in any other system of the measures of ratios, the *modulus* of the system, is the measure.

CCXXXVIII. If the sum a be supposed to be lent for so long a term of years before its interest b is to be paid, that the said interest shall be greater than the principal, it is evident that the literal parts of the series of the foregoing series $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.a^2} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4}$ is greater than the original sum a .

N^o 2

$$+ \frac{b^5}{2.3.4.5.a^4}$$

$$+ \frac{b^3}{2.3.4.5.a^4} + \&c. \text{ ad infinitum, to wit, } a, b, \frac{bb}{a}, \frac{b^3}{aa}, \frac{b^4}{a^3}, \frac{b^5}{a^4} \&c.$$

The said series will in this case diverge in its first terms, but afterwards converge in all the remaining terms.

will continually increase. And hence it may seem, perhaps, at first sight, that the terms of the series will in these cases continually diverge, and that consequently their sum, or the series itself, will become infinitely great, and therefore will not exhibit truly the value of the aforesaid limit. But, if we consider the matter with a little attention, we shall perceive that the series must in all cases be a converging one, whatever be the proportion in which the interest b may exceed the original sum a , though the convergency will not, in these cases, begin from the first term a , as it does when the interest b is less than a . For, as, upon this supposition

the literal parts of the terms, to wit, $a, b, \frac{bb}{a}, \frac{b^3}{aa}, \frac{b^4}{a^3}, \frac{b^5}{a^4}$, &c. will

diverge, or increase, continually in the same given proportion of b to a and the numeral parts of the third, fourth, fifth, sixth, and other follow-

ing terms, to wit, $\frac{1}{2}, \frac{1}{2.3}, \frac{1}{2.3.4}, \frac{1}{2.3.4.5}$, &c. do, in all cases, decrease

in proportions which are continually and indefinitely increasing, to wit, in the proportions of 2 to 1, 3 to 1, 4 to 1, and so on, it is evident that the proportion of the numeral parts of two contiguous terms in the series (in which the latter of the said numeral parts will be less than the former of them,) must always, in some part or other of the series, become greater than the proportion of the literal parts of the same terms, (in which the latter of the said literal parts will be greater than the former of them,) and consequently that the latter of the said two contiguous terms will, upon the whole, be less than the former of them. And as the terms in which this happens all the following terms of the series will continually decrease in a greater and greater proportion. Thus, for

example, if b is $= 10a$, the first ten terms of the series $a + b +$

$$+ \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c. \text{ ad infinitum, or (if}$$

denote the numeral parts of these terms, to wit, 1, 1, $\frac{1}{2}, \frac{1}{2.3}, \frac{1}{2.3.4},$

$\frac{1}{2.3.4.5}$, &c. by the capital letters A, B, C, D, E, F , &c. as

$$\text{Art. 237,) of the series } Aa + Bb + \frac{Cbb}{a} + \frac{Db^3}{aa} + \frac{Eb^4}{a^3} + \frac{Fb^5}{a^4} + \&c.$$

&c. *ad infinitum*, or of the series $Aa + Bb + \frac{Bbb}{2a} + \frac{Cbb^2}{3aa} + \frac{Dbb^3}{4a^2}$

$$+ \frac{Eb^4}{5a^3} + \frac{Fb^5}{6a^4} + \frac{Gb^6}{7a^5} + \frac{Hb^7}{8a^6} + \frac{Ib^8}{9a^7} + \frac{Kb^{10}}{10a^8} + \frac{Lb^{11}}{11a^9}$$

$$+ \frac{Mb^{12}}{12a^{10}} + \frac{Nb^{13}}{13a^{11}} + \text{&c.} \text{ will diverge, or increase; because the}$$

proportion of b to a , or of 10 to 1, (in which the literal part of every term exceeds the literal part of the term that immediately precedes it,) greater than the proportions of 2 to 1, 3 to 1, 4 to 1, 5 to 1, to 1, 7 to 1, 8 to 1, and 9 to 1, in which the numeral parts of third, fourth, fifth, sixth, seventh, eighth, ninth, and tenth terms are exceeded by the numeral parts of the terms that immediately precede them: but the eleventh term $\frac{Kb^{10}}{10a^8}$ will not be greater than the tenth term

$$\frac{Ib^9}{9a^7}, \text{ but exactly equal to it, being equal to } \left(K \times \frac{b^{10}}{10a^8} = \frac{I}{9} \times \frac{b^{10}}{10a^8} \right)$$

$$= \frac{1}{9} \times \frac{b^9}{a^7} \times \frac{b}{10a} = \frac{Ib^9}{9a^8} \times \frac{b}{10a} = \frac{Ib^9}{9a^8} \times \frac{1}{10} \times \frac{b}{a} = \frac{Ib^9}{9a^8} \times \frac{1}{10}$$

$$\left(\frac{1}{1} = \right) \frac{Ib^9}{9a^8} \times \frac{10}{10}, \text{ which is } = \frac{Ib^9}{9a^8}; \text{ and the twelfth term}$$

$$\frac{Lb^{11}}{11a^9} \text{ will be less than the eleventh term } \frac{Kb^{10}}{10a^8}, \text{ being } \left(= L \times \frac{b^{11}}{11a^9} \right)$$

$$= \frac{K}{10} \times \frac{b^{11}}{11a^9} = \frac{K}{10} \times \frac{b^{10}}{a^8} \times \frac{b}{11a} = \frac{Kb^{10}}{10a^8} \times \frac{b}{11a} = \frac{Kb^{10}}{10a^8} \times \frac{1}{11}$$

$$\times \frac{b}{a} = \frac{Kb^{10}}{10a^8} \times \frac{1}{11} \times \left(\frac{10}{1} \right) = \frac{Kb^{10}}{10a^8} \times \frac{10}{11}, \text{ which is less than}$$

$$\frac{Kb^{10}}{10a^8} \text{ in the proportion of 10 to 11. And in like manner it will be}$$

$$\text{found, that the thirteenth term } \frac{Mb^{12}}{12a^{10}} \text{ is less than the twelfth term}$$

$$\frac{Lb^{11}}{11a^9} \text{ in the proportion of 10 to 12, and that the fourteenth term}$$

$$\frac{Nb^{13}}{13a^{11}} \text{ is less than the thirteenth term } \frac{Mb^{12}}{12a^{10}} \text{ in the proportion of 10}$$

$$\text{to 13, and that every following term is less than the term that immediately}$$

diately preceeds it in the increasing proportions of 10 to 14, 10 to 15, 10 to 16, and 10 to every following higher number whatsoever. So that

from the eleventh term $\frac{Kb^{10}}{10a^9}$ the foregoing series will be a converging

series. And the same convergency must take place, (though farther from the beginning of the series,) if we should suppose the interest b to exceed the original sum a in the proportion of ten thousand to one, or any other and greater proportion, how great soever, instead of exceeding it only in the proportion of 10 to 1. We may therefore conclude that the said series

$$a + b + \frac{bb}{2a} + \frac{b^2}{2.3.aa} + \frac{b^3}{2.3.4.a^2} + \frac{b^4}{2.3.4.5.a^3} + \text{&c. ad infinitum}$$

will always exhibit the true value of the aforesaid limit of the quantity to which the original sum a may be made to increase by compound interest, in the same time as at simple interest it increases to $a + b$, whatever be the proportion of the interest b to the said original sum a .

But it is now time to return from this digression, concerning the increase of money at compound interest, to the consideration of the values of annuities depending upon lives, which is the principal subject of this tract.

[End of the inquiry concerning the increase of a sum of money by means of compound interest.]

Concerning remote life-annuities, that are to commence at the distance of 30 years.

CCXXXIX. Now the next thing I propose to enter upon, concerning this subject of Life-annuities, is the computation of the values of certain remote life-annuities, which seem to deserve particular consideration: I mean life-annuities that are to commence at the distance of 30 years, or whereof the first payments are to be made at the end of 31 years;—I say, at the end of 31 years, and not at the end of 30 years and a half, because I shall now return to the supposition that has been made throughout all the former part of this book, as far as Art. 200, “that the payments of annuities are to be made only at the end of every year,” and shall not trouble either my reader or myself any more with the perplexings, but not very importantly, consideration of the difference between the values of annuities paid yearly, and annuities paid half-yearly. Now these particular remote life-annuities, which are to take place at the distance of 30 years, seem to me to be more interesting than

than any others (as, for instance, than life-annuities that are to commence at the distance of 20, or 25, or 35, or 40, years,) because I conceive that it would be a very useful and convenient measure, both for the Publick, and the individuals whom it would concern, if the Parliament were to establish such annuities, which the people should be at liberty to purchase at their full and proper values according to the several ages of the purchasers. For, as the Parliament has, within these few years past, thought fit to establish annuities for a term of 30 years certain, it seems reasonable to suppose that it would be a great facilitation to the younger part of the proprietors of those annuities to be able, for a moderate sum of money, (such as about two years annuity,) to purchase an additional interest in them for their own lives, and thereby to rid themselves of the uneasy apprehension, that, without living to an uncommonly great old age, they might possibly outlive the income that supports them. If this objection to these annuities for 30 years were removed, by establishing such additional life-annuities as have been just now mentioned, it would probably induce more people to become purchasers of them, and consequently would tend to enhance their value, and enable the Government to get a better price for them. And thus, when the Government and the individuals who should purchase these 30 years annuities, or any other annuities for the same term that may hereafter created, would find their account in the establishment of the said additional remote life-annuities.

Nor would the establishment of such remote life-annuities retard, in any considerable degree, the diminution of the national debt arising from the expiration of the annuities for 30 years; because it is almost certain that much the greater part of the proprietors of those annuities will be dead before the expiration of the said term of 30 years; and of the few that will live beyond it, and who consequently might, if they had become purchasers of these additional life-annuities, continue to enjoy their respective annuities beyond it, much the greater part will die in the course of the next ten or fifteen years: and it must also be remembered that they will have paid to the Government a full and fair price for the said prolongation of their said annuities during the time they do enjoy them.

For these reasons I am inclined to think that the measure of establishing such a set of remote life-annuities is not liable to any objection but that of the difficulty (as it may be supposed) of distinctly ascertaining the several values according to the several ages of the persons who shall purchase them. But this difficulty is a small one, and may be easily removed by computing a few tables of those values according to the most received and customary rates of interest at which money is at this time

time (July 1781) borrowed by the Government, and will, probably, be borrowed by them when the blessing of peace shall be restored to us. And with this view I have procured the four following tables of the values of such remote life-annuities to be computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon suppositions that the interest of money is 5 per cent. $4\frac{1}{2}$ per cent. 4 per cent. and $3\frac{1}{2}$ per cent. It does not seem likely that Government should be able to borrow money at a lower interest than $3\frac{1}{2}$ per cent. for many years to come, even though peace should be speedily re-established.

Of the manner of computing the values of these remote life-annuities.

CCXL. The values of remote life-annuities may be either computed in the manner explained above in Problem II, Coroll. 3, Art. 3, page 33, without any reference to the values of immediate life-annuities for the lives of persons of the ages at which the remote annuities are to take place; or they may be derived from the values of those immediate life-annuities by an easy operation or two of multiplication and division. And, as the values of those immediate life-annuities have been already computed, and are contained in the foregoing tables, I conceive the latter method of obtaining the values of the said remote annuities to be upon the whole the most convenient. This method may be explained in the manner following.

A method of deriving them from the values of immediate life-annuities contained in the foregoing tables in Art. 201, pages 221, 222, &c. — 232.

CCXLI. Let $\bar{R}$ be the value sought of an annuity of one pound a year, that is to commence at the end of m years, (or so that the first payment of it shall become due at the end of $m+1$ years,) in case a person who is now of the age of N years shall be then alive, and who is to continue during the whole remainder of the said person's life. And let $\bar{V}$ be the value of an immediate annuity of one pound a year for the life of a person of the age of $N+m$ years. Let P be the number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life, as being alive at the age of N years, and F the number of persons represented there as living at the age of $N+m$ years, and F' the number living at the age of $N+m+1$ years, and F'' , F''' , F^{IV} , F^V , F^{VI} , F^{VII} , &c. the numbers living at the ages of $N+m+3$ years, $N+m+4$ years, $N+m+5$ years, $N+m+6$ years, &c.

to the product that arises by multiplying V , (or the value of an immediate annuity of one pound a year for the life of a person of the age of $N+m$ years,) first into the fraction $\frac{1}{pm}$, and then into the fraction $\frac{F}{P}$. Q.E.F.

CCXLII. Thus, for example, if it is required to find the value of a remote annuity of one pound a year for the life of a person of the age of 10 years, to commence at the distance of 30 years, or when the said annuitant shall be 40 years of age, or so that the first payment of it shall become due at the end of 31 years, or when the said annuitant shall be 41 years old, the interest of money being $3\frac{1}{2}$ per cent. we must proceed in the manner following.

In the first place we must look out in the foregoing tables, (Table XV, page 224,) the value, V , of an immediate annuity of one pound a year for the life of a person of the age of ($N+m$, or $10+30$, or) 40 years; which we shall find to be $\pounds 16,104,542$.

Secondly, since the interest of money is supposed to be $3\frac{1}{2}$ per cent. we shall have $r = 1.035$, and $\frac{1}{r} = \frac{1}{1.035} = 0.966,183$, and $\frac{1}{pm} = \frac{1}{730} = 0.001356,278$; as appears by Mr. Sauri's second table of compound interest, pages 60 and 61.

And, lastly, it appears from Monsieur de Parcieux's table of the probabilities of the duration of human life, that P , or the number of persons living at the age of (N , or) 10 years, out of an original number of 1000 living at the age of 3 years, is 880, and that F , or the number of persons living at the age of ($N+m$, or $10+30$, or) 40 years, out of the same original number, is 657.

Therefore $\frac{F}{P} \times \frac{1}{pm} \times \frac{F}{V}$, or $\frac{F}{P} \times \frac{1}{r^3} \times \frac{F}{V}$, is $= \frac{657}{880} \times 0.001356,278 \times \pounds 16,104,542 \left(= \frac{234,074,646}{880} \times \pounds 16,104,542 = 0.265,993 \right)$
 $\times \pounds 16,104,542$

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$\pounds 16,104,542) = 4,283,695$; or the present value of a remote annuity of one pound a year for the life of a person of the age of 10 years, that is to commence at the end of 30 years, or when the annuitant shall be 40 years of age, or so that the first payment of it shall become due at the end of 31 years, or when the said annuitant shall be 41 years old, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. is $\pounds 4,283,695$, or $4l. 5s. 3d.$ or somewhat more than four years and a quarter's annuity. Q.E.I.

And in this manner all the values of remote life-annuities contained in the four following tables have been computed.

T A B L E XXIV.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years inclusively, that is to commence only at the end of 30 years, or whereby the first payment is to be received at the end of 31 years, after the time of purchasing it; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
3	$\pounds 4,444,037$	16	$\pounds 3,748,438$
4	$4,483,991$	17	$3,653,162$
5	$4,487,811$	18	$3,555,256$
6	$4,471,977$	19	$3,455,031$
7	$4,440,391$	20	$3,352,405$
8	$4,397,007$	21	$3,251,937$
9	$4,346,451$	22	$3,149,313$
10	$4,283,695$	23	$3,044,957$
11	$4,208,738$	24	$2,938,658$
12	$4,121,688$	25	$2,830,830$
13	$4,032,235$	26	$2,721,424$
14	$3,940,261$	27	$2,610,440$
15	$3,845,704$	28	$2,498,339$

The Principles of the Doctrine of

Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
29	£ 2.385,100	47	£ 0.453,064
30	2.270,749	48	0.383,586
31	2.155,438	49	0.320,944
32	2.038,874	50	0.264,962
33	1.921,611	51	0.216,020
34	1.803,961	52	0.173,893
35	1.685,836	53	0.137,512
36	1.567,839	54	0.106,163
37	1.450,584	55	0.079,874
38	1.332,772	56	0.058,249
39	1.217,431	57	0.041,158
40	1.105,337	58	0.027,702
41	0.996,861	59	0.017,798
42	0.892,829	60	0.010,135
43	0.793,643	61	0.005,251
44	0.699,717	62	0.002,352
45	0.611,500	63	0.000,813
46	0.528,105	64	0.000,000.

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T A B L E XXV.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years inclusively, that is to commence only at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4 per cent.

Years of Age.	Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.
3	£ 3.587,85	24	£ 2.427,34	45	£ 0.517,24
4	3.623,63	25	2.341,00	46	0.447,80
5	3.630,32	26	2.253,22	47	0.383,98
6	3.621,19	27	2.163,84	48	0.325,40
7	3.599,31	28	2.073,38	49	0.272,51
8	3.567,87	29	1.981,77	50	0.225,18
9	3.530,60	30	1.889,02	51	0.183,76
10	3.483,36	31	1.795,12	52	0.148,06
11	3.426,10	32	1.700,08	53	0.117,20
12	3.358,91	33	1.604,31	54	0.090,57
13	3.289,62	34	1.507,87	55	0.068,20
14	3.218,16	35	1.410,77	56	0.049,79
15	3.144,42	36	1.313,53	57	0.035,21
16	3.068,30	37	1.216,66	58	0.023,72
17	2.993,65	38	1.119,08	59	0.014,98
18	2.916,67	39	1.023,35	60	0.008,69
19	2.837,63	40	0.930,14	61	0.004,51
20	2.756,45	41	0.839,74	62	0.002,009
21	2.676,12	42	0.752,89	63	0.000,700
22	2.595,36	43	0.669,94	64	0.000,000
23	2.512,24	44	0.591,26		

TABLE XXVI.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years, inclusively, that is to commence only at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $4\frac{1}{2}$ per cent.

Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
3	£ 2.907,57	24	£ 2.009,15	45	£ 0.437,99
4	2.939,17	25	1.939,82	46	0.379,55
5	2.947,25	26	1.869,12	47	0.325,76
6	2.942,54	27	1.797,02	48	0.276,32
7	2.927,52	28	1.723,83	49	0.231,62
8	2.904,73	29	1.649,54	50	0.191,56
9	2.877,17	30	1.574,14	51	0.156,46
10	2.841,47	31	1.497,61	52	0.126,19
11	2.797,56	32	1.419,93	53	0.099,98
12	2.745,46	33	1.341,48	54	0.077,33
13	2.691,56	34	1.262,26	55	0.058,29
14	2.635,79	35	1.182,30	56	0.042,60
15	2.578,05	36	1.102,01	57	0.030,15
16	2.518,24	37	1.021,84	58	0.020,34
17	2.459,53	38	0.940,89	59	0.012,86
18	2.398,78	39	0.861,31	60	0.007,47
19	2.336,23	40	0.788,67	61	0.003,88
20	2.271,78	41	0.708,23	62	0.001,73
21	2.208,42	42	0.635,62	63	0.000,604
22	2.143,56	43	0.566,16	64	0.000,000
23	2.077,16	44	0.500,17		

T A B L E XXVII.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years, inclusively, that is to commence at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 5 per cent.

Years of a remote life annuity of one pound a year, that is to commence at the end of 30 years.	Years of age.	Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.	Years of age.	Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.
£ 2:364,81	24	£ 1:666,37	45	£ 0:371,26
2:393,50	25	1:610,16	46	0:322,03
2:400,38	26	1:553,46	47	0:276,64
2:399,10	27	1:495,12	48	0:234,87
2:388,90	28	1:435,76	49	0:197,04
2:372,38	29	1:375,37	50	0:163,10
2:351,96	30	1:313,93	51	0:133,33
2:324,88	31	1:251,42	52	0:107,63
2:291,05	32	1:187,80	53	0:085,35
2:250,48	33	1:123,40	54	0:066,08
2:208,88	34	1:058,21	55	0:049,85
2:164,67	35	0:992,23	56	0:036,46
2:119,27	36	0:925,83	57	0:025,83
2:072,10	37	0:859,37	58	0:017,43
2:025,74	38	0:792,10	59	0:011,03
1:977,62	39	0:725,83	60	0:006,410
1:927,94	40	0:661,07	61	0:003,326
1:876,60	41	0:598,01	62	0:001,488
1:826,07	42	0:536,84	63	0:000,520
1:774,23	43	0:478,99	64	0:000,000
1:721,01	44	0:423,57		

OBSERVATIONS
ON THE
NATIONAL DEBT,
AND

The most likely Methods of paying off a Part of it.

ART. CCXLIII.

WHEN the blessing of peace shall be restored to us (which, in our present friendless and declining condition, we ought surely to wish for upon almost any terms;) it is to be hoped that our rulers will set themselves seriously to work to reduce to a more moderate quantity the immense load of debt under which the nation now labours, and which will then be at least two hundred and twenty millions of pounds sterling, even if a peace should be made as soon as possible. For at this present time, November, 1781, it amounts to more than a hundred and ninety-eight millions, two hundred thousand pounds; or at least it will do so on the first of next January, 1782; as will appear from an accurate statement of it which has been printed in the Publick Advertiser of the second of October last, and which the reader may see in the note below.* And

* An account of the taxes laid on since the beginning of the American War.

	Computed produce per annum.	£
1776.		
Stamps on deeds,	— — — — —	30,000
Ditto on news-papers,	— — — — —	18,000
Ditto on cards,	— — — — —	6,000
Additional duty on coaches, &c.	— — — — —	19,000
		73,000
1777.		
Tax on servants,	— — — — —	105,000
Stamps,	— — — — —	55,000
Additional duty on glais,	— — — — —	45,000
Duty on sales by auction,	— — — — —	37,000
		242,000

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to this sum we must add at least 25 millions more for the expence attending the conclusion of the war, before every thing shall be completely settled on a peace establishment: so that at the final re-establishment of peace the whole debt will be at least two hundred and twenty millions. Now, when the important business of reducing this enormous debt to a less sum shall be undertaken, it seems probable that one, or more, of the following methods will be adopted for that purpose.

7be

1778.					
Tax upon house rents,	—	—	—	—	£.264,000
Additional duty on wines,	—	—	—	—	72,000
					336,000
1779.					
tax upon taxes, viz. an additional surcharge of 5 per cent. on					
customs and excise,	—	—	—	—	314,000
tax upon post horses,	—	—	—	—	164,000
					478,000
1780.					
an additional tax upon malt,	—	—	—	—	310,000
additional duty on British low wines,	—	—	—	—	20,617
ditto on British spirits,	—	—	—	—	34,557
ditto on brandy,	—	—	—	—	35,310
ditto on rum,	—	—	—	—	70,958
second additional duty on wines,	—	—	—	—	72,000
additional duty on coals exported,	—	—	—	—	12,899
additional 5 per cent. on the above laid taxes,	—	—	—	—	46,193
additional duty upon salt,	—	—	—	—	69,000
additional stamp duties,	—	—	—	—	21,000
duty on licences to sell tea,	—	—	—	—	9,082
					701,616
1781.					
five per cent. on excise, except malt, soap, candles, and hides,					150,000
discount of the customs,	—	—	—	—	167,000
tobacco one penny three farthings per pound,	—	—	—	—	61,000
sugar halfpenny per pound,	—	—	—	—	326,000
Since laid,	—	—	—	—	
duty on paper,	—	—	—	—	100,000
ditto on almanacks,	—	—	—	—	10,000
					110,000
					2,644,616

The exact national debt up to July 5, 1781, is £.177,206,000.
The annual interest raised on the publick is £.6,889,000.

far the funded debt, and the taxes laid, in order to discharge the interest to the publick creditors. The debt unfunded may be computed as here under:

P P

Navy

The first method of employing a given sum of money every year in the reduction of the national debt.

The first method is to employ the annual Surplus of the Public Revenue, which can be spared for this good work, in repaying to some of the national creditors a part of the capital which is due to them, and also to employ the annual savings of the interest upon the capital so discharged (which, it is evident, will increase every year) in the same manner, that is, in discharging, or repaying, some more of the said capital due to the publick creditors. Thus, for example, if the sum that can be spared out of the publick revenue for this purpose should be a million of pounds sterling a year, and some of the publick funds, as, for example, the 4 per cent. annuities, should rise to par, or above par, so that 100 pounds in those annuities shall sell for a hundred pounds, or more, in the money, the said surplus million of the publick revenue may be employed

Navy debt on the 1st of January, 1782, about	£.9,000,000
Army extraordinaries,	3,000,000
Vote of credit of last session,	1,000,000
Ordnance debt,	1,000,000
Money to be voted for navy extras,	1,000,000
Exchequer bills in circulation, about	4,000,000
Borrowed from the Bank of England,	2,000,000

21,000,000

Suppose, when this sum comes to be funded, that the loan or bargain with the publick may, as it has for the two or three last years, be negotiated at 5 and a half per cent. the annual interest to be paid on 21 millions will be

1,155,000

RECAPITULATION.

Principal funded on the 5th of July, 1781, —	177,206,000
Principal which will remain unfunded on the 1st of January, 1782,	21,000,000
Total of the national debt on the said last mentioned day,	198,206,000
Interest paid, for which provisions have been made by taxes, 5th July, 1781,	6,889,000
Interest to be paid for the debts not yet funded, which will stand due on the 1st of January, 1782,	1,155,000
	8,044,000

So that on the 1st of January, 1782, the national debt, funded and unfunded, amounts to one hundred ninety-eight millions, and a considerable fraction, and the interest to eight millions, which is nearly double to what was paid by the people in taxes, previous to the breaking out of the present war, the annual interest on the 1st of January, 1776, being in or about four millions three hundred thousand pounds.

paying off a million of the capital of the said annuities: and thus the national debt will be diminished by the sum of one million by this first payment. But by this first diminution of the debt it is evident the publick will have less interest to pay to the publick creditors, at the beginning of the next year than it had before by the interest of the million that has been so paid off, that is, (if the annuities are 4 per cents,) by 40,000 pounds. Therefore this sum of 40,000 pounds, which, before the discharge of the said million, used to be paid away every year to the publick creditors as the interest of it, may then be employed, together with the annual million of this year originally destined to this purpose, in discharging, or repaying, some more of the capital of the said 4 per cent. annuities. And in like manner there will be in every following year a greater and greater sum of money, by means of these savings of the interest of the discharged debt, that may be so applied. And in the course of fifty, sixty, years the amount of these annual payments to the publick creditors, or the portion of the national debt extinguished by them, will be exceeding great, being evidently equal to the amount of an annual sum of one million of pounds for the same period of time, improved at compound interest. In fifty years time it would be more than a hundred and fifty-two millions; and in sixty years it would be nearly two hundred and fifty-eight millions: as appears from Mr. Smart's third table of compound interest, pages 70 and 72. And consequently, as the annual interest of 2 hundred and fifty-two millions, at 4 per cent. is 152 times 40,000 pounds, or 6,080,000 pounds, and the annual interest of two hundred and thirty-eight millions, at the same rate, is 238 times 40,000 pounds, or 9,520,000 pounds; the quantity of the annual interest of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed by the operation of this annual surplus million in the course of fifty years would be £.6,080,000 *per annum*; and the quantity of those perpetual annuities that would be redeemed by the operation of this annual million in the course of sixty years would be £.9,520,00 *per annum*; which more than the whole interest of the present national debt, though increased to so alarming a quantity. So great would be the effect of the useful application of only the moderate sum of one million of pounds *per annum* for the space of only 60 years to this important purpose!

CCLXV. This first method of employing the supposed surplus part of the publick revenue is only fit to be applied to such of the public funds as all rise to *par* or above *par*, as I have supposed may be the case with the 4 per cent. annuities. But the greatest part of the national debt consists of 3 per cent. annuities; which will not, in all probability, rise at the peace to any thing like their *par*, or nominal value, they being at this time, November, 1781, at the very low price of 56 per cent. But,

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perhaps,

perhaps, they may rise, upon a peace, to £75 per cent. Now, if they should then rise to this value, or to any other value less than a hundred pounds per cent. it is not to be expected that the Government will reduce them at the full price of 100 pounds per cent. or pay the publick creditors a real hundred pounds for every nominal hundred pounds of their stock in these annuities; because the nation is not under any obligation to reduce them at that, or any other, price, provided it continues to pay the interest of them with punctuality. The Government will therefore chuse to continue to pay the interest due upon these funds, unless the owners of them will consent to be paid off at a less price than the *par*, or nominal value, of their stocks, and will accept a price equal to, or a little exceeding, the price at which those stocks shall then be found to sell at the publick market. And many of the owners of these stocks will, doubtless, be glad to accept of such a price for them. This brings me to the consideration of the second method in which it is probable that a given annual sum of money that can be spared from the publick revenue for that purpose, may be employed in diminishing the national debt, and redeeming the perpetual annuities that are now due to the publick creditors as the interest of it.

A second method of employing a given sum of money every year in the reduction of the national debt.

CCXLVI. This second method of employing a given annual sum of money in the reduction, or diminution, of the national debt consists in buying up with it, from the owners of some of the publick stocks that shall be sold at the publick market for less than their *par*, or nominal value, a part of their said stocks, with the full and free consent of the owners, at the market-price, or a price a little exceeding the market-price of the said stocks; and in employing the interest that would thereby be every year redeemed to the nation, in the same manner.

Thus, for example, if we suppose, as before, that the annual surplus of the publick revenue that can be spared for this purpose, is a million of pounds sterling, and that the value of the 3. per cent. annuities should rise after the conclusion of a peace, to 74 or 75 pounds per cent. the five million of pounds sterling *per annum* may be employed in buying up a part of the capital of the said 3 per cent. annuities, with the consent of the respective owners of it, at the price of £75 per cent. and the interest that would thereby be redeemed to the nation at the end of every year may also be employed in the same manner.

CCXLVII. If a million of pounds sterling were to be so employed, Of the effect it is evident that 750,000 pounds of it would buy up, or extinguish, a method of diminishing the national debt. million of the said capital of the 3 per cent. annuities, and consequently that the whole million so employed would extinguish a million and the third part of a million, or £1,333,333, of the said capital; of which capital of £1,333,333, the annual interest is 40,000 pounds.

This would be the effect of employing a single million of pounds sterling in this manner, or of the first year's execution of the supposed project of employing a million of pounds in this way for several years together. But at the end of the said first year it is evident that the Government would have the 40,000 pounds, which used before to pay the interest of the capital of £1,333,333 so extinguished, to employ in the same manner, over and above the annual million belonging to this year; which was originally destined to this purpose; so that the quantity of the said capital of the 3 per cent. annuities that would be extinguished by the second operation of this method would be so much as could be purchased at the price of £75 per cent. by the sum of 1,040,000 pounds, that is,

$$\frac{100}{75} \times £1,040,000, \text{ or } \frac{4}{3} \times £1,040,000, \text{ or } \frac{4,160,000}{3}, \text{ or } £1,386,666; \text{ of which sum the annual interest is } 41,600 \text{ pounds.}$$

Therefore at the beginning of the third year the Government would have the three following sums to employ in this manner; to wit, first, the original million belonging to the said third year; 2dly, the 40,000 pounds of the interest of the national debt which was redeemed by the first operation; and, 3dly, the 41,600 pounds of the interest of the same debt which was redeemed by the second operation; which sums together amount to 1,081,600 pounds. And this sum would be sufficient to extinguish $\frac{4}{3} \times £1,081,600$, or £1,442,133 of the said capital of the 3 per cent. annuities; the interest of which capital is 43,264 pounds *per annum*.

Therefore the sums that the Government will have to employ in this manner at the beginnings of the first, second, third, and fourth years during the time that this method shall be pursued, will be as follows; to wit,

1st,	£1,000,000;	
2dly,	£1,000,000 + £40,000, or £1,040,000,	
3dly,	£1,000,000 + £40,000 + £41,600, or £1,081,600; and	
4thly,	£1,000,000 + £40,000 + £41,600 + £43,264, or	
	£1,124,864,	

These

These sums, £1,000,000, £1,040,000, £1,081,600, and £1,124,864, are in continual geometrical proportion, the common ratio of the terms being that of 100 to 104; and the second, third and fourth terms are the values of a million of pounds at the ends of one, two, and three years, when improved at compound interest at the rate of 4 per cent. And in like manner it will be found that the sums which the Government will have to employ in the same manner at the beginnings of the fifth, sixth, seventh, and other following years during which this method shall be pursued, will the values of a million of pounds at the ends of four, five, six, and every following number of years, when the said million is improved during those years at compound interest at the rate of 4 per cent. Therefore the total amount of all the annual sums which the Government would have to employ in this manner during any given number of years, in consequence of having set apart an original annual sum of a million of pounds for this purpose, would be equal to the total amount of an annual sum of a million of pounds at the end of the same number of years, if the said annual sum were, as fast as it was received by the person to whom it was due, to be immediately lent out and improved at compound interest at the rate of 4 per cent. But we have already observed that it appears from Mr. Smart's tables of compound interest, pages 70 and 72, that the total amount of an annual payment of a million of pounds, so improved at compound interest at the rate of 4 per cent. for a period of fifty years, is more than a hundred and fifty-two millions; and that the amount of it for a period of sixty years is very nearly two hundred and thirty-eight millions. Therefore the total amount of all the sums which the Government would have to employ in thus buying up the stock of the 3 per cent. annuities at the price of £75 per cent. if an annual sum of a million of pounds sterling *per annum*, together with the interest that would be continually redeemed by it, were to be faithfully applied to this purpose for a period of fifty years, would be more than a hundred and fifty-two millions; and the amount of all the sums which the Government would have to employ in this manner in the course of sixty years would be very nearly two hundred and thirty-eight millions. And, as every million of this money would be sufficient to extinguish £1,333,333 of the capital of the said 3 per cent. annuities, it follows that the amount of all the said annual sums that would, according to this second method of employing the said annual surplus million, be applied to the extinguishment of the capital of the said 3 per cent. annuities, in the course of fifty years, would be sufficient to have extinguished in that time more than 152 times £1,333,333, or than £202,666,666 of that capital, and consequently to have redeemed for the nation more than 152 times 40,000 pounds *per annum*, or 6,080,000 pounds *per annum* of the perpetual annuities which pay the interest of that capital: and the amount of the said annual sums, that might be so employed, in the course of sixty years would be sufficient to have extinguished,

very nearly, 238 times £1,333,333, or, very nearly, £317,332,54, of the said capital of the 3 per cent. annuities, and consequently to have redeemed for the nation very nearly 238 times 40,000 pounds *per annum*, or very nearly 9,520,000 pounds *per annum* of the perpetual annuities which pay the interest of the said capital: which latter sum, £9,520,000, is greater than the whole interest of the present national debt, though increased of late to so alarming a quantity.

CCXLVIII. It is evident that the two foregoing methods of applying an annual sum of a million of pounds, together with the interest continually redeemed by it, for a given number of years, to the diminution of the national debt, to wit, by paying off at *par* those stocks which carry an interest of 4 per cent. when the price of those stocks shall rise to *par*, and by buying up at the price of £75 per cent. (with the consent of the owners of them,) those stocks which shall carry an interest of 3 per cent. would be equally beneficial to the publick, or that the quantity of the interest of the national debt that would be redeemed by either of them in a given time would be exactly the same as the quantity of the said interest that would be redeemed by the other in the same time.

The two foregoing methods of diminishing the national debt would be equally efficacious and beneficial to the nation.

Expediency of appropriating by act of parliament, in the strictest manner possible, the sum of one million of pounds sterling per annum out of the sinking fund, during a term of fifty, or sixty, years, to the discharge of the national debt.

CCXLIX. The foregoing very great operation of only one million of pounds sterling a year, with the interest continually redeemed by it, in the course of so moderate a period of time as fifty, or sixty, years, when strictly applied to this purpose of diminishing the national debt without any interruption, in either of the two foregoing methods, ought, one would think, to induce the Parliament to appropriate that sum out of the sinking fund to this important purpose in the strictest manner that can be devised, for the space of fifty, or sixty, years, and to forbear to interrupt its operation during that period upon any account, or occasion, whatsoever, however urgent. And it seems the more reasonable to expect that such a measure will soon be adopted, because the sinking fund has of late years produced no less a sum than three millions of pounds sterling *per annum*: and our ministers of state, as well as the owners of property in the publick funds, ought to recollect that the whole of the said fund, as its name imports, was once appropriated by Parliament to this very purpose, of *sinking*, or diminishing, the national debt, in the manner now recommended for one third of it.

A third

A third method of employing a given annual sum of money in the reduction of the national debt.

CCL. A third method of employing the said annual surplus million, or other part of the publick revenue that can be spared for this purpose, to the diminution of the national debt, would be to convert some of the *perpetual* annuities now due to the publick creditors as the interest of the said debt, into *temporary* annuities, or annuities that should continue only for a limited number of years, as, for example, for 100 years, or 80 years, or 60 years, or 30 years, but which should be greater than the perpetual annuities in the room of which they should be substituted, in a certain just and reasonable proportion that was adapted to the change made in their duration: the said change being made with the consent of the proprietors of the perpetual annuities that should be so converted. For without this consent a measure of this kind would be a breach of the national faith: but with such consent of the proprietors concerned it would be a perfectly just measure, and might, I imagine, be sometimes an expedient one. And, if it were adopted, the said annual surplus million, or other part of the publick revenue that could be spared for this purpose, might be employed in paying the additions that would be necessary to be made to the said perpetual annuities in compensation for the abridgement of their duration.

A remark on the said third method of diminishing the national debt.

CCLI. By this third method of proceeding the national debt would not be diminished gradually, or every year a little, as in the two former methods; but, on the contrary, the interest annually paid to the publick creditors would be somewhat increased during the continuance of the new temporary annuities that had been substituted in the room of some of the former perpetual ones. But at the end of that time the whole of the said temporary annuities would cease at once, and consequently the perpetual annuities, in lieu of which they had been substituted, would be redeemed to the nation.

The said third method would be equally efficacious with either of the two former in diminishing the national debt.

CCLII. This third method of diminishing the national debt will have precisely the same effect at the end of the term during which the new temporary annuities which are substituted in lieu of the former perpetual annuities, are to continue, as either of the two former methods; that is, the quantity of the interest of the national debt which will be redeemed by this third method at the end of the said limited number of years, by employing a million of pounds sterling, or any other given sum of money, every year in this manner, will be the same as the quantity of

the said interest that would be redeemed in the same course of time by employing the same million of pounds, or other annual sum, together with the savings of interest annually produced by it, in either of the two former methods.

CCLIII. Thus, for example, if the interest of money is supposed, as before, to be 4 per cent. and the term during which the new temporary annuities, (which are substituted in lieu of the former perpetual annuities,) are to continue, is sixty years, we shall find that the quantity of the interest of the national debt, or of the perpetual annuities, that would be redeemed by the application of a million of pounds every year during the said sixty years according to this third method, or by employing the said million in paying the additions which would have been made to the perpetual annuities which had been converted into temporary ones, would be exactly the same as the quantity of the interest of the national debt, or of the perpetual annuities, that would be redeemed in the same period of sixty years by employing the same sum of a million every year, together with the annual savings of interest produced by it, in either of the two former methods; which sum we have seen above to be £9,520,000 *per annum*. This may be shewn in the manner following.

When the interest of money is 4 per cent. or a perpetual annuity of four pounds a year is worth a hundred pounds, and consequently perpetual annuity of one pound a year is worth twenty-five pounds, temporary annuity of one pound a year, that is to continue only or sixty years, will be worth only £22.6234; as appears from Mr. Mart's fourth table of compound interest, page 80. Therefore a temporary annuity for 60 years that shall be worth 25 pounds, or shall be equal in value to a perpetual annuity of one pound a year, must be greater than a like temporary annuity of one pound a year in the proportion of 25 to £22.6234, and therefore will be = $\text{£}1 \times \frac{25}{22.6234}$, or £1.1050,

something less than 1*l.* 2*s.* 1*d.* $\frac{1}{2}$ a year. And consequently, if the government were to convert some of the perpetual annuities, now due to the publick creditors, into temporary annuities for 60 years, (with the consent of the proprietors of them,) and were to allow to each proprietor the sum of 1*l.* 2*s.* 1*d.* $\frac{1}{2}$ *per annum* during the said term of 60 years, instead of every sum of 1*l.* *per annum* that had formerly been due him for ever, such a bargain ought to be considered as fair and equal both sides.

Now, since for every perpetual annuity of one pound a year that should thus changed into a temporary annuity for 60 years, it would be necessary

cessary to allow the proprietor of it an annuity of 1*l.* 2*s.* 1*d.*, or £1.10*s.*0, a year during the said term of 60 years: it follows that for every million of pounds *per annum* of the said perpetual annuities that should be thus changed into temporary annuities for a term of 60 years, it would be necessary to allow to the proprietors of the said million of perpetual annuities a million times £1.10*s.*0 *per annum*, or £1,105,000, or a million and a hundred and five thousand pounds *per annum*, to be continued during the said term of 60 years; or, in other words, an annual additional sum of £105,000 would be required to enable the Government to convert £1,000,000 *per annum* of the perpetual annuities into temporary annuities for 60 years. Therefore an annual additional sum of £1,000,000 would be sufficient to enable the Government to convert into annuities for 60 years a quantity of the perpetual annuities that is as much greater than £1,000,000 a year as £1,000,000 is greater than £105,000; which greater quantity is $(£1,000,000 \times \frac{1,000,000}{105,000})$, or $£1000,000 \times \frac{1000}{105}$ or £1,000,000 $\times$ 9.52. or) £9,520,000. Therefore the quantity of the interest of the national debt, or of the perpetual annuities, that would be redeemed to the nation in the course of 60 years by employing the sum of a million of pounds *per annum* during that time in this third manner would be £9,520,000 *per annum*. QED.

In one respect this third method of applying a given sum of money to the diminution of the national debt seems to be preferable to either of the two former methods of applying it to the same purpose.

CCLIV. It appears therefore that these three different methods of applying a given annual sum of money to the diminution of the national debt, or to the redemption of the publick revenue that is mortgaged for the payment of the interest of it, are equally efficacious, if they are pursued with equal steadiness. But in this last respect, I mean the probability of their being pursued with steadiness for a considerable length of time they do not seem to be quite equal; but the third method seems rather to have the advantage of the former two. For, if either the first method of applying a given annual sum of money to this purpose, to wit, "that of employing it every year in repaying, at their *par*, or nominal value, some of those stocks, or branches of the national debt, which shall have risen at the publick market either to, or above, their said *par*, or nominal value; or the second method of applying it to the same purpose, to wit, "that of buying up at the market price, or at a price very little exceeding the market price, some of the publick stocks that were under their *par*, nominal value, with the consent of the respective owners of them;" I apprehend that, whenever any particular emergence should arise that required a greater expenditure of publick money than ordinary, the ministers of state might be tempted to propose to the Parliament, and to

Parliament to adopt the proposal, that the operation of diminishing the national debt should be suspended for a year, and the said annual sum that had been allotted to this purpose, should be applied to the discharge of the extra-ordinary expence that was made necessary by the supposed emergency: because by so doing they would avoid the odium and difficulty of raising the said extra-ordinary money by laying fresh taxes upon the people. The experience of the nation affords sufficient grounds for such an apprehension. For we have had but too many instances of such a disposition both in our ministers of state and in our parliaments ever since the year 1733, when Sir Robert Walpole first began to divert the sinking fund from its original destination of *sinking*, or diminishing, the national debt, and employed it in defraying the current services of the year: without which change in the application of it, the continual operation of that large and useful fund from the year 1733 to the present year, 1781, would have extinguished almost all our publick debts soon after they had been contracted, though they have now increased to such an enormous quantity. But, if the aforesaid annual surplus million were to be applied to the diminution of the national debt in the third method above-mentioned, that is, “by paying with it the additions that should have been made to some of the perpetual annuities now due to the publick creditors, upon a conversion of the said annuities (with the consent of the creditors to whom they were due,) into greater annuities for a limited number of years,” it could never afterwards be diverted from this destination, and applied to any other purpose, without an absolute breach of the national faith; which, I presume, we may consider as a kind of moral impossibility. For, as such a measure would not only be exorbitantly unjust and cruel towards the numerous unhappy individuals who would be the immediate sufferers by it, but likewise most ruinous to the general credit and commerce of the kingdom, and likely to produce the most dreadful scenes of internal misery and confusion; we may reasonably hope that our ministers of state, and our parliaments, will always look upon it with the utmost horror and detestation, and will consequently think themselves bound in duty to prevent it by the most vigorous and extensive exertions of their right of imposing new taxes that may become necessary for that purpose, however disagreeable and unpopular such exertions may appear. And therefore, I think, we may consider such an event as morally impossible. And, if we are right in so considering it, we may conclude that the aforesaid third method of applying any given annual sum to the diminution of the national debt will deserve to be preferred to both the former methods of applying it to the same purpose, because of the possibility, and, perhaps, even probability, that those former methods of applying it may occasionally be suspended and interrupted.

A fourth method of employing a given sum of money every year in the reduction of the national debt.

CCLV. A fourth method of applying a given annual sum of money to the diminution of the national debt, or the redemption of the interest of it, might be as follows. Some of the perpetual annuities now due to the publick creditors might, with the consent of the owners of them, be converted into life-annuities for the lives of their respective owners, or for the lives of other persons whom they should name, and who may be called their *nominees*: which life-annuities, it is evident, must be greater than the corresponding perpetual annuities, in the room of which they would have been substituted, in various proportions suited to the several ages of the persons for whose lives they would be to continue. And the said annual sum of money, that could be spared out of the publick revenue for the purpose of diminishing the national debt, might be employed in the payment of the additions necessary to be made to the said perpetual annuities upon their being so converted into life-annuities.

This fourth method of diminishing the national debt, if properly pursued, would be equally effectual for that purpose with either of the three preceding methods.

CCLVI. In this method of diminishing the national debt, it is evident, there will be continual savings of interest arising to the nation every year by the deaths of some of the life-annuitants. Now, if these savings are continually employed, for the same purpose of diminishing the national debt, in either the first or the second method above-mentioned, that is to say, either in paying off at *par* some part of those stocks which shall have risen to *par*, or above *par*, or in buying up at the market price, or at a price very little greater than the market price, (with the consent of the respective owners of it) some part of those stocks which shall continue under *par*; the quantity of annual interest that would be redeemed to the nation by applying a given sum of money every year in this fourth method during the life of the longest liver of all the said life-annuitants, would be the very same that would be redeemed by applying the same sum of money to the diminution of the national debt during the same length of time in either of the three preceding methods. Of which it may not be amiss to give an example: which may be done as follows.

An example of this third method of applying a given sum of money every year to the diminution of the national debt.

CCLVII. It has been shewn above that a million of pounds sterling *per annum*, applied constantly to the diminution of the national debt, or the redemption of the interest of it, for the space of 60 years, in either of

the three foregoing methods, will redeem an annual interest of 9,520,000 pounds. We will therefore now inquire what quantity of annual interest the same annual sum of a million of pounds sterling will redeem in the same course of time, if applied in this fourth method; making the same suppositions as before, to wit, that the stocks which carry an interest of 4 per cent. will rise to *par*, and that those which carry an interest of 3 per cent. will rise to £75 per cent. and using the table of the values of life-annuities when the interest of money is 4 per cent. which is given above in page 225, for the valuation of the life-annuities which we are now going to consider, and Monsieur de Parcieux's table of the probabilities of the duration of human life (upon which the said table in page 225 is founded,) for the purpose of ascertaining the numbers of the life-annuitants that will die off in every year of the whole term, and the consequent savings of interest that will accrue every year to the publick by their deaths.

CCLVIII. Now, that this calculation (which will unavoidably run into considerable length,) may not be too tedious and intricate, we will suppose that all the persons for whose lives the new annuities, that are to be substituted instead of the former perpetual annuities, shall be granted, are of the same age. And further,—as it is supposed that the whole operation of this fourth method is to continue during the space of 60 years;—and in Monsieur de Parcieux's table of the probabilities of the duration of human life, only one person, out of the whole original number of 1000 persons living at the age of 3 years, is supposed to live to the age of 94 years, and that one person is supposed to die before he attains the age of 95 years;—we will suppose the several persons, for whose lives the said new annuities shall be granted, to be all of the age of 35 years: for then it may be supposed that the longest liver of them will live almost to the very end of the said term of 60 years during which this operation is to continue.

CCLIX. Now it appears above by Table xvi, page 225, that, when the interest of money is 4 per cent. or the value of a perpetual annuity of four pounds a year is a hundred pounds, and consequently the value of a perpetual annuity of one pound a year is twenty-five pounds, the value of an annuity of one pound a year for the life of a person of the age of 35 years is £16.084,014. Therefore an annuity for the life of a person of the age of 35 years that shall be worth 25 pounds, or shall be equal in value to a perpetual annuity of one pound a year, will be greater than an annuity of one pound a year for the life of the same person in the proportion of £25 to £16.084,014, and therefore will be equal

that would accrue to the publick by the deaths of some of the life-annuitants in the course of the said term.

equal help of the annual savings

equal to £1 × $\frac{25,000,000}{16.084,014}$, or £1.554,338. This therefore is the life-

annuity which each of the persons aged 35 years for whose lives we suppose the said life-annuities to be granted, ought to receive in lieu of the perpetual annuity of one pound a year in the room of which it is substituted. Therefore for every million of pounds *per annum* of the perpetual annuities, now due to the publick creditors, which should be thus converted into life-annuities for the lives of persons of the age of 35 years, it would be necessary that Government should establish life-annuities to the amount of a million times £1.554,338 *per annum*, or to the amount of £1,554,338, *per annum*; which would require an additional annual interest of £554,338: so that an additional annual interest of 554,338 pounds would be sufficient to enable the Government to convert a million of pounds *per annum* of the perpetual annuities into life-annuities for the lives of persons of the age of 35 years. Therefore an additional annual interest of £1000,000 will be sufficient to enable the Government to convert into life-annuities for the lives of persons of the age of 35 years a quantity of the perpetual annuities that is greater than £1000,000 *per annum* in the proportion of £1000,000 to £554,338, that is, a quantity of the said perpetual annuities that is equal to (£1000,000 × $\frac{554,338}{1000,000}$, or) £1,803,953 *per annum*. Therefore at the end of the said term of 60 years, when all the said life-annuitants (who were of the age of 35 years at the beginning of the said term,) will be dead, the said sum of £1,803,953 *per annum* of the perpetual annuities will be entirely redeemed to the nation by means of this conversion of them into life-annuities.

Of the savings
that would ac-
cure every year
to the publick
by the deaths
of some of the
life-annui-
tants;

CCLX. This would be the quantity of the perpetual annuities that would be redeemed by this operation in the course of 60 years, if all the aforesaid life-annuitants had lived to within a day of the end of the whole 60 years, or if, when they had died off in the different years of the said term according to the course of nature, the savings which would have accrued every year to the publick by their deaths had not been applied to the same purpose of diminishing the national debt, but had been spent upon some other service. But it was supposed above that these savings were to be employed continually to the same purpose of diminishing the national debt in either the first or the second of the methods above-described. We must now therefore inquire what these savings would amount to in every year of this whole term, and what quantity of the perpetual annuities they would respectively be sufficient to redeem in the course of the said term, if they were applied to that use in either the first or the second method above-mentioned. These inquiries may be made in the manner following.

CCLXI. The number of persons represented in Monsieur de Parcieux's table as living at the age of 35 years is 694. The money to be paid annually by the Government in life-annuities, so long as the life-annuitants shall be all alive, in lieu of the £1,803,953 *per annum* of perpetual annuities which are to be redeemed, is the said sum of £1,803,953 *per annum* together with the annual million of pounds which is destined to this purpose, and consequently is £2,803,953 *per annum*. Therefore, if we suppose these life-annuities to be distributed equally amongst 694 persons, all of the age of 35 years, each of the said persons will be possessed

of a life-annuity of $\left(\frac{2,803,953}{694}, \text{ or } \right) £404.27 \text{ per annum}$. And con-

sequently, if we suppose them to be equally distributed amongst 694 persons, all of the same age of 35 years, each of the said persons will be possessed of a life-annuity of £404.027 *per annum*: and, if we suppose them to be equally distributed amongst 6940 persons, all of the same age of 35 years, each of the said persons will be possessed of a life-annuity of £40.4027, or 40l. 8s. $\frac{1}{4}d.$ *per annum*. Let us make this last supposition.

CCLXII. Then, since it appears by Monsieur de Parcieux's table of the probabilities of the duration of human life, that out of 694 persons of the age of 35 years, all living at the same time, only 686 persons will be living at the end of a year, or at the age of 36 years, it follows that out of the said 6940 persons above-mentioned, of the age of 35 years, all living at the same time, for whose lives these annuities of £40.4027, or 40l. 8s. $\frac{1}{4}d.$ each, are supposed to be granted, only 6860 will be living at the end of a year, or at the age of 36 years. And consequently the annuities that would have been payable to the other 800 life-annuitants at the end of the year, if the said annuitants had been then living, will be saved to the publick, and will be ready in the publick treasury to be employed, in any manner that may be thought fit, towards the diminution of the national debt. The amount of these annuities is 800 times £40.4027, or £32,322.1600. Now, if this sum £32,322.1000 is immediately laid out either in paying off some of the capital of the 4 per cent. annuities at *par*, according to the first method above-described, or in buying up some of the capital of the 3 per cent. annuities at the price of £75 per cent. according to the second method above-described; and the interest of the capital thereby extinguished, or the portion of the perpetual annuities that will be redeemed at the end of the next, or second, year, by thus employing the said sum of £32,322.1600, be likewise employed in the same manner; and the interest of the capital extinguished by this second operation, or the portion of the perpetual annuities that will thereby be redeemed

Of the manner made at the end of the first year by the deaths of some of the life-annuitants. in which the said saving is to be employed during the remaining 59 years of the said term of 60 years.

Of the quantity of the capital of the national debt that will have been extinguished by means of the said saving at the end of the said term of 60 years.

at the end of the third year, be likewise employed in the same manner, and all the following portions of the perpetual annuities that will be thus redeemed by means of this first sum of £32,322.1600, that is saved to the nation at the end of the first year by, the death of the aforesaid 800 life-annuitants, be likewise employed in the same manner during the whole remainder of the said term of 60 years;—I say, if these successive sums of money be so employed without interruption during the remainder of the said term of 60 years, the quantity of the capital of the national debt which will be thereby extinguished at the end of the said term of 60 years, or at the end of 59 years from the time when this first saving of £32,322.1600 will have accrued, will be equal in value to the sum, to which the said first saving of £32,322.1600 would have increased in the same period of 59 years, if it had been improved all the time at compound interest at the rate of 4 per cent. and therefore it will, (according to Mr. Smart's first table of compound interest, page 56,) be equal in value to the sum of £32,322.1600 $\times$ 10.115,026, or £326,939.4887; the interest of which at 4 per cent. is $\left(\frac{4}{100} \times £326,939.4887, \text{ or } \right) £13077.5795$ *per annum*. Therefore the quantity of the interest of the national debt, or of the perpetual annuities now due to the publick creditors, which would be redeemed at the end of the whole term of 60 years by means of this first saving of £32,322.1600, which will accrue to the publick at the end of the first year of the said term by the deaths of the aforesaid 800 life-annuitants, would be £13,077.5795 *per annum*.

Of the saving made at the end of the second year by the deaths of life-annuitants.

CCLXIII. In the course of the second year of this term the 68600 life-annuitants of the age of 36 years, who are living at the end of the first year, will be reduced to 67800. Therefore 800 more of the said life-annuitants will have died in the course of this second year. And consequently at the end of this second year there will be a saving to the Government of twice 800 payments of £40.4027 each, which would have then become due to the 1600 life-annuitants who will have died in the two preceding years, if they had lived till that time. There will therefore be in the hands of the Government at the end of this second year the sum of 1600 $\times$ £40.4027, or £64,644.3200, to be employed during the remainder of the said term of 60 years, that is, during 58 years, in diminishing the national debt; which will be sufficient to extinguish as much of the capital of the said debt as is equal in value to the sum to which this second saving of £64,644.3200 will increase in the course of 58 years, if constantly improved during that time at compound interest at the rate of 4 per cent. Now the amount of £64,644.3200 at the end of 58 years, when improved during that time at compound interest at 4 per cent, appear

appears by Mr. Smart's first table of compound interest, page 56, to be $\pounds 64,644.3200 \times 9.725,986$, or $\pounds 628,729.7513$; the interest of which at 4 per cent. is $\left(\frac{4}{100} \times \pounds 628,729.7513\right)$, or $\pounds 25,149.1900$ per annum.

Therefore the quantity of the capital of the national debt $\pounds 25,149.1900$. That would be extinguished at the end of the said term of 60 years by means of this second saving of $\pounds 64,644.3200$ (which would have accrued to the publick at the end of the second year of the said term by the deaths of the said 1600 life-annuitants who would have died in the two preceeding years,) will be equal in value to $\pounds 628,729.7513$; and the quantity of the interest of the said debt, or of the perpetual annuities now due to the publick creditors, that would then be redeemed to the nation by means of this second saving, would be the sum of $\pounds 25,149.1900$ per annum.

Of the quantity of the capital of the national debt that will be made at the ends of the third, fourth, and fifth years, by the means of this second saving, at the end of the said term of 60 years.

CCLXIV. In the same manner we must proceed to investigate the savings that will accrue to the publick at the ends of the third, fourth, fifth, sixth, and every following year of the whole term of 60 years, and the quantities of the capital of the national debt that will be extinguished by each of the said savings at the end of the said term of 60 years, if they are constantly employed for that purpose in either the first or the second method above-mentioned, during the several remaining years of the said term after the times when they will have respectively accrued. And, in order to these investigations, it will be necessary in the first place to ascertain, by the help of Monsieur de Parcieux's table of the probabilities of the duration of human life, the numbers of life-annuitants who will be alive at the ends of the several remaining years of this term, except the last year of it, (at the end of which they will be all dead,) and hence to determine the numbers of those who will have died in each of the said years, and from whose deaths the several savings to the publick will arise.

Of the savings that will be made at the ends of the third, fourth, and fifth years, by the means of this second saving, at the end of the said term of 60 years.

CCLXV. Now of the aforesaid original number of 69400 life-annuitants, of the age of 35 years, who were supposed to be living at the beginning of the said term of 60 years, and for whose lives the said annuities, of $\pounds 4027$ to each, were supposed to be granted, it appears from Monsieur Parcieux's table of probabilities that the following numbers will be living at the subsequent ages of 38 years, 39 years, 40 years, 41 years, &c. to the age of 94 years, inclusive, or at the ends of the third, fourth, fifth, sixth, and every following year of the said term of 60 years; viz,

R r

67100,

67100,	59900,	48900,	34700,	15400,	2200,
66400,	59000,	47600,	32900,	13600,	1600,
65700,	58100,	46300,	31000,	11800,	1100,
65000,	57100,	45000,	29100,	10100,	700,
64300,	56000,	43700,	27100,	8500,	400,
63600,	54900,	42300,	25100,	7100,	200,
62900,	53800,	40900,	23100,	5900,	100,
62200,	52600,	39500,	21100,	4800,	
61500,	51400,	38000,	19200,	3800,	
60700,	50200,	36400,	17300,	2900,	

The numbers of life-annuitants who will have died in each of the said years, and in the last year.

Therefore the numbers who will have died in each of those years, and in the last year of the said period of 60 years, or from the age of 95 years to the age of 96 years, will be as follows; to wit,

700,	800,	1300,	1700,	1900,	700,
700,	900,	1300,	1800,	1800,	600,
700,	900,	1300,	1900,	1800,	500,
700,	1000,	1300,	1900,	1700,	400,
700,	1100,	1300,	2000,	1600,	300,
700,	1100,	1400,	2000,	1400,	200,
700,	1100,	1400,	2000,	1200,	100,
700,	1200,	1400,	2000,	1100,	100,
700,	1200,	1500,	1900,	1000,	
800,	1200,	1600,	1900,	900,	

CCLXVI. Consequently the savings of interest that will be made by the publick at the ends of those years in consequence of the death of life-annuitants that will have happened in the preceding years of the first term, will be as follows; to wit,

At the end of the third year

$$\frac{1600 + 700}{2} \times £40.4027,$$

$$\text{or } 2300 \times £40.4027 = £92,926.2100.$$

At the end of the fourth year

$$\frac{2300 + 700}{2} \times £40.4027,$$

$$\text{or } 3000 \times £40.4027 = £121,208.1000.$$

At

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At the end of the fifth year,

$$\frac{3000 + 700}{3700} \times £40.4027, \\ \text{or } 3700 \times £40.4027, = £149,489.9900.$$

At the end of the sixth year,

$$\frac{3700 + 700}{4400} \times £40.4027, \\ \text{or } 4400 \times £40.4027, = £177,771.8800.$$

At the end of the seventh year,

$$\frac{4400 + 700}{5100} \times £40.4027, \\ \text{or } 5100 \times £40.4027, = £206,053.7700.$$

At the end of the eighth year,

$$\frac{5100 + 700}{5800} \times £40.4027, \\ \text{or } 5800 \times £40.4027, = £234,335.6600.$$

At the end of the ninth year,

$$\frac{5800 + 700}{6500} \times £40.4027, \\ \text{or } 6500 \times £40.4027, = £262,617.5500.$$

At the end of the tenth year,

$$\frac{6500 + 700}{7200} \times £40.4027, \\ \text{or } 7200 \times £40.4027, = £290,899.4400.$$

At the end of the eleventh year,

$$\frac{7200 + 700}{7900} \times £40.4027, \\ \text{or } 7900 \times £40.4027, = £319,181.3300.$$

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At the end of the twelfth year,

$$\frac{7900 + 800}{} \times £40.4027,$$

$$\text{or } 8700 \times £40.4027, = £351,503.4900.$$

At the end of the thirteenth year,

$$\frac{8700 + 800}{} \times £40.4027,$$

$$\text{or } 9500 \times £40.4027, = £383,825.6500.$$

At the end of the fourteenth year,

$$\frac{9500 + 900}{} \times £40.4027,$$

$$\text{or } 10,400 \times £40.4027, = £420,188.0800.$$

At the end of the fifteenth year,

$$\frac{10,400 + 900}{} \times £40.4027,$$

$$\text{or } 11,300 \times £40.4027, = £456,550.5100.$$

At the end of the sixteenth year,

$$\frac{11,300 + 1000}{} \times £40.4027,$$

$$\text{or } 12,300 \times £40.4027, = £496,953.2100.$$

At the end of the seventeenth year,

$$\frac{12,300 + 1100}{} \times £40.4027,$$

$$\text{or } 13,400 \times £40.4027, = £541,396.1800.$$

At the end of the eighteenth year,

$$\frac{13,400 + 1100}{} \times £40.4027,$$

$$\text{or } 14,500 \times £40.4027, = £585,839.1500.$$

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At the end of the nineteenth year,

$$\frac{14,500 + 1100}{\quad} \times £40.4027,$$

$$\text{or } 15,600 \times £40.4027, = £630,282,1200.$$

At the end of the twentieth year,

$$\frac{15,600 + 1200}{\quad} \times £40.4207,$$

$$\text{or } 16,800 \times £40.4027, = £678,765.3600.$$

At the end of the twenty-first year,

$$\frac{16,800 + 1200}{\quad} \times £40.4027,$$

$$\text{or } 18,000 \times £40.4027, = £727,248.6000.$$

At the end of the twenty-second year,

$$\frac{18,000 + 1200}{\quad} \times £40.4027,$$

$$\text{or } 19,200 \times £40.4027, = £775,731.8400.$$

At the end of the twenty-third year,

$$\frac{19,200 + 1300}{\quad} \times £40.4027,$$

$$\text{or } 20,500 \times £40.4027, = £828,255.3500.$$

At the end of the twenty-fourth year,

$$\frac{20,500 + 1300}{\quad} \times £40.4027,$$

$$\text{or } 21,800 \times £40.4027, = £880,778.8600.$$

At the end of the twenty-fifth year,

$$\frac{21,800 + 1300}{\quad} \times £40.4027,$$

$$\text{or } 23,100 \times £40.4027, = £933,302.3700.$$

At

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At the end of the twenty-sixth year,

$$\begin{array}{l} \underline{23,100 + 1300} \times £40.4027, \\ \text{or } 24,400 \times £40.4027, = £985,825.8800. \end{array}$$

At the end of the twenty-seventh year,

$$\begin{array}{l} \underline{24,400 + 1300} \times £40.4027, \\ \text{or } 25,700 \times £40.4027, = £1,038,349.3900. \end{array}$$

At the end of the twenty-eighth year,

$$\begin{array}{l} \underline{25,700 + 1400} \times £40.4027, \\ \text{or } 27,100 \times £40.4027, = £1,094,913.1700. \end{array}$$

At the end of the twenty-ninth year,

$$\begin{array}{l} \underline{27,100 + 1400} \times £40.4027, \\ \text{or } 28,500 \times £40.4027, = £1,151,476.9500. \end{array}$$

At the end of the thirtieth year,

$$\begin{array}{l} \underline{28,500 + 1400} \times £40.4027, \\ \text{or } 29,900 \times £40.4027, = £1,208,040.7300. \end{array}$$

At the end of the thirty-first year,

$$\begin{array}{l} \underline{29,900 + 1500} \times £40.4027, \\ \text{or } 31,400 \times £40.4027, = £1,268,644.7800. \end{array}$$

At the end of the thirty-second year,

$$\begin{array}{l} \underline{31,400 + 1600} \times £40.4027, \\ \text{or } 33,000 \times £40.4027, = £1,333,289.1000. \end{array}$$

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At the end of the thirty-third year,

$$\frac{33,000 + 1700}{\text{or } 34,700} \times \text{£}40.4027, = \text{£}1,401,973.6900.$$

At the end of the thirty-fourth year,

$$\frac{34,700 + 1800}{\text{or } 36,500} \times \text{£}40.4027, = \text{£}1,474,698.5500.$$

At the end of the thirty-fifth year,

$$\frac{36,500 + 1900}{\text{or } 38,400} \times \text{£}40.4027, = \text{£}1,551,463.6800.$$

At the end of the thirty-sixth year,

$$\frac{38,400 + 1900}{\text{or } 40,300} \times \text{£}40.4027, = \text{£}1,628,228.8100.$$

At the end of the thirty-seventh year,

$$\frac{40,300 + 2000}{\text{or } 42,300} \times \text{£}40.4027, = \text{£}1,709,034.2100.$$

At the end of the thirty-eighth year,

$$\frac{42,300 + 2000}{\text{or } 44,300} \times \text{£}40.4027, = \text{£}1,789,839.6100.$$

At the end of the thirty-ninth year,

$$\frac{44,300 + 2000}{\text{or } 46,300} \times \text{£}40.4027, = \text{£}1,870,645.0100.$$

At

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At the end of the fortieth year,

$$\frac{46,300 + 2000}{\text{}} \times £40.4027,$$

$$\text{or } 48,300 \times £40.4027, = £1,951,450.4100,$$

At the end of the forty-first year,

$$\frac{48,300 + 1900}{\text{}} \times £40.4027,$$

$$\text{or } 50,200 \times £40.4027, = £2,028,215.5400,$$

At the end of the forty-second year,

$$\frac{50,200 + 1900}{\text{}} \times £40.4027,$$

$$\text{or } 52,100 \times £40.4027, = £2,104,980.6700,$$

At the end of the forty-third year,

$$\frac{52,100 + 1900}{\text{}} \times £40.4027,$$

$$\text{or } 54,000 \times £40.4027, = £2,181,745.8000,$$

At the end of the forty-fourth year,

$$\frac{54,000 + 1800}{\text{}} \times £40.4027,$$

$$\text{or } 55,800 \times £40.4027, = £2,254,470.6600,$$

At the end of the forty-fifth year,

$$\frac{55,800 + 1800}{\text{}} \times £40.4027,$$

$$\text{or } 57,600 \times £40.4027, = £2,327,195.5200,$$

At the end of the forty-sixth year,

$$\frac{57,600 + 1700}{\text{}} \times £40.4027,$$

$$\text{or } 59,300 \times £40.4027, = £2,395,880.1100,$$

At the end of the forty-seventh year,

$$\frac{59,300 + 1600}{\quad} \times £40.4027,$$

$$\text{or } 60,900 \times £40.4027, = £2,460,524.4300.$$

At the end of the forty-eighth year,

$$\frac{60,900 + 1400}{\quad} \times £40.4027,$$

$$\text{or } 62,300 \times £40.4027, = £2,517,088.2100.$$

At the end of the forty-ninth year,

$$\frac{62,300 + 1200}{\quad} \times £40.4027,$$

$$\text{or } 63,500 \times £40.4027, = £2,565,571.4500.$$

At the end of the fiftieth year,

$$\frac{63,500 + 1100}{\quad} \times £40.4027,$$

$$\text{or } 64,600 \times £40.4027, = £2,610,014.4200.$$

At the end of the fifty-first year,

$$\frac{64,600 + 1000}{\quad} \times £40.4027,$$

$$\text{or } 65,600 \times £40.4027, = £2,650,417.1200.$$

At the end of the fifty-second year,

$$\frac{65,600 + 900}{\quad} \times £40.4027,$$

$$\text{or } 66,500 \times £40.4027, = £2,686,779.5500.$$

At the end of the fifty-third year,

$$\frac{66,500 + 700}{\quad} \times £40.4027,$$

$$\text{or } 67,200 \times £40.4027, = £2,715,061.4400.$$

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At the end of the fifty-fourth year,

$$\overline{67,200} + \overline{600} \times £40.4027, \\ \text{or } 67,800 \times £40.4027, = £2,739,303.0600.$$

At the end of the fifty-fifth year,

$$\overline{67,800} + \overline{500} \times £40.4027, \\ \text{or } 68,300 \times £40.4027, = £2,759,504.4100.$$

At the end of the fifty-sixth year,

$$\overline{68,300} + \overline{400} \times £40.4027, \\ \text{or } 68,700 \times £40.4027, = £2,775,665.4900.$$

At the end of the fifty-seventh year,

$$\overline{68,700} + \overline{300} \times £40.4027, \\ \text{or } 69,000 \times £40.4027, = £2,787,786.3000.$$

At the end of the fifty-eighth year,

$$\overline{69,000} + \overline{200} \times £40.4027, \\ \text{or } 69,200 \times £40.4027, = £2,796,866.8400.$$

At the end of the fifty-ninth year,

$$\overline{69,200} + \overline{100} \times £40.4027, \\ \text{or } 69,300 \times £40.4027, = £2,799,907.1100.$$

And at the end of the sixtieth and last year,

$$\overline{69,300} + \overline{100} \times £40.4027, \\ \text{or } 69,400 \times £40.4027, = £2,803,947.3800.$$

Note. This last saving £2,803,947.3800, ought to be equal to the whole sum allotted to the payment of the life-annuities; since at the time of its accruing to the publick all the life-annuitants are supposed to be dead. It ought therefore to be equal to $\frac{£1000000}{£1,803,953}$, or £2,803,953, which was the sum allotted to that purpose. And so we find it very nearly is, since it differs from it by less than £6, which, upon so large a sum as £2,803,953, is not worth attending to. And it would have been more nearly equal to £2,803,953, if in dividing £2,803,953 by 694 (in Art. CCLXI) we had carried the quotient to more than six places of figures. For, if we had carried it only to one figure more, we should have found it to be 4040.278. And, in consequence of this increase of this quotient, the annuity which would have belonged to each of the foresaid 69400 life-annuitants for his life, would have been £40.40278, instead of £40.40273; and $69400 \times £40.40278 = £2,803,952.93200$, which differs from £2,803,953 by less than $\frac{1}{10}$ of a pound, or than two shillings. But the numbers £40.4027 and £2,803,947.3800 are near enough to the truth to answer the purpose of this computation.

CCLXVII. Now all these savings are to be employed, as fast as they rise, for the purpose of diminishing the national debt, in either the first or the second method above-mentioned. By this means the last saving, £2,803,947.3800, will either extinguish £2,803,947.3800 of the capital of the 4 per cent. annuities, (which are supposed to sell at their *par*, or nominal value,) or, if employed in the second method above-described, it will extinguish a proportionally greater capital of the 3 per cent. annuities, which are supposed to sell at the price of £75 per cent. and in either case will redeem to the publick the interest of 4 per cent. upon it, or the

annual sum of $\left(\frac{4}{100} \times £2,803,947.3800, \text{ or } \frac{£11,215,789.5200}{100}, \text{ or } \right)$

£112,157.8952. But all the other savings will have time to perform more than one operation of this kind towards the diminution of the national debt, to wit, a new operation at the end of every year of the term that remaining after the time when it accrued: and, in consequence of these repeated operations, the quantity of the national debt which each of these savings will have extinguished at the end of the said term of 60 years, will be equal in value to the amount of such saving at the end of the said term, improved in the mean time at compound interest at the rate of four per cent.

Of the effect of the said savings in diminishing the national debt.

We must therefore now compute the amounts of these savings at the end of the said term, if so improved at compound interest in the mean time, or during the remaining years of the said term after the times at which they will have respectively accrued, that is, during 57 years, 56 years, 55 years, 54 years, &c. to the last year of the said term. Now these amounts will be as follows.

A computati-
on of the a-
mounts of all
the foregoing
savings, ex-
cept the last,
at the end of
the said term
of 60 years, if
improved in
the mean time
at compound
interest at the
rate of 4 per
cent.

CCLXVIII. The amount of £92,926.21, improved at compound interest at 4 per cent. during 57 years, is = £92,926.21 x 9.351,910 = £869,037.5525.

The amount of £121,208.1000, improved in the same manner during 56 years, is = £121,208.1000 x 8.992,221 = £1,089,930.0221.

The amount of £149,489.9900, improved in the same manner during 55 years, is = £149,489.9900 x 8.646,367 = £1,292,545.3163.

And the amounts of all the other savings, improved in the same manner during the remaining years of the said term of 60 years after they shall have respectively accrued to the publick, will be as follows.

£177,771.8800	x	8.313,814	=	£1,477,962.3447.
£206,053.7700	x	7.994,052	=	£1,647,204.5521.
£234,335.6600	x	7.686,588	=	£1,801,241.6721.
£262,617.5500	x	7.390,950	=	£1,940,993.1811.
£290,899.4400	x	7.106,683	=	£2,067,330.1049.
£319,181.3300	x	6.833,349	=	£2,181,077.4221.
£351,503.4900	x	6.570,528	=	£2,309,563.5231.
£383,825.6500	x	6.317,815	=	£2,424,939.4489.
£420,188.0800	x	6.074,822	=	£2,552,567.7925.
£456,550.5100	x	5.841,175	=	£2,666,791.4252.
£496,953.2100	x	5.616,515	=	£2,791,145.1576.
£541,396.1800	x	5.400,495	=	£2,923,807.3631.
£585,839.1500	x	5.192,783	=	£3,042,135.5788.
£630,282.1200	x	4.993,061	=	£3,147,037.0723.
£678,765.3600	x	4.801,020	=	£3,258,766.0686.
£727,248.6000	x	4.616,366	=	£3,357,245.7105.

£775,731.849

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£775,731.8400	x	4.438,813	=	£3,443,328.5759.
£828,255.3500	x	4.268,089	=	£3,535,067.5285.
£880,778.8600	x	4.103,932	=	£3,614,656.5484.
£933,302.3700	x	3.946,089	=	£3,682,894.2159.
£985,825.8800	x	3.794,316	=	£3,740,534.9097.
£1,038,349.3900	x	3.648,381	=	£3,788,294.1858.
£1,094,913.1700	x	3.508,058	=	£3,841,018.9053.
£1,151,476.9500	x	3.373,133	=	£3,884,084.8987.
£1,208,040.7300	x	3.243,397	=	£3,918,155.6795.
£1,268,644.7800	x	3.118,651	=	£3,956,460.3117.
£1,333,289.1000	x	2.998,703	=	£3,998,138.0240.
£1,401,973.6900	x	2.883,368	=	£4,042,406.0745.
£1,474,698.5500	x	2.772,469	=	£4,088,556.0142.
£1,551,463.6800	x	2.665,836	=	£4,135,947.7308.
£1,628,228.8100	x	2.563,394	=	£4,173,645.4215.
£1,709,034.2100	x	2.464,715	=	£4,212,282.2529.
£1,789,839.6100	x	2.369,918	=	£4,241,773.0758.
£1,870,645.0100	x	2.278,768	=	£4,262,765.9881.
£1,951,450.4100	x	2.191,123	=	£4,275,867.8767.
£2,028,215.5400	x	2.106,849	=	£4,273,143.8822.
£2,104,980.6700	x	2.025,816	=	£4,264,203.5209.
£2,181,745.8000	x	1.947,900	=	£4,249,822.6438.
£2,254,470.6600	x	1.872,981	=	£4,222,580.7112.
£2,327,195.5200	x	1.800,943	=	£4,191,146.4813.
£2,395,880.1100	x	1.731,676	=	£4,148,888.0853.
£2,460,524.4300	x	1.665,073	=	£4,096,952.7942.
£2,517,088.2100	x	1.601,032	=	£4,029,938.7710.
£2,565,571.4500	x	1.539,454	=	£3,949,579.2309.
£2,610,014.4200	x	1.480,244	=	£3,863,458.1851.
£2,650,417.1200	x	1.423,311	=	£3,772,367.8414.

£2,686,779.5500.

31.84

£2,686,779.5500	x	1.368,569	=	£3,677,943.2019
£2,715,061.4400	x	1.315,931	=	£3,572,833.5168
£2,739,303.0600	x	1.265,319	=	£3,466,092.2085
£2,759,504.4100	x	1.216,652	=	£3,357,356.5594
£2,775,665.4900	x	1.169,858	=	£3,247,134.4788
£2,787,786.3000	x	1.124,864	=	£3,135,880.4485
£2,795,866.8400	x	1.081,600	=	£3,024,009.5741
£2,799,907.1100	x	1.040,000	=	£2,911,993.3944

The addition of the amounts of all the yearly savings at the end of the said term of 60 years, into one sum.

CCLXIX. Having thus found the amounts of the several yearly savings at the end of the said term of 60 years, we must now add these several amounts together; which may be done as follows.

The amounts of the savings made at the ends of the first ten years, the second ten years, and the third ten years, of the said term, are as follows.

£	326,939.4887	£	2,181,077.4221	£	3,357,245.7105
	628,729.7513		2,309,563.5231		3,443,328.5759
	869,037.5525		2,424,939.4489		3,535,067.5285
	1,089,930.0221		2,552,567.7925		3,614,656.5484
	1,292,545.1668		2,666,791.4252		3,682,894.2159
	1,477,962.3447		2,791,145.1576		3,740,534.9097
	1,647,204.5521		2,923,807.3631		3,788,294.1858
	1,801,241.6721		3,042,135.5788		3,841,018.9053
	1,940,993.1811		3,147,037.0723		3,884,084.8987
	2,067,330.1049		3,258,766.0686		3,918,155.6795
	£3,141,913.8363		27,297,830.8522		36,805,281.1582

And

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And the amounts of the savings made at the ends of the fourth ten years, the fifth ten years, and the sixth ten years, are as follows.

£ 3,956,460.3117	£ 4,273,143.8822	£ 3,772,367.8414
3,998,138.0240	4,264,203.5209	3,677,043.2019
4,042,406.0745	4,249,822.6438	3,572,833.5158
4,088,556.0142	4,222,580.7112	3,466,092.2085
4,135,947.7308	4,191,146.4813	3,357,356.5594
4,173,645.4215	4,148,888.0853	3,247,134.4788
4,212,282.2529	4,096,952.7942	3,135,880.4485
4,241,773.0758	4,029,938.7710	3,024,009.5741
4,262,765.9881	3,949,579.2309	2,911,903.3944
4,275,867.8767	3,863,458.1851	2,803,947.3800
41,387,842.7702	41,289,714.3059	32,968,568.6028

And the sum total of these six several sums, or the sum total of the amounts that will arise from all the savings made at the ends of the 60 years of the said term, at the end of the said term, is 192,891,151.5256.

£ 13,141,913.8363
27,297,830.8522
36,805,281.1582
41,387,842.7702
41,289,714.3059
32,968,568.6028
192,891,151.5256

CCLXX. Therefore the quantity of the capital of the national debt The quantity it would have been extinguished by means of all these savings at the end of the said term of 60 years, will be equal in value to the said sum of £192,891,151.5256; of which the annual interest at 4 per cent. is

$$\frac{£ 192,891,151.5256}{100} \times 4 = £ 7,715,646.0610$$

Therefore yearly savings.

The quantity Therefore the quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of the said interest, that would be redeemed to the publick at the end of the said term of 60 years by means of all these savings, (so that at the end of the next, or sixth, year it would be free to be disposed of by the Parliament in any other way,) is $\pounds 7,715,646.0610$ *per annum*. QEI.

The whole CCLXXI. This sum of $\pounds 7,715,646.0610$ *per annum* must therefore be added to $\pounds 1,803,953$ *per annum*, which was the quantity of the perpetual annuities that was supposed to be converted into life-annuities for the lives of 69400 persons, all of the age of 35 years, and which will therefore be now wholly redeemed to the publick by the deaths of all the said life-annuitants: and the sum of both, or the sum of $\pounds 9,519,599.0610$, or (neglecting the fraction .0610,) $\pounds 9,519,599$ *per annum* will be the whole quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the said term of 60 years (so as to be free at the end of the next, or sixth, year to be disposed of by the Parliament in any other way,) by the faithful application of a million of pounds sterling a year to that salutary purpose during that period according to the fourth method above-described. QEI.

Agreement CCLXXII. This sum of $\pounds 9,519,599$ *per annum*, which is the quantity of the interest of the national debt that would be redeemed to the publick in the course of 60 years by the application of a million of pounds sterling *per annum* in this fourth method, or in the way of life-annuities, and is the same with the quantity of the interest of the said debt that would have been redeemed to the publick in the same time by applying a million of pounds *per annum* to the same purpose in either of the three former methods. For that was shewn above, (in Art. cclxiv, cclxvii, cclviii,) to be $\pounds 9,520,000$ *per annum*; which differs from $\pounds 9,519,599$ *per annum*, by only $\pounds 401$ *per annum*, which is a mere trifle upon so large a sum. And, if we had computed the said former quantity more accurately, we should have found it to approach still nearer to $\pounds 9,519,599$. For it is not exactly equal to $\pounds 9,520,000$ *per annum*, or $238 \times \pounds 40,000$ *per annum*, (as it is there, for the sake of round numbers, supposed to be,) but more nearly to $237.9906 \times \pounds 40,000$ *per annum*, that is, to $\pounds 9,519,624.0000$, or $\pounds 9,519,624$ *per annum*; which differs from $\pounds 9,519,599$ *per annum* by only $\pounds 25$ *per annum*. They may therefore be considered as equal; agreeably to what is asserted in Art. cclvi.

CCLXXIII. We may therefore conclude that this fourth method of applying a million of pounds sterling, or any other given sum of money, every year, during a given number of years, to the purpose of diminishing the national debt, by converting some of the perpetual annuities, now due to the publick creditors, into life-annuities, would be just as efficacious in redeeming the interest of the said debt, and consequently just as beneficial to the nation, as either of the three former methods of applying the same sum of money to the same purpose during the same period of time; to prove which was the principal object of the foregoing computation. But this, it must be remembered, is true only upon a supposition that the savings of the interest of the national debt, that will accrue to the publick every year by the deaths of some of the life-annuitants, are immediately employed in discharging a part of the said debt in either the first or second method above-described, or in some other method that is equivalent to them. For, if those savings are not so employed, but are diverted to other uses, the conversion of the perpetual annuities, now due to the publick creditors, into life-annuities would be evidently a *much less* efficacious method of diminishing the national debt than either of the three former methods above-mentioned. Thus, for example, we have seen that, if the said savings are employed in the manner above-described, the sum of one million of pounds sterling, a year will be sufficient to redeem to the publick, in the course of 60 years, the sum of £9,519,599 *per annum* of the interest of the national debt; but without the help of the said savings, it will redeem in that time only £1,803,953 *per annum*. Therefore,

whenever any proposal is made to pay off any part of the national debt by means of life-annuities, great care must be taken that the savings of the interest of the said debt, that will continually accrue to the publick by the deaths of some of the life-annuitants, shall be constantly applied to the same purpose; or otherwise the nation must (as Dr. Price has justly observed in his Essay on Publick Credit and the National Debt in the third chapter of his Treatise on Reverfionary Payments, first edition, page 19,) lose greatly by all schemes of this kind.

CCLXXIV. This fourth method of employing a given sum of money every year in the diminution of the national debt, “by converting some of the perpetual annuities, that now pay the interest of it, into life-annuities, and allotting the said annual sum to the payment of the additions thereupon made to the perpetual annuities so converted,” partakes in some degree of the advantage mentioned in Art. ccliv as belonging to the said method, to wit, that of securing to the publick the faithful application of such annual sum to the purposes of its destination, without suspension or interruption. For it would then be impossible to with-hold any

A comparison between this fourth method and the third method, with respect to the advantage of securing to the publick the faithful application of the said annual sum of money to the purposes of its destination.

part of the life-annuities so created from the persons to whom they would be due, without an absolute breach of the publick faith; which, (as we before observed,) may be considered as a moral impossibility. But this advantage will relate only to so much of the publick revenue as will be due at any time to the life-annuitants that are still alive; and not to that other part of it which was allotted to the payment of the annuities of the other life-annuitants who will have died since the establishment of them, and which will be every year increasing by the deaths of more of the said life-annuitants. For this part of the revenue, which will be thus saved every year to the publick, will be as liable to be diverted, by the ministers of state and the Parliament, from its original destination of diminishing the national debt, in order to defray the expence of some temporary measure of Government, as the money that should be allotted to the diminution of the national debt in either the first or the second method above-described: whereas in the third method of applying a given sum of money every year to the diminution of the national debt, "by converting some of the perpetual annuities, that now pay the interest of it, into greater temporary annuities that should continue for a certain limited number of years, and allotting the said annual sum to the payment of the additions therupon made to the perpetual annuities so converted," the whole of the said annual sum is effectually secured from being diverted from its original and proper destination, and applied to any other purpose whatsoever, during the whole of the said term.

A fifth method of employing a given sum of money every year in the reduction of the national debt.

CCLXXV. A fifth method of applying a given sum of money every year to the diminution of the national debt, would be to combine the foregoing third and fourth methods together, by converting some of the perpetual annuities now due to the publick creditors, into greater temporary annuities of a mixt nature, or that should continue both for a certain moderate number of years at all events, or whether the proprietors of them lived to the end of the said term, or not, and should, in case of their dying before the end of the said term, be paid to their executors or other personal representatives, and should likewise continue during the lives of such of the said proprietors, or of other persons to be named by the said proprietors, as should live beyond the end of the said term, and by employing the said annual sum of money in paying the additions that it would be necessary to make to the perpetual annuities so converted, in compensation for the abridgement of their duration: such conversion being made (as in the second, third, and fourth methods above-mentioned,) with the consent of the proprietors of the annuities so converted.

CCLXXVI. In this fifth method of employing a given sum of money every year for the purpose of diminishing the national debt, it is evident there would be no savings accruing to the public till after the expiration of the limited term of years during which the new annuities, into which the perpetual annuities had been converted, were to continue at all events, whether the proprietors of the said perpetual annuities, or their *nominées*, were living or dead. But after the expiration of the said term there would accrue to the public very large savings every year by the deaths of some of the persons for whose lives the said annuities should have been granted. Now these savings ought to be employed, as fast as they arose, in diminishing the national debt in either the first or the second method above-described, or in some other method equivalent to them.

Of the savings that would accrue every year to the public, after the expiration of a certain term, by the deaths of life-annuitants.

CCLXXVII. Thus, for example, it might be expedient to convert million, or two millions, or, perhaps, three millions, of pounds, sterling, or *annua* of the perpetual annuities, now due to the public creditors, with the consent of the proprietors of such annuities,) into annuities which should continue for 30 years at all events, and likewise, after the expiration of the said 30 years, during the lives of their respective proprietors, or of other persons that should be named by the said proprietors, and who may be called their *nominées*. And then the million of pounds, sterling, a year, or other given annual sum that could be spared out of the public revenue for this purpose, or a part of the said sum, might be employed in paying the additions which it would be necessary to make to the perpetual annuities so converted, in order to make amends to the proprietors of them for this abridgement of the time of their continuance. And, as there would accrue to the public at the end of the said term of 30 years, during which the said annuities were to continue at all events, great saving of interest by the deaths of the many life-annuitants who would have died in the course of the said term;—and other and still greater savings of interest would accrue in like manner to the public at the end of every following year after the expiration of the said term by the deaths of life-annuitants, until all the said life-annuitants were dead;—these several savings should, as fast as they accrued, be employed, for the same purpose of diminishing the national debt, in either the first or the second method above-mentioned.

CCLXXVIII. This fifth method of paying off a part of the national debt, by converting some of the perpetual annuities, now due to the public creditors, into these compound, temporary, annuities, (which are to continue at all events during a certain given number of years, and further for the lives of certain life-annuitants,) would have exactly the same effect as the four preceding methods.

same effect towards diminishing the national debt, in any given period of time, as either of the four preceding methods; that is, if a given sum of money was every year to be faithfully applied to the diminution of the national debt in this fifth method during a given number of years;—and the savings of interest accruing to the publick, by the deaths of the life-annuitants, at the expiration of the said term certain (during which the annuities were to continue at all events,) and at the ends of all the following years, after the expiration of the said term, during the lives of any of the said life-annuitants, were, as soon as they accrued, to be faithfully employed, for the same purpose of diminishing the national debt, in either the first or the second method above-described;—the quantity of the interest of the national debt, or of the perpetual annuities now allowed to the payment of it, that would be redeemed to the publick at the end of any supposed number of years, by employing the said given sum of money in this fifth method, would be the same as the quantity of the said interest that would have been redeemed to the publick by the same annual sum of money, if it had been applied to the same useful purpose during the same number of years in either of the four preceding methods. Of this it may not be amiss to give the following example.

An example of the said fifth method of applying a given sum of money every year to the diminution of the national debt.

Preliminary
suppositions.

CCLXXIX. Let us suppose (as in the foregoing articles,) that the sum of money that is to be applied every year to the diminution of the national debt in this fifth method, is a million of pounds, sterling, a year; and that the period during which it is to be so applied, is 60 years. And let us suppose further, that the persons upon whose lives the new annuities (which are, according to this fifth method, to be established in lieu of some of the perpetual annuities now due to the publick creditors,) are to depend, are all, at the time of establishing these annuities, of the age 35 years; and that they will die off in the course of the following years in the proportions expressed in Monsieur de Parcieux's table of the probabilities of the duration of human life; and consequently that the longer liver of them will live almost through the whole of the said period of 60 years, and die a few days before the end of it. And let the number of years during which the said annuities are to continue at all events, whether the said life-annuitants are alive or dead, (and during which they are to be paid to the said life-annuitants, if they are alive, and, if they are dead, to their executors or other personal representatives,) be 30 years. And let the interest of money be supposed (as in the foregoing articles) to be 4 per cent. and the price at which the 3 per cent. annuities sell in the publick market, to be £75 per cent.

CCLXXX. Th

CCLXXX. These things being premised, we must inquire, in the next place, what quantity of the perpetual annuities, which now are employed in paying the interest of the national debt, might be converted into these compound, temporary, annuities (which are at the same life-annuities and annuities for a term certain of 30 years,) by the help of the said sum of a million of pounds, sterling, a year. Now this quantity may be determined in the manner following.

CCLXXXI. It appears from Mr. Smart's fourth table of compound interest, page 78, that the value of an annuity of one pound a year for term certain of 30 years, is £17.292,03. And it appears above in table XXV, page 285, that the value of a remote life-annuity of one pound a year for the life of a person of the age of 35 years, that is to commence at the distance of 30 years, or whereof the first payment is to be received at the end of 31 years, is £1.410,77. It follows therefore that the value of a compound, temporary, annuity of one pound a year, that is to continue for 30 years certain, and also for the life of a person of the age of 35 years, must be £17.292,03 + £1.410,77, or £18.702,80. Now, when the interest of money is 4 per cent. (as it is here supposed to be,) the value of a perpetual annuity of one pound a year is £25. Therefore, in order to make an annuity of one pound a year of the compound and temporary kind here described, or that is to continue only for 30 years certain and during the life of a person of the age of 35 years, equal in value to a perpetual annuity of one pound a year, we must increase in the proportion of £25 to £18.702,80; by which means it will become equal to $\left(\frac{25.00000}{18.70280} \right) \times £1$, or £1.336,698. Therefore a compound, temporary, annuity of £1.336,698 a year, that is to continue for 30 years certain and also for the life of a person of the age of 35 years, will be equal in value to a perpetual annuity of one pound a year: and consequently, if the Government were to convert some of the perpetual annuities, now due to the publick creditors as the interest of their debts, to compound, temporary, annuities of the kind here described, or that should continue for 30 years certain and further for the lives of persons of the age of 35 years, they ought to allow to the said publick creditors, those perpetual annuities should be so converted into these temporary ones, an annuity of £1.336,698 a year of the kind here described, for every perpetual annuity of £1 a year of which they had been before possessed. Therefore for every million of pounds *per annum* of the perpetual annuities at should be so converted, it would be necessary to allow to the proprietors of them the sum of a million times £1.336,698 *per annum*, or of £1,336,698 *per annum* of these compound and temporary annuities: and consequently

consequently an additional annual sum of £336,698 would be required to enable the Government to convert £1,000,000 *per annum* of the perpetual annuities, now due to the public creditors, as the interest of their debt into these compound, temporary, annuities for 30 years certain and for the lives of persons of the age of 35 years. Therefore an additional annual sum of £1,000,000 would be sufficient to enable the Government to convert into these compound, temporary, annuities for 30 years certain and for the lives of persons of the age of 35 years, a quantity of the perpetual annuities that is greater than £1,000,000 *per annum* in the same proportion as £1,000,000 is greater than £336,698, that is, a quantity of the perpetual annuities that is equal to $(£1,000,000 \times \frac{1,000,000}{336,698})$

$$\frac{£1,000,000,000,000}{336,698}, \text{ or } £2,970,020 \text{ per annum. Q.E.I.}$$

CCXXXII. It is evident therefore that at the end of the said term of 60 years, when all the said life-annuitants (who were of the age of 35 years at the beginning of the said term,) will be dead, the said sum of £2,970,020 *per annum* of the perpetual annuities, now due to the public creditors, will be intirely redeemed to the publick by means of this conversion of them into these compound, temporary, annuities.

Of the savings that would accrue to the publick at the end of every year, after the first 30 years, by the deaths of life-annuitants.

CCLXXXIII. This would be the quantity of the said perpetual annuities that would be redeemed to the nation by this operation in the course of 60 years, if all the aforesaid life-annuitants had lived to within day of the end of the whole 60 years, or if, when they had died off the different years of the said term of 60 years according to the course of nature, the savings which would have accrued every year to the publick by their deaths during the last 30 years of the said term, had not been applied to the same purpose of diminishing the national debt, but had been spent upon some other service. But it was supposed above that the savings were to be employed continually, as fast as they arose, for the same purpose of diminishing the national debt, in either the first, or the second of the methods above-described. We must therefore now proceed to inquire, what these savings would amount to in every year of the said term of 60 years after the first 30 years of it, and what quantity of the perpetual annuities, now due to the publick creditors, they would be sufficient to redeem in the course of the latter half of the said term, if they were constantly employed for that purpose, until the end of the said term, either the first or the second method above-mentioned. These inquiries may be made in the manner following.

CCLXXXIV. T

CCLXXXIV. The number of persons represented in Monsieur de Parcieux's table of probabilities as living at the age of 35 years, is 694. The money that is to be paid annually by the Government in the aforesaid compound annuities (for the lives of several persons of the age of 35 years, and, in case of their dying within the space of 30 years, to the executors of such deceased life-annuitants for the remainder of the said term of 30 years,) in lieu of the £2,970,020 *per annum* of perpetual annuities which are to be thereby redeemed, is the said sum of £2,970,020 or *annum* together with the annual million of pounds which is destined to this purpose, and consequently is £3,970,020 *per annum*. Therefore, if we suppose these compound annuities to be distributed equally amongst 694 persons, all of the age of 35 years, each of the said persons will be entitled of an annuity, for 30 years certain and also for his life, of $\frac{£3,970,020}{694}$, or £5720.4899, or, very nearly, £5720.4900 *per annum*. and consequently, if we suppose them to be equally distributed amongst 694 persons, all of the same age of 35 years, each of the said persons will be possessed of an annuity, of the same kind, or £572.0490 *per annum*: and, if we suppose them to be equally distributed amongst 69400 persons, all of the same age of 35 years, each of the said persons will be possessed of an annuity, of the same kind, of £57.2049, or 57*l.* 4*s.* 1*d.* $\frac{2}{3}$ *per annum*. Let us make this last supposition.

CCLXXXV. Then, since it appears by Monsieur de Parcieux's table of the probabilities of the duration of human life, that out of 694 persons the age of 35 years, all living at the same time, only 380 persons will be living at the end of 31 years, or at the age of 66 years, it follows that out of the said 69400 persons above-mentioned, of the age of 35 years, living at the same time, (for whose lives, and likewise for a term certain of 30 years, these annuities of £57.2049, or 57*l.* 4*s.* 1*d.* $\frac{2}{3}$ each, are supposed to be granted,) only 38000 will be living at the end of 31 years, at the age of 66 years. And consequently the annuities of £57.2049, 57*l.* 4*s.* 1*d.* $\frac{2}{3}$ each, which would have been payable to the other 31400 life-annuitants at the end of the said 31 years, the said annuitants had been then living, will be saved to the publick, and will be ready in the publick treasury to be employed, in any manner it may be thought fit, towards the diminution of the national debt. Of the manner in which the said sum of £1,796,233.8600 is immediately to be employed, according to the first method above-described, or in buying up the capital of the 3 per cent. annuities at the price of £75 per cent. of the saving made at the end of the 31st year by the deaths of some of the life-annuitants.

cent. according to the second method above-described; and the interest of the capital thereby extinguished, or the portion of the perpetual annuity that will be redeemed at the end of the next, or 32d, year, by thus employing the said sum of £1,796,233.8600, be likewise employed in the same manner; and the interest of the capital extinguished by this second operation, or the portion of the perpetual annuities that will thereby be redeemed at the end of the 33d year, be likewise employed in the same manner; and all the following portions of the perpetual annuities that will be thus redeemed by means of this first sum of £1,796,233.8600, that is saved to the nation at the end of the 31st year by the deaths of the aforesaid 31400 life-annuitants, be likewise employed in the same manner during the whole remainder of the said term of 60 years; — I say, these successive sums of money be so employed without interruption during the remainder of the said term of 60 years, the quantity of the capital thereby extinguished by the national debt that will be thereby extinguished at the end of the said term of 60 years, or at the end of 29 years from the time when this first saving of £1,796,233.8600 will have accrued, will be equal in value to the sum to which the said first saving of £1,796,233.8600 would have increased in the same period of 29 years, if it had been improved all the time at compound interest at the rate of 4 per cent. and therefore it is (according to Mr. Smart's first table of compound interest, page 54) equal in value to the sum of £1,796,233.8600 $\times$ 3.118,651,

£5,601,826.5237; the interest of which at 4 per cent. is $\left(\frac{4}{100}\right)$

£5,601,826.5237, or $\frac{£22,407,306.0948}{100}$, or) £224,073.0609 *per ann.*

The quantity *num.* Therefore the quantity of the interest of the national debt, or of the interest of the perpetual annuities now due to the publick creditors, which would be redeemed at the end of the whole term of 60 years by means of this first saving of £1,796,233.8600, which would have accrued to the publick at the end of the 31st year of the said term by the deaths of the aforesaid 31,400 life-annuitants, would be £224,073.0609 *per annum.*

Of the saving made at the end of the thirty-second year by the deaths of life-annuitants.

CCLXXXVI. In the course of the thirty-second year of this century the 38,000 life-annuitants of the age of 66 years who were living at the end of the thirty-first year, or at the age of 66 years, will be reduced to 36,400. Therefore 1600 more of the said life-annuitants will have died in the course of this thirty-second year. And consequently at the end of this thirty-second year there will be a saving to the Government of (31,400 $\times$ 1,600, or) 33,000 payments of £57.2049 each, which would have become due to the 33,000 life annuitants who will have died in the thirty-

no preceding years, if they had lived till that time. There will therefore be in the hands of Government at the end of this thirty-second year the sum of $33,000 \times \pounds 57.2049$, or $\pounds 1,887,761.7000$, to be employed during the remainder of the said term of 60 years, that is, during 28 years, in diminishing the national debt; which will be sufficient to extinguish as much of the capital of the said debt as is equal in value to the sum to which this second saving of $\pounds 1,887,761.7000$ will increase in the course of 28 years, if it be constantly improved during that time at compound interest at the rate of 4 per cent. Now the amount of $\pounds 1,887,761.7000$ at the end of 28 years, when improved during that time at compound interest at the rate of 4 per cent. appears by Mr. Smart's first table of compound interest, page 54, to be $\pounds 1,887,761.7000 \times 2.998,703$, or $\pounds 5,660,836.6730$; the interest of which at 4 per cent. is $\left(\frac{4}{100} \times \pounds 5,660,836.6730 \right.$, or $\pounds 226,643,346.6920$, or $\left. \right) \pounds 226,433.4669$ *per annum*.

Therefore the quantity of the capital of the national debt which would be extinguished at the end of the whole term of 60 years by means of this second saving of $\pounds 1,887,761.7000$, (which would have accrued to the publick at the end of the thirty-second year of the said term by the annuity of the aforesaid 33,000 life-annuitants who would have died in the preceding 32 years,) would be equal in value to the sum of $\pounds 5,660,836.6730$; and the quantity of the interest of the said debt, or of the perpetual annuities now due to the publick creditors, which would be redeemed at the end of the said term of 60 years by means of the said second saving, would be the sum of $\pounds 226,433.4669$ *per annum*. Q E I.

CCLXXXVII. In the same manner we must proceed to investigate the savings that will accrue to the publick at the ends of the thirty-third, thirty-fourth, the thirty-fifth, the thirty-sixth, and every following year of the whole term of 60 years, and the quantities of the capital of the national debt that will be extinguished by each of the said savings at the end of the said term of 60 years; if they are constantly employed for that purpose, in either the first or the second method above described, during several remaining years of the said term after the times when they will be respectively accrued. And, in order to these investigations, it will be necessary, in the first place, to ascertain, by the help of Monsieur de Moivre's table of the probabilities of the duration of human life, the numbers of life-annuitants who will be alive at the ends of the several remaining years of this term, and thence to determine the numbers of the who will have died in each of the said years, and from whose deaths several new and successive savings to the publick will arise.

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CCLXXXVIII.

Of the savings that will be made at the ends of the 33d, 34th, 35th, and other following years of the said term of 60 years, by the deaths of life-annuitants.

The numbers of life-annuitants who will be living at the ends of all the said years, except the last year.

CCLXXXVIII. Now of the aforesaid original number of 69,400 life-annuitants, all of the age of 35 years, who were supposed to be living at the beginning of the said term of 60 years, and for whose lives (as well as for a term certain of 30 years,) the said annuities, of £57.2049 each, were supposed to be granted, it appears from Monsieur de Parcieux's table of probabilities that the following numbers will be living at the subsequent ages of 68 years, 69 years, 70 years, 71 years, &c. as far as the age of 94 years inclusive, or at the ends of the thirty-third, the thirty-fourth, the thirty-fifth, the thirty-sixth, and every following year of the said term of 60 years, except the last; to wit,

34,700,	21,100,	8,500,	1,600,
32,900,	19,200,	7,100,	1,100,
31,000,	17,300,	5,900,	700,
29,100,	15,400,	4,800,	400,
27,100,	13,600,	3,800,	200,
25,100,	11,800,	2,900,	100,
23,100,	10,100,	2,200,	

The numbers of life-annuitants who will have died in each of the said years, and in the last year.

Therefore the numbers of the said life-annuitants who will have died in each of those years, to wit, the 33d, the 34th, the 35th, the 36th, &c. and in the last year of the said period of 60 years, or from the age of 94 years to the age of 95 years, are as follows; to wit,

1700,	2000,	1600,	600,
1800,	1900,	1400,	500,
1900,	1900,	1200,	400,
1900,	1900,	1100,	300,
2000,	1800,	1000,	200,
2000,	1800,	900,	100,
2000,	1700,	700,	and 100.

The savings that will be made at the ends of the said years in consequence of the deaths of the said life-annuitants.

CCLXXXIX. Therefore the savings of interest that will be made by the publick at the ends of those years in consequence of the deaths of life-annuitants that will have happened in the course of them respectively will be as follows; to wit,

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At the end of the thirty-third year,

$$\frac{33,000 + 1,700}{\circ} \times £57.2049,$$

$$\text{or } 34,700 \times £57.2049, = £1,985,010.0300.$$

At the end of the thirty-fourth year,

$$\frac{34,700 + 1800}{\circ} \times £57.2049,$$

$$\text{or } 36,500 \times £57.2049, = £2,087,978.8500.$$

At the end of the thirty-fifth year,

$$\frac{36,500 + 1900}{\circ} \times £57.2049,$$

$$\text{or } 38,400 \times £57.2049, = £2,196,668.1600.$$

At the end of the thirty-sixth year,

$$\frac{38,400 + 1900}{\circ} \times £57.2049,$$

$$\text{or } 40,300 \times £57.2049, = £2,305,357.4700.$$

At the end of the thirty-seventh year,

$$\frac{40,300 + 2000}{\circ} \times £57.2049,$$

$$\text{or } 42,300 \times £57.2049, = £2,419,767.2700.$$

At the end of the thirty-eighth year,

$$\frac{42,300 + 2000}{\circ} \times £57.2049,$$

$$\text{or } 44,300 \times £57.2049, = £2,534,177.0700.$$

At the end of the thirty-ninth year,

$$\frac{44,300 + 2000}{\circ} \times £57.2049,$$

$$\text{or } 46,300 \times £57.2049, = £2,648,586.8700.$$

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At the end of the fortieth year,

$$\frac{46,300 + 2000}{\text{or } 48,300} \times £57.2049, = £2,762,996.6700.$$

At the end of the forty-first year,

$$\frac{48,300 + 1900}{\text{or } 50,200} \times £57.2049, = £2,871,685.9800.$$

At the end of the forty-second year,

$$\frac{50,200 + 1900}{\text{or } 52,100} \times £57.2049, = £2,980,375.2900.$$

At the end of the forty-third year,

$$\frac{52,100 + 1900}{\text{or } 54,000} \times £57.2049, = £3,089,064.6000.$$

At the end of the forty-fourth year,

$$\frac{54,000 + 1800}{\text{or } 55,800} \times £57.2049, = £3,192,033.4200.$$

At the end of the forty-fifth year,

$$\frac{55,800 + 1800}{\text{or } 57,600} \times £57.2049, = £3,295,002.2400.$$

At the end of the forty-sixth year,

$$\frac{57,600 + 1700}{\text{or } 59,300} \times £57.2049, = £3,392,250.5700.$$

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At the end of the forty-seventh year,

$$\frac{59,300 + 1600}{\times} \times £57.2049,$$

$$\text{or } 60,900 \times £57.2049, = £3,483,778.4100.$$

At the end of the forty-eighth year,

$$\frac{60,900 + 1400}{\times} \times £57.2049,$$

$$\text{or } 62,300 \times £57.2049, = £3,563,865.2700.$$

At the end of the forty-ninth year,

$$\frac{62,300 + 1200}{\times} \times £57.2049,$$

$$\text{or } 63,500 \times £57.2049, = £3,632,511.1500.$$

At the end of the fiftieth year, A

$$\frac{63,500 + 1100}{\times} \times £57.2049,$$

$$\text{or } 64,600 \times £57.2049, = £3,695,436.5400.$$

At the end of the fifty-first year, A

$$\frac{64,600 + 1000}{\times} \times £57.2049,$$

$$\text{or } 65,600 \times £57.2049, = £3,752,641.4400.$$

At the end of the fifty-second year, A

$$\frac{65,600 + 900}{\times} \times £57.2049,$$

$$\text{or } 66,500 \times £57.2049, = £3,804,125.8500.$$

At the end of the fifty-third year, A

$$\frac{66,500 + 700}{\times} \times £57.2049,$$

$$\text{or } 67,200 \times £57.2049, = £3,844,169.2800.$$

As

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At the end of the fifty-fourth year,

$$\underline{67,200} + \underline{600} \times £57.2049,$$

$$\text{or } 67,800 \times £57.2049, = £3,878,492.2200.$$

At the end of the fifty-fifth year,

$$\underline{67,800} + \underline{500} \times £57.2049,$$

$$\text{or } 68,300 \times £57.2049, = £3,907,094.6700.$$

At the end of the fifty-sixth year,

$$\underline{68,300} + \underline{400} \times £57.2049,$$

$$\text{or } 68,700 \times £57.2049, = £3,929,976.6300.$$

At the end of the fifty-seventh year,

$$\underline{68,700} + \underline{300} \times £57.2049,$$

$$\text{or } 69,000 \times £57.2049, = £3,947,138.1000.$$

At the end of the fifty-eighth year,

$$\underline{69,000} + \underline{200} \times £57.2049,$$

$$\text{or } 69,200 \times £57.2049, = £3,958,579.0800.$$

At the end of the fifty-ninth year,

$$\underline{69,200} + \underline{100} \times £57.2049,$$

$$\text{or } 69,300 \times £57.2049, = £3,964,299.5700.$$

And at the end of the sixtieth and last year,

$$\underline{69,300} + \underline{100} \times £57.2049,$$

$$\text{or } 69,400 \times £57.2049, = £3,970,020.0600.$$

CCXC. All these savings are to be laid out in diminishing the national debt in either the first or the second method above-described; by which means the portion of the national debt that will have been extinguished at the end of the said term of 60 years in consequence of each of these savings, except the last, to wit, £3,970,020.0600, or (neglecting the fraction .0600,) £3,970,020, will be equal in value to a sum that is greater than such saving, namely, to the sum which is the amount to which such saving will have increased at the end of the said term, if we suppose it to be improved in the mean time at compound interest at the rate of 4 per cent. We must therefore, in the next place, compute the amounts of these savings at the end of the said term, if so improved at compound interest in the mean time, or during the remaining years of the said term after the times at which they will have respectively accrued, that is, during 27 years, 26 years, 25 years, 24 years, &c. to the last year of the said term. Now these amounts will be as follows.

CCXCI. The amount of £1,985,010.0300, improved at compound interest at the rate of 4 per cent. during 27 years, is = £1,985,010.0300
 $\times 2.883368 = £5,723,514.4001$.

The amount of £2,087,978.8500, improved in the same manner during 26 years, is £2,087,978.8500 $\times 2.772,469 = £5,788,856.6342$.

The amount of £2,196,668.1600, improved in the same manner during 25 years, is £2,196,668.1600 $\times 2.665,836 = £5,855,957.0609$.

And the amounts of all the following savings, improved in the same manner during the remaining years of the said term of 60 years after the times at which they will have respectively accrued to the publick, will be as follows.

£2,305,357.4700	$\times$	2.563,304	=	£5,909,332.0242.
£2,419,767.27	$\times$	2.464,715	=	£5,964,036.6868.
£2,534,177.0700	$\times$	2.369,918	=	£6,005,791.8533.
£2,648,586.8700	$\times$	2.278,768	=	£6,035,515.0045.
£2,762,996.6700	$\times$	2.191,123	=	£6,054,065.5525.
£2,871,685.9800	$\times$	2.106,849	=	£6,050,208.7352.
£2,980,375.2900	$\times$	2.025,816	=	£6,037,691.9484.
£3,089,064.6000	$\times$	1.947,900	=	£6,017,188.9343.

£3,192,033.4200

£3,192,033.4200	x	1.872,981	=	£5,978,617.9647.
£3,295,002.2400	x	1.800,943	=	£5,934,111.2191.
£3,392,250.5700	x	1.731,676	=	£5,874,278.8980.
£3,483,778.4100	x	1.665,073	=	£5,800,745.3684.
£3,563,865.2700	x	1.601,032	=	£5,705,862.3409.
£3,632,511.1500	x	1.539,454	=	£5,592,083.8199.
£3,695,436.5400	x	1.480,244	=	£5,476,147.7657.
£3,752,641.4400	x	1.423,311	=	£5,341,175.8406.
£3,804,125.8500	x	1.368,569	=	£5,206,208.7104.
£3,844,169.2800	x	1.315,931	=	£5,058,661.5248.
£3,878,492.2200	x	1.265,319	=	£4,907,529.8973.
£3,907,094.6700	x	1.216,652	=	£4,753,574.5444.
£3,929,976.6300	x	1.169,858	=	£4,597,514.6004.
£3,947,138.1000	x	1.124,864	=	£4,439,993.5517.
£3,958,579.0800	x	1,081,600	=	£4,281,599.1329.
And £3,964,299.5700	x	1,040,000	=	£4,122,871.5528.

CCXCII. Having thus found the amounts to which the several years of the amounts savings made at the ends of the thirty-first year and every following year of all the year-ly savings at the end of the said term of 60 years, except the last, will have increased at the end of the said term by being improved in the mean time at compound interest at the rate of 4 per cent. we must now add all these amounts together with the saving of £3,970,020.0600, which will accrue at the end of the last year, into one sum: which may be done as follows.

The amounts of the savings made in the first period of six years after the expiration of the term certain of 30 years, and in the second period of six years after the expiration of the said term, and in the third period of six years after the expiration of the said term, will be as follows.

£	5,601,826.5237	£	5,964,036.6868	£	6,017,188.9343
	5,660,836.6730		6,005,791.8533		5,978,617.9647
	5,723,514.4001		6,035,515.0045		5,934,111.2191
	5,788,856.6342		6,054,065.5525		5,874,278.8980
	5,855,957.0609		6,050,408.7352		5,800,745.3684
	5,909,332.0242		6,037,691.9484		5,705,862.3409
	34,540,323.3161		36,147,309.7807		35,310,804.7234

And the amounts of the savings made in the fourth period of six years and the fifth period of six years after the expiration of the said term certain of 30 years, will be as follows.

£	£
5,592,083.8199	4,753,574.5444
5,476,147.7657	4,597,514.6004
5,341,175.8406	4,439,993.5517
5,206,208.7104	4,281,599.1329
5,058,661.5248	4,122,871.5528
4,907,529.8973	3,970,020.0600
<u>31,581,807.5587</u>	<u>26,165,573.4422</u>

Therefore the sum total of the amounts of all these savings at the end of the said term of 60 years, is £163,745,818.8231.

£
34540,323.3161
36,147,309.7807
35,310,804.7254
31,581,807.5587
26,165,573.4422
<u>163,745,818.8231</u>

CCXCIII. Therefore the quantity of the capital of the national debt that would have been extinguished, by means of all these savings, at the end of the said term of 60 years, will be equal in value to the said sum £163,745,818.8231; the annual interest of which at 4 per cent. is

$$\frac{4}{100} \times £163,745,818.8231, \text{ or } \frac{654983,275.2924}{100}, \text{ or } £6,549,832.7529.$$

Therefore the quantity of the interest of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed

the publick at the end of the said term of 60 years by means of all these savings, (so that at the end of the next, or 61st, year it would be to be disposed of by the Parliament in any other way,) would be £549,832.7529 *per annum*, or (neglecting the fractional part .7529,) £549,832 *per annum*. Q.E.I.

X x

CCXCIV. This

The quantity of the national debt that will have been extinguished by means of all the foregoing savings, at the end of the said term of 60 years.

The quantity of the interest of the national debt that will be then reckoned to the publick by means of the said savings.

The whole quantity of the interest of the national debt that will be redeemed to the publick at the end of the said term of 60 years by the foregoing operation.

CCXCIV. This sum of £6,549,832 *per annum* must therefore be added to £2,970,020 *per annum*; which was the quantity of the perpetual annuities, now due to the publick creditors, which was supposed to be converted into these compound, temporary, annuities for 30 years certain and for the lives of 69400 persons, all of the age of 35 years; and which will therefore be now wholly redeemed to the publick by the expiration of the said term of 30 years, and the deaths of all the said life-annuitants: and the sum of both, or the sum of £9,519,852 *per annum*, will be the whole quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the said term of 60 years, (so as to be free at the end of the next, or 61st, year to be disposed of by the Parliament in any other way,) by the faithful application of a million of pounds, sterling, a year to that salutary purpose during that period according to this fifth method. Q E I.

Agreement between the effect of the foregoing scheme, "of life-annuities with a term certain annexed to them," and the effects of the four preceding methods.

CCXCV. This sum of £9,519,852 *per annum* may be considered as equal to the quantity of the perpetual annuities, now due to the publick creditors, which would have been redeemed to the publick in the same space of 60 years by employing the same annual sum of a million of pounds, sterling, for the same purpose in either of the four preceding methods; which quantity we have seen in the foregoing pages to be £9,520,000 *per annum*, or, more accurately, $\left(\frac{4}{100} \times £237,990,685.24\right.$
or $\frac{£951,962,740.96}{100}$, or) £9,519,627.4096, or (neglecting the fraction .4096,) £9,519,627 *per annum*. For the difference between them is only £225 *per annum*; which, upon so large a sum as £9,519,627, is a mere trifle.

General conclusion concerning the efficacy of this fifth method of applying a given sum of money every year to the diminution of the national debt.

CCXCVI. We may therefore conclude that this fifth method of employing a million of pounds, sterling, or any other given sum of money, every year to the purpose of diminishing the national debt, "by converting some of the perpetual annuities, now due to the publick creditors, into these compound, temporary, annuities, which are to continue at all events during a certain term of years, and are also to be extended contingently beyond the said term and during the lives of certain life-annuitants," would be just as efficacious in redeeming the perpetual annuities that are now due to the publick creditors as the interest of the said debt, and therefore would be just as beneficial to the nation as either of the four preceding methods of applying the same sum of money to the same purpose.

purpose during the same period of time: which was the principal object of the foregoing computation. But this, it must be remembered, is true only upon a supposition that the annual savings which will accrue to the publick at the end of every year after the expiration of the said term certain during which the said annuities are to continue at all events,) by the deaths of the said life-annuitants, are immediately employed in discharging a part of that debt in either the first or the second method above-described, or in some other method that is equivalent to them. For, if those savings are not so employed, but are diverted to other uses, the conversion of the perpetual annuities, now due to the publick creditors, into compound, temporary, annuities of the kind described in this fifth method, would be evidently a much *less* efficacious method of diminishing the national debt than either of the four preceding methods. Thus, for example, we have seen that, by employing the said savings in the manner above-described, the sum of one million of pounds, sterling, a year, employed in this fifth method, will be sufficient to redeem to the publick, in the course of 60 years, the sum of £9,519,852 *per annum* of the interest of the national debt; but without the help of the said savings, it will redeem in that time only £2,970,020 *per annum* of the said interest. Therefore, whenever any proposal is made to pay off any part of the national debt by means of these compound, temporary, annuities, (which are to continue for a certain number of years at all events, and afterwards during the lives of certain life-annuitants,) great care must be taken that the savings of the publick revenue, which will accrue every year to the publick, (after the expiration of the term certain, during which such annuities are to continue at all events,) by the deaths of the said life-annuitants, shall be constantly applied to the same purpose; or otherwise the nation must (as Dr. Price has observed concerning the conversion of the perpetual annuities, now due to the publick creditors, into life-annuities,) lose greatly by all schemes of this kind.

A sixth method of employing a given sum of money every year in the reduction of the national debt, by means of life-annuities with benefit of survivorship during a certain term of years.

CCXCVII. A sixth method of applying a given sum of money every year to the diminution of the national debt, might be grafted on the fourth method, in which some of the perpetual annuities, now due to the publick creditors, are supposed to be converted into life-annuities. If, instead of supposing (as we did in the said fourth method,) that, upon the deaths of the said life-annuitants, their annuities shall revert to the publick, and be immediately employed, for the purpose of diminishing the

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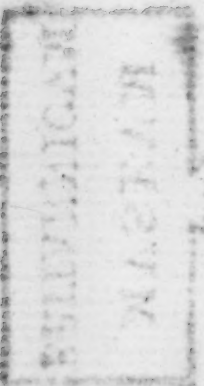
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the national debt, in either the first, or the second, method above-mentioned, we may suppose them to accrue to the surviving life-annuitants, and to be equally divided amongst them, until their annuities (which, it is evident, will, upon this plan, be continually increasing,) shall have increased to ten times their original quantity, or to some other very great sum, or during a certain number of years less than the whole number of years during which it is probable that some few of the said life-annuitants may live: and then the annuities of those life-annuitants who shall die after the expiration of the said term of years, or after the annuities of the surviving life-annuitants shall have been increased to their greatest allowed magnitude, may be supposed to accrue to the publick, (as in the aforesaid fourth method,) and to be employed, for the purpose of diminishing the national debt, in either the first, or the second, method above-mentioned. And, if this plan were adopted, the annual sum of money, which could be spared from the publick revenue for this purpose, might be employed in paying to the proprietors of the perpetual annuities so converted into temporary ones, the additions which it would be necessary to make to the said perpetual annuities in compensation for the said abridgement of the duration.

In this method, as well as in the four preceding methods, it is supposed that the proprietors of the perpetual annuities that are so to be converted into temporary ones, have previously consented to such conversion.

CCXCVIII. Thus, for example, if a million of pounds sterling, *per annum* could be spared from the publick revenue for this purpose, the Government might convert a proportionable quantity of the perpetual annuities, now due to the publick creditors, with the consent of the proprietors of such annuities, into annuities for the lives of the said proprietors, or of such persons as they should name, with the benefit succeeding to each other's annuities by survivorship during the space 30 years; after which the annuities of the then surviving life-annuitants should no longer accrue, upon the deaths of the said life-annuitants, their surviving companions, but should belong to the publick, and be employed every year in the diminution of the national debt in either first, or the second, method above-mentioned. And the said million pounds *per annum* would, in such case, be employed in paying the additions which it would be necessary to make to the perpetual annuities that should be so converted, as a compensation to the proprietors of them for abridgement of the time of their continuance.



CCXCIX. If a given sum of money were to be employed every year for the purpose of diminishing the national debt in this sixth method, its effect would be exactly the same in a given period of time as it would be, if employed for the same purpose during the same period in either of the preceding methods; that is, the quantity of the interest of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed to the publick by the application of a given annual sum of money to the diminution of the national debt during a given number of years in this sixth method would be exactly the same as that which would be redeemed to the publick by applying the same sum of money every year to the same purpose during the same number of years in either of the five preceding methods. Of this I will now proceed to give an example.

An example of the said sixth method of employing a given sum of money every year in the reduction of the national debt.

CCC. Let the sum of money to be employed in the reduction of the national debt according to this sixth method be (as in the former examples,) a million of pounds, sterling, a year; and the period during which it is to be so employed, be also the same as was chosen in the former examples, to wit, a period of 60 years. And let the proprietors of the perpetual annuities which are to be converted into life-annuities with benefit of survivorship according to this sixth method, or the *nominees* of the said proprietors, during whose lives the new life-annuities are to continue, be all of the age of 35 years; so that it may reasonably be expected, (according to Monsieur de Parcieux's table of the probabilities of the duration of human life,) that the last survivor of them will live almost through the whole period of 60 years, and die a few days before the end of it. And let the number of the life-annuitants for whose lives the new annuities are to continue, be supposed to be the same in the examples above given of the foregoing fourth and fifth methods, to wit, 69,400. And let the said 69,400 life-annuitants, (who are all of the age of 35 years at the time of granting the said life-annuities,) be supposed to die off every year in the proportions set forth in Monsieur de Parcieux's table of the probabilities of the duration of human life. And, lastly, let the interest of money be supposed to be 4 per cent. and the price at which the 3 per cent. annuities sell at the publick market, to be £75 per cent.

CCCI. These

An investigation of the quantity of the perpetual annuities, now due to the public, due to the public creditors, which might be converted into these compound life annuities, with benefit of survivorship, by the help of a million of pounds, sterling, *per annum*.

CCCI. These things being premised, we must, in the next place, inquire what quantity of the perpetual annuities, now due to the public creditors, might, by the help of a million of pounds, sterling, a year, be converted into temporary annuities of the kind above-described. Now this may be determined in the manner following.

Let us first suppose that these 69,400 life-annuitants (who are all of the age of 35 years,) receive from the Government, for an adequate consideration, grants of annuities of £10 a piece, of the kind just now described, or that shall increase by survivorship during the space of 30 years, and then shall continue at their last magnitude, (to which they will have increased in the course of the said 30 years,) during the lives of each of the then surviving life-annuitants respectively. The sum of all these annuities will be £694,000 *per annum*; which, it is evident, the Government will be obliged to pay for 30 years together to such of the said life-annuitants as shall be living to receive it. But at the end of the said term of 30 years the Government will have less than £694,000 to pay every year to the said surviving life-annuitants; because then the right of succeeding to each other's annuities by survivorship is supposed to cease, and the annuities of the life-annuitants who shall die after the end of the said 30 years are supposed to accrue to the public. The Government will therefore have a saving at the end of every year after the expiration of the said 30 years, which may be employed in diminishing the national debt in either the first or the second method above-mentioned: and these savings will be greater and greater every year untill all the said life-annuitants are dead, until the end of the whole term of 60 years. The number of life-annuitants will be reduced in the course of 30 years from 69,400 to 39,500; so that at the end of the 30th year the sum of £694,000 will be to be divided between 39,500 persons, instead of 69,400 persons, and consequently each of the said 39,500 persons will receive, for his share of the said £694,000 the sum of $\frac{£694,000}{39,500}$, or) £17.569,620. And this sum of £17.569,620

Division of a compound life-annuity of £10 a year, for the life of a person of the age of 35 years, with benefit of survivorship during 30 years, into two parts.

which he will be intitled to receive at the end of the said 30th year, he will continue to receive at the end of every following year of his whole life. We may therefore divide the whole annuity which each of the said 69,400 life-annuitants will be intitled to according to the foregoing supposition into the two following parts, to wit, 1st, an annuity of £10 a year for the first 30 years of his life, and no longer, but accompanied with the benefit of succeeding by survivorship to the annuities of such of his fellow life-annuitants as should die in the course of the said 30 years; by which means the said annuity of £10 a year will gradually increase from £10 a year to £17.569,620 a year; and secondly, a remote life-annuity of £17.569,620 that is to commence at the end of 30 years, (or whereof the first payment

is to be received at the end of 31 years,) and that is to continue during the remainder of his life. Now the values of these two annuities may be found in the manner following.

Accomputation of the value of the first part of such compound life-annuity.

CCCL. As the several annuities of those life-annuitants who shall die every year during a term of 30 years, are supposed to be divided amongst their surviving companions, and not to accrue to the publick, it is evident that the price which ought to be paid to the publick by the whole body of the said 69,400 life-annuitants for the enjoyment of the said life-annuities during the said term of 30 years, in case they shall live so long, accompanied with the aforesaid benefit of survivorship during the said term, will be the same that ought to be paid by them for the same annuities, if the said annuities were to be granted to them for a term certain of 30 years, as to become payable to their executors, or other personal representatives; in case the said life-annuitants themselves should happen to die in the course of the said term: because it would be a matter of indifference to the publick, whether the annuities of the life-annuitants that died in the course of the said term of 30 years, were made payable, during the several remaining years of the said term, to the executors, or other personal representatives, of the said deceased life-annuitants, or to their surviving companions, who would, in such case, supply the place of their executors. and consequently, as all the said 69,400 life-annuitants are supposed to be of the same age of 35 years, and there is no reason that any one of them should pay more for his annuity than any other; the price which each of them ought to pay for the enjoyment of his life-annuity during the said term of 30 years, or such part of the said term as he shall happen to live through, when accompanied with this benefit of succeeding, by survivorship, to his proportional share of the annuities of those of his companions who shall die in the course of the said term, will be the same as he ought to pay for the purchase of the same annuity for the same term of 30 years, when it is to be enjoyed at all events during the whole of the said term, either by himself or his executor, or other personal representative, but without any chance of augmentation. Now it appears by Mr. Smart's table of compound interest, page 78, that when the interest of money is 4 per cent. the value of an annuity of one pound a year for a term of 30 years certain (or not depending on the continuance of any life, or other contingency,) is £17,492,033. Therefore the value of an annuity of £10 a year for the same term of 30 years is £172,920,333. and consequently the value of a life-annuity of £10 a year for the life of one of the aforesaid 69,400 life-annuitants of the age of 35 years, during the same term of 30 years, if he shall so long live, but not to continue after the said term, accompanied with the above-mentioned benefit of succeeding, by survivorship, to the annuities of such of his fellow life-annuitants as shall die in the course of the said term, will also be £172,920,333. Q E I.

A computation of the value of the second part of such compound life-annuity.

CCCIII. The other annuity, of which we were to determine the value is a remote life-annuity of £17.569,620 *per annum*, for the life of a person of the age of 35 years, that is to commence at the end of 30 years, whereof the first payment is to be received at the end of 31 years, the interest of money being 4 per cent. Now the value of a life-annuity of only one pound a year of this remote kind for the life of a person of the age of 35 years, when the interest of money is 4 per cent. appears above (in Table xxv, page 285,) to be £1.410,77. Therefore the value of life-annuity of £17.569,620 *per annum* of this remote kind for the life of a person of the age of 35 years, is = 17.569,620 × £1.410,77, or £24.786,692. QEI.

The value of the whole of the said compound life-annuity of 10l. *per annum*.

CCCIV. Therefore the value of both the foregoing annuities, or of the whole annuity which is supposed to be granted by the Government to each of the aforesaid 69,400 life-annuitants of the age of 35 years, is £172.920,33 + £24.786,69, or £197.707,02. Therefore £197.707,02 is the price which each of the said 69,400 life-annuitants ought to pay to the Government for a grant of an annuity of £10 a year of the kind above-described. QEI.

The said compound life-annuity of 10l. *per annum* is equal in value to a perpetual annuity of £7.908,28 *per annum*.

CCCV. Now, when the interest of money is 4 per cent. the interest of £197.707,02 will be $\left(\frac{4}{100} \times £197.707,02, \text{ or } \frac{790.828,08}{100}, \text{ or } £7.908,28 \text{ per annum}\right)$; and consequently a perpetual annuity of £7.908,28 *per annum* is worth the sum of £197.707,02. Therefore a perpetual annuity of £7.908,28 *per annum* is equal in value to a life-annuity of the kind above-described of £10 *per annum* for the life of a person of the age of 35 years. Therefore the 69,400 life-annuities of £10 each which are above supposed to be granted by the Government to 69,400 persons, of the age of 35 years, will be equal in value to 69,400 perpetual annuities of £7.908,28 *per annum*, or to a perpetual annuity of 69,400 × £7.908,28 or £548,834.63200 *per annum*. Therefore, if the yearly sum of £548,834.63200 of the perpetual annuities, now due to the public creditors, were the property of the said 69,400 persons of the age of 35 years, to whom we supposed above that the Government had granted life-annuities of £10 each of the kind above-described, and the said 69,400 life-annuitants were to resign to the Government their right to the said annual sum of £548,834.63200 of the perpetual annuities, in exchange for, and as the price of, their said life-annuities of £10 each, or the annual sum of £694,000 amongst them, such a bargain would be fair and equi-

on both sides. Therefore the difference between £694,000 *per annum* and £548,834.63200 *per annum*, that is, the sum of £145,165.36800 *per annum*, would be the addition which it would be necessary for the Government to make to the said sum of £548,834.63200 of the perpetual annuities, upon their being converted into life-annuities of the kind above-described, as a compensation to the proprietors of them for the abridgement of the time of their continuance. And consequently, since the additional sum of £145,165.36800 *per annum* would be sufficient to enable the Government to convert the sum of £548,834.63200 *per annum* of the perpetual annuities into life-annuities of the kind above-described, the additional sum of a million of pounds *per annum* would be sufficient to enable the Government to convert into life-annuities of the same kind a quantity of the perpetual annuities that is greater than £548,834.63200 *per annum* in the same proportion in which £1000,000 *per annum* is greater than £145,165.36800 *per annum*, or a quantity of the perpetual annuities that is equal to $\frac{1000,00}{145,165.36800} \times £548,834.63200$, or $\frac{548,834.632,000}{145,165.368,00}$, or £3,780,754 *per annum*. Therefore the quantity which we proposed in Art. cccci as necessary to be determined, to wit, the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into life-annuities of the kind above-described by the help of a fund of a million of pounds *per annum*, is the sum of £3,780,754 *per annum*. QEI.

CCCVI. Let us now therefore suppose that the Government were to employ the said fund of a million of pounds *per annum* in converting £3,780,754 *per annum* of the perpetual annuities into life-annuities of the kind above-described; and that the number of persons of the age of 35 years, to whom, or for whose lives, the said life-annuities were to be granted, was 69,400. Then it is evident that the whole sum of money which would be paid every year to these 69,400 life-annuitants, or the survivors of them, for the space of 30 years, would be £3,780,754 *per annum* together with the said additional sum of £1,000,000 *per annum*, which would be necessary to enable the Government to make this conversion; that is, it would be £4,780,754 *per annum*. And consequently the life-annuity which would be due to each of the said 69,400 life-annuitants at first, or before any of his companions had died, would be equal to $\frac{£4,780,754}{69,400}$, or £68.886,94 *per annum*.

The quantity of the perpetual annuities mentioned in Art. 301 as necessary to be determined, is the sum of £3,780,754 *per annum*.

The original quantity of each man's life-annuity, upon a supposition that the whole sum of £3,780,754 *per annum* of the perpetual annuities is converted into these life-annuities, and that the number of the life-annuitants continues (as before) to be 69,400, is £68.88694 *per annum*.

Y Y

CCCVII. The *per annum*.

Another manner of determining the quantity of the said original life-annuity.

CCCVII. The quantity of each of these life-annuities may likewise be determined in the following manner.

Since, when each of the 69,400 persons of the age of 35 years was supposed to have a life-annuity, of the kind above-described, of £10 a year granted to him by Government, the additional annual sum of money which was required to enable the Government to convert a portion of the perpetual annuities due to the publick creditors, that was equivalent to the annual sum of £694,000 (which was thus supposed to be granted, in life-annuities of £10 a piece, to the said 69,400 life-annuitants,) into the said life-annuities, was £145,165.36800 *per annum*, it follows that, when an additional sum of £1,000,000 *per annum* is employed for the same purpose, and the number of the life-annuitants, who are to receive these life-annuities from Government, still continues the same as before, to wit, 69,400, (as is the case on the present supposition,) the life-annuity which each of the said persons will be intitled to upon this second supposition will be greater than the life-annuity of £10, (to which he was intitled upon the former supposition,) in the proportion £1,000,000 to £145,165.36800, and therefore will be equal to £10 x

$$\frac{1,000,000}{145,165.36800}, \text{ or } \frac{£10,000,000}{145,165.36800}$$

£68.88695, which agrees with the number found before, to wit, £68.88694, to six places of figures, and differs from it only by an unit in the seventh figure. We may therefore be confident that the said first six figures are true, or that the quantity of each of the original life-annuities belonging to the said 69,400 life-annuitants, or that which he would be intitled to at the end of the first year, if none of his companions had died in the course of it, is £68.8869 *per annum*.

Of the increase of the said original life-annuity of £68.88694 *per annum*, by the benefit of survivorship, in the course of 30 years.

CCCVIII. We must now proceed to inquire to what quantity the original life-annuity, belonging to each of the said 69,400 life-annuitants, will increase by the deaths of some of the said life-annuitants, in the course of the said term of 30 years. Now, since, when each of the original life-annuities was supposed to be £10 a year, it was found (in Art. cccvi.) to increase, in the course of the said 30 years, to £17.569,620 a year, it follows that the original life-annuity of £68.8869, or, more accurately, £68.88694 *per annum*, to which each of the said 69,400 life-annuitants of the age of 35 years will be intitled upon this second supposition, will increase in the course of the said 30 years to a quantity that will exceed £68.886,94 *per annum* in the same proportion as £17.569,620 exceeds £10

that is, to the quantity $\frac{17.569,620}{10} \times \frac{\pounds 68.88694}{10}$, or $\frac{\pounds 1210,317,368,762,8}{10}$,

or) $\pounds 121.031,736,876,28$, or (neglecting the seven last figures,) $\pounds 121.0317$. Therefore at the end of the 30th year of the said term of 60 years, each of the survivors, out of the original 69,400 life-annuitants, will be intitled to receive from the Government the sum of $\pounds 121.0317$; and he will also be intitled to receive the same sum at the end of every following year during his whole life; but without any further increase, because it is supposed that from this time the annuities of the several life-annuitants, who shall die in the following years of the said term of 60 years, are to accrue to the publick.

CCCIX. That this sum of $\pounds 121.0317$ is the true quantity of the increased annuity, to which each of the surviving life-annuitants will be intitled at the end of the 30th year of the said term of 60 years, will likewise appear from the following consideration. The number of the said life-annuitants will, in the course of the said 30 years, be reduced from 69,400 to 39,500; and consequently the sum total of all the money paid at the end of every year, during the said 30 years, to all the surviving life-annuitants together, to wit, the sum $\pounds 4,780,754$, will be divided at the end of the 30th year amongst 39,500 persons; and consequently the annuity belonging to each of the said 39,500 persons will be $\frac{\pounds 4,780,754}{39,500}$, or $\pounds 121.0317$ per annum. QE1.

CCCX. We must therefore, in the next place, inquire into the savings which will now accrue to the publick at the end of every following year after the 30th, by the deaths of the remaining life-annuitants: which savings are to be employed, for the purpose of diminishing the national debt, in either the first, or the second, method above-mentioned.

Of the savings that will accrue to the publick after the expiration of the said term of 30 years.

CCCXI. Now it appears from Monsieur de Parcieux's table of the probabilities of the duration of human life, that, out of the 39,500 life-annuitants who will be living at the end of the 30th year of the said term of 60 years, and who will be then of the age of 65 years, only 38,000 will be living at the age of 66 years, or at the end of the 31st year of the said term, and that the numbers living at the subsequent ages of 67 years, 68 years, 69 years, 70 years, &c. or at the ends of the thirty second year, the thirty-third year, the thirty-fourth year, the thirty-fifth year, and every following year of the said term of 60 years, will be as follows; to wit,

Y y 2

36,400,

36,400,	21,100,	7,100,	700,
34,700,	19,200,	5,900,	400,
32,900,	17,300,	4,800,	200,
31,000,	15,400,	3,800,	100,
29,100,	13,600,	2,900,	000,
27,100,	11,800,	2,200,	
25,100,	10,100,	1,600,	
23,100,	8,500,	1,100,	

The numbers of the said life-annuitants that will have died in each of the said years.

CCCXII. Therefore the numbers of the said life-annuitants who will have died in the course of the 31st year, the 32d year, the 33d year, the 34th year, and every following year of the whole period of 60 years, in which they will all have died, will be as follows; to wit,

1,500,	2,000,	1,600,	500,
1,600,	2,000,	1,400,	400,
1,700,	1,900,	1,200,	300,
1,800,	1,900,	1,100,	200,
1,900,	1,900,	1000,	100,
1,900,	1,800,	900,	and 100.
2,000,	1,800,	700,	
2,000,	1,700,	600,	

The savings that will accrue to the publick at the ends of each of those years by the deaths of the said life-annuitants.

CCCXIII. Therefore at the end of the 31st year there will accrue to the Government a saving of the annuities of the 1500 life-annuitants who will have died in the course of the said year, or which would have become due to them at the end of the said year, if they had been then alive; that is, a saving of 1500 times £121.0317, or of the sum £181,547.5500. And at the end of the thirty-second year there will accrue to the Government a still greater saving, to wit, a saving of the annuities of the 1500 life-annuitants who will have died in the course of the 31st year, and likewise of the annuities of the 1600 life-annuitants who will have died in the course of the 32d year, that is, a saving of 1500 times £121.0317, or 3100 times £121.0317, or of the sum £375,198.2700. And in like manner we shall find the savings at the end of the 33d year, the 34th year, the 35th year, the 36th year, and every following year, of the whole term of 60 years, to be as follows; to wit,

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At the end of the thirty-third year,

$$\frac{3,100}{1700} \div \times £121.0317,$$

$$\text{or } 4,800 \times £121.0317, = £580,952.1600.$$

At the end of the thirty-fourth year,

$$\frac{4,800}{1800} \div \times £121.0317,$$

$$\text{or } 6,600 \times £121.0317, = £798,809.2200.$$

At the end of the thirty-fifth year,

$$\frac{6,600}{1900} \div \times £121.0317,$$

$$\text{or } 8,500 \times £121.0317, = £1,028,769.4500.$$

At the end of the thirty-sixth year,

$$\frac{8,500}{1900} \div \times £121.0317,$$

$$\text{or } 10,400 \times £121.0317, = £1,258,729.6800.$$

At the end of the thirty-seventh year,

$$\frac{10,400}{2000} \div \times £121.0317,$$

$$\text{or } 12,400 \times £121.0317, = £1,500,793.0800.$$

At the end of the thirty-eighth year,

$$\frac{12,400}{2000} \div \times £121.0317,$$

$$\text{or } 14,400 \times £121.0317, = £1,742,856.4800.$$

At the end of the thirty-ninth year,

$$\frac{14,400}{2000} \div \times £121.0317,$$

$$\text{or } 16,400 \times £121.0317, = £1,984,919.8800.$$

At

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At the end of the fortieth year,

$$\frac{16,400 + 2000}{} \times £121.0317,$$

$$\text{or } 18,400 \times £121.0317, = £2,226,983.2800.$$

At the end of the forty-fifth year,

$$\frac{18,400 + 1900}{} \times £121.0317,$$

$$\text{or } 20,300 \times £121.0317, = £2,456,943.5100.$$

At the end of the forty-second year,

$$\frac{20,300 + 1900}{} \times £121.0317,$$

$$\text{or } 22,200 \times £121.0317, = £2,686,903.7400.$$

At the end of the forty-third year,

$$\frac{22,200 + 1900}{} \times £121.0317,$$

$$\text{or } 24,100 \times £121.0317, = £2,916,863.9700.$$

At the end of the forty-fourth year,

$$\frac{24,100 + 1800}{} \times £121.0317,$$

$$\text{or } 25,900 \times £121.0317, = £3,134,721.0300.$$

At the end of the forty-fifth year,

$$\frac{25,900 + 1800}{} \times £121.0317,$$

$$\text{or } 27,700 \times £121.0317, = £3,352,578.0900.$$

At the end of the forty-sixth year,

$$\frac{27,700 + 1700}{} \times £121.0317,$$

$$\text{or } 29,400 \times £121.0317, = £3,558,331.9800.$$

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At the end of the forty-seventh year,

$$\frac{29,400 + 1600}{\quad} \times £121.0317,$$

$$\text{or } 31,000 \times £121.0317, = £3,751,982.7000.$$

At the end of the forty-eighth year,

$$\frac{31,000 + 1400}{\quad} \times £121.0317,$$

$$\text{or } 32,400 \times £121.0317, = £3,921,427.0800.$$

At the end of the forty-ninth year,

$$\frac{32,400 + 1200}{\quad} \times £121.0317,$$

$$\text{or } 33,600 \times £121.0317, = £4,066,665.1200.$$

At the end of the fiftieth year,

$$\frac{33,600 + 1100}{\quad} \times £121.0317,$$

$$\text{or } 34,700 \times £121.0317, = £4,199,799.9900.$$

At the end of the fifty-first year,

$$\frac{34,700 + 1000}{\quad} \times £121.0317,$$

$$\text{or } 35,700 \times £121.0317, = £4,320,831.6900.$$

At the end of the fifty-second year,

$$\frac{35,700 + 900}{\quad} \times £121.0317,$$

$$\text{or } 36,600 \times £121.0317, = £4,429,760.2200.$$

At the end of the fifty-third year,

$$\frac{36,600 + 700}{\quad} \times £121.0317,$$

$$\text{or } 37,300 \times £121.0317, = £4,514,482.4100.$$

At

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At the end of the fifty-fourth year,

$$\frac{37,300}{+ 600} \times £121.0317,$$

$$\text{or } 37,900 \times £121.0317, = £4,587,101.4300.$$

At the end of the fifty-fifth year,

$$\frac{37,900}{+ 500} \times £121.0317,$$

$$\text{or } 38,400 \times £121.0317, = £4,647,617.2800.$$

At the end of the fifty-sixth year,

$$\frac{38,400}{+ 400} \times £121.0317,$$

$$\text{or } 38,800 \times £121.0317, = £4,696,029.9600.$$

At the end of the fifty-seventh year,

$$\frac{38,800}{+ 300} \times £121.0317,$$

$$\text{or } 39,100 \times £121.0317, = £4,732,339.4700.$$

At the end of the fifty-eighth year,

$$\frac{39,100}{+ 200} \times £121.0317,$$

$$\text{or } 39,300 \times £121.0317, = £4,756,545.8100.$$

At the end of the fifty-ninth year,

$$\frac{39,300}{+ 100} \times £121.0317,$$

$$\text{or } 39,400 \times £121.0317, = £4,768,648.9800.$$

And at the end of the sixtieth and last year,

$$\frac{39,400}{+ 100} \times £121.0317,$$

$$\text{or } 39,500 \times £121.0317, = £4,780,752.1500.$$

CCCXIV. Now all these savings are to be employed, as fast as they arise, for the purpose of diminishing the national debt, in either the first or the second method above-mentioned. By this means the last saving, £4,780,752.1500, (which accrues to the publick at the end of the whole period of 60 years,) will either extinguish £4,780,752.1500 of the capital of the 4 per cent. annuities (which are supposed to sell at their *par*, or nominal value,) or, if employed in the second method above-described, it will extinguish a proportionably greater capital of the 3 per cent. annuities, which are supposed to sell at the price of £75 per cent. and in either case it will redeem to the publick the interest of 4 per cent. upon it, or the

annual sum of $\left(\frac{4}{100} \times £4,780,752.1500, \text{ or } \frac{£19,123,008.6000}{100}, \text{ or } \right)$

£19,1230.0860. But all the other savings will have time to perform more than one operation of this kind towards the diminution of the national debt, to wit, a new operation at the end of every year of the term that is remaining after the time when it accrued: and, in consequence of these repeated operations, the quantity of the national debt which each of these savings will have extinguished at the end of the said term of 60 years, will be equal in value to the amount of such saving at the end of the said term, if improved in the mean time at compound interest at the rate of per cent.

We must therefore now compute the amounts of these savings at the end of the said term, if so improved at compound interest in the mean time, or during the remaining years of the said term after the times at which they will have respectively accrued, that is, during 29 years, 28 years, 27 years, 26 years, &c. to the last year of the said term. Now these amounts will be as follows.

CCCXV. The amount of £181,547.5500, improved at compound interest at 4 per cent. during 29 years, is = £181,547.5500 $\times$ 3.118,651 £566,183.4483.

The amount of £375,198.2700, improved in the same manner during years, is = £375,198,2700 $\times$ 2.998,703 = £1,125,108.1778.

And the amounts of all the other savings, improved in the same manner during the remaining years of the said term of 60 years after they shall have respectively accrued to the publick, will be as follows.

Z z £580,952.1600

A computation of the amounts of all the said savings, except the last, at the end of the said term of 60 years, if improved in the mean time at compound interest at the rate of 4 per cent.

£580,952.1600	x	2.883,368	will be =	£1,675,098.8676.
£798,809.2200	x	2.772,469	=	£2,214,673.7993.
£1,028,769.4500	x	2.665,836	=	£2,742,530.6375.
£1,258,729.6800	x	2.563,304	=	£3,226,506.8236.
£1,500,793.0800	x	2.464,715	=	£3,699,027.2161.
£1,742,856.4800	x	2.369,918	=	£4,130,426.9433.
£1,984,919.8800	x	2.278,768	=	£4,523,171.9051.
£2,226,983.2800	x	2.191,123	=	£4,879,594.2854
£2,456,943.5100	x	2.106,849	=	£5,176,408.9871.
£2,686,903.7400	x	2.025,816	=	£5,443,172.5869.
£2,916,863.9700	x	1.947,900	=	£5,681,759.3271.
£3,134,721.0300	x	1.872,981	=	£5,871,272.9294.
£3,352,578.0900	x	1.800,943	=	£6,037,802.0431.
£3,558,331.9800	x	1.731,676	=	£6,161,878.0897.
£3,751,982.7000	x	1.665,073	=	£6,247,325.0902.
£3,921,427.0800	x	1.601,032	=	£6,278,330.2407.
£4,066,665.1200	x	1.539,454	=	£6,260,443.8856.
£4,199,799.9900	x	1.480,244	=	£6,216,728.7363.
£4,320,831.6900	x	1.423,311	=	£6,149,887.2735.
£4,429,760.2200	x	1.368,569	=	£6,062,432.5145.
£4,514,482.4100	x	1.315,931	=	£5,940,747.3522.
£4,587,101.4300	x	1.265,319	=	£5,804,146.5943.
£4,647,617.2800	x	1.216,652	=	£5,654,532.8589.
£4,696,029.9600	x	1.169,858	=	£5,493,688.2149.
£4,732,339.4700	x	1.124,864	=	£5,323,238.3055.
£4,756,545.8100	x	1.081,600	=	£5,144,679.9480.
£4,768,648.9800	x	1.040,000	=	£4,959,394.9392.

CCCXVI. Having thus found the amounts to which the several The addition of the amounts of all the aforefaid yearly savings, at the end of the faid term of 60 years, except the laft, will have increafed at the end of the faid term by being improved in the mean time at compound intereft at the rate of 4 per cent. we muft now add up all thefe amounts, together with the faving of £4,780,752.1500, (which will accrue at the end of the 60th, or laft, year,) into one fum: which may be done as follows.

The amounts of the favings made in the firft period of fix years after the expiration of the above-mentioned term of 30 years, and in the fecond period of fix years after the expiration of the faid term, and in the third period of fix years after the expiration of the faid term, will be as follows.

£ 560,183.4483,	£ 3,699,027.2161,	£ 5,681,759.3271,
1,125,108.1778,	4,130,426.9433,	5,871,272.9294,
1,675,098.8676,	4,523,171.9051,	6,037,802.0431,
2,214,673.7993,	4,879,594.2854,	6,161,878.0897,
2,742,530.6375,	5,176,408.9871,	6,247,325.0902,
3,226,506.8236.	5,443,172.5869.	6,278,330.2407,
11,550,101.7541	27,851,801.9239	36,278,367.7202

And the amounts of the favings made in the fourth period of fix years after the expiration of the faid term of 30 years, and in the fifth and laft period of fix years after the expiration of the faid term, will be as follows.

£ 6,260,443.8856,	£ 5,654,532.8589,
6,216,728.7363,	5,493,688.2149,
6,149,887.2735,	5,323,238.3055,
6,062,432.5145,	5,144,679.9480,
5,940,747.3522,	4,952,394.9392,
5,804,146.5943.	4,780,752.1500.
36,434,386.3564	31,356,286.4165

Z z 2

And

And the sum total of these five several sums, or the sum total of the amounts that will arise from all the savings made at the ends of all the years of the said term of 60 years after the 30th year, at the end of the said term of 60 years, is £143,470,944.17 11.

£	
11,550,101.7541,	
27,851,801.9239,	
36,278,367.7202,	
36,434,386.3564,	
31,356,286.4165,	
143,470,944.17 11.	

The quantity of the capital of the national debt that will have been extinguished, by means of all the foregoing savings, at the end of the said term of 60 years.

The quantity of the interest of the national debt that will be then redeemed to the publick by means of the said savings.

The whole quantity of the interest of the national debt that will be redeemed to the publick, at the end of the said term of 60 years, by the foregoing operation.

CCCXVII. Therefore the quantity of the capital of the national debt that would have been extinguished, by means of all these savings, at the end of the said term of 60 years, will be equal in value to the said sum of £143,470,944.17 11; of which the annual interest at 4 per cent. $\left(\frac{4}{100} \times £143,470,944.17 11, \text{ or } \frac{573,883,776.6844}{100}, \text{ or } £5,738,837.7668\right)$

Therefore the quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of the said interest, that would be redeemed to the publick at the end of the said term of 60 years means of all these savings, (so that at the end of the next, or 61st year, it would be free to be disposed of by the Parliament in any other way,) is £5,738,837.7668 *per annum*. Q.E.I.

CCCXVIII. This sum of £5,738,837.7668 *per annum* must therefore be added to £3,780,753 *per annum*, which was the quantity of the perpetual annuities, now due to the publick creditors, which was supposed to be converted into life annuities of the kind above-described for the lives of 69400 persons, all of the age of 35 years, and which will therefore now wholly redeemed to the publick by the deaths of all the said life annuitants: and the sum of both, or the sum of £9,519,590.7668, (neglecting the fraction .7668) £9,519,590 *per annum* will be the whole quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the said term of 60 years, (so as to be free at the end of the next, or 61st, year to be disposed of by the Parliament in

other way,) by the faithful application of a million of pounds, sterling, *per annum* to that salutary purpose, during that period, according to the method above-described. Q.E.I.

CCCXIX. This sum of £9,519,590 *per annum*, which is the quantity of the interest of the national debt that would be redeemed to the publick the course of 60 years by the application of a million of pounds, sterling, *per annum* to that purpose in this sixth method, or in the way of life-annuities with a right of succession by survivorship to the annuities of the exalted life-annuitants for a term of 30 years, is the same with the quantity of the interest of the said debt that would have been redeemed to the publick in the same time by applying the same sum of a million of pounds *per annum* to the same purpose in either of the five former methods. For the sum that would have been so redeemed in either the first or the second method is £9,519,624 *per annum*; which differs from the present sum of £9,519,590 *per annum* by only £34 *per annum*, which, upon so large a sum as £9,519,624, is perfectly inconsiderable. And the quantities of the said interest that would be redeemed by the application of the same sum of a million of pounds *per annum* to the same purpose during the same period of sixty years, in either the third, or the fourth, or the fifth, method above-mentioned, have been before shewn to be very nearly equal to the same sum of £9,519,624 *per annum*. Therefore the quantity of the said interest which would be redeemed by this sixth method may be considered as equal to that which would be redeemed in the same time by the same annual sum of money in either of the five preceding methods; agreeably to what is asserted above in Art. CCXCIX. Q.E.D.

CCCXX. This sixth method of employing a given sum of money every year in the diminution of the national debt, “by means of the above-mentioned life-annuities with the benefit of survivorship during a certain number of years,” partakes in a greater degree than the fourth method, (which consisted in establishing life-annuities without the benefit of survivorship,) of the advantage mentioned in Art. CCIV as belonging to the third method, to wit, that of securing to the publick the faithful application of such annual sum to the purposes of its destination, without suspension or interruption. For it would then be impossible to withhold any part of the original life-annuities so created, but also any part of the additions that would accrue to the surviving life-annuitants every year by the deaths of some of their companions in the course of the given term of years during which the said benefit of survivorship had been granted

A comparison between this sixth method and the third and fourth methods above-mentioned, with respect to the advantage of securing to the publick the faithful application of the said annual sum of money to the purposes of its destination.

granted to them, without an absolute breach of the publick faith; which (as we observed above in Art. ccliv) may be considered as a moral impossibility. And consequently during the whole of the said term throughout which such benefit of survivorship was to continue, the annual sum of money allotted to the purpose of diminishing the national debt would be tied down to the purpose of its destination. But, when the said term was at an end, the subsequent savings that would arise from the deaths of the life-annuitants who would die after the said term, would be as liable to be diverted, by the ministers of state and the parliament, from its original destination of diminishing the national debt, and to be employed in defraying the expence of some temporary measure of Government, as the money that should be allotted to the same purpose, of diminishing the national debt, in either the first or the second method above-described: where in the *third* method of applying a given sum of money every year to the diminution of the national debt, "by converting some of the perpetual annuities that now pay the interest of it, into greater temporary annuities that should continue for a certain limited number of years, and allowing the said annual sum to the payment of the additions thereupon made to the perpetual annuities so converted," the whole of the said annual sum, effectually secured from being diverted from its original and proper destination, and applied to any other purpose whatsoever, during the whole of the said term.

A seventh method of employing a given sum of money every year to the reduction of the national debt, by means of life-annuities, which should continue at all events for a certain term of years, and, in case of the deaths of the life-annuitants, should become payable to their executors, or other personal representatives, during the remainder of the said term, and which should afterwards be accompanied with the benefit of survivorship during a second term of years.

CCCXXI. A seventh method of applying a given sum of money every year to the diminution of the national debt would be to combine the foregoing fifth and sixth methods together in the following manner.

A portion of the perpetual annuities, now due to the publick creditors might (with the consent of the proprietors of such annuities) be converted into equivalent life-annuities of the following kind. The said annuities should continue for a certain moderate term of years at all events, so that in case the proprietors themselves should die before the expiration of the said term, the annuities should be payable to the executors, or other

final representatives, of the said proprietors during the remaining years of it. At the end of the said term of years the annuities payable to the executors of the deceased proprietors should cease; but those that were payable to the surviving proprietors should continue during their lives, (as in the fifth method above-described,) and should also, during a second term of years, increase continually (as in the last, or sixth, method above-described) by a division of the annuities of the life-annuitants that died in every year after the end of the first term, amongst their surviving companions. And after the end of this second term of years there should be no further increase of the said life-annuities by survivorship; but the then surviving life-annuitants should receive during the remainder of their lives the same annuities as they had received at the end of the last year of the said second term. And the savings that would accrue to the publick after the end of the said second term by the deaths of the few remaining life-annuitants, should be applied, as fast as they arose, to the diminution of the national debt in either the first or the second method above-described.

Thus, for example, if the life-annuitants for whose lives these annuities were to be granted, were all of the age of 35 years, and consequently a few of the longest liverers of them would (according to Monsieur de Parcieux's table of the probabilities of the duration of human life,) live to almost the end of a period of 60 years, the said period of 60 years might be divided into three parts, to wit, 1st, a term of 30 years, 2dly, a term of 20 years, and, 3dly, a term of 10 years. And during the first of these terms the annuities might be made payable not only to the said life-annuitants themselves, if they were living, but, in case of their decease, to their executors, or other personal representatives; but at the end of the said first term of 30 years the payments of these annuities to the representatives of the said deceased life-annuitants should intirely cease, and only those due to the surviving life-annuitants should be continued. And during the second period of 20 years the surviving life-annuitants should not only continue to receive their original life-annuities, but should succeed, by survivorship, to the life-annuities of such of their companions as should die in the course of the said 20 years; by which means the annuities of the few persons who should live to the end of the said 20 years, or to the age of 85 years, would be increased in the proportion of the number of the said life-annuitants who would be living at the said age of 85 years to the number of them that had been living at the beginning of the said term of 20 years, or at the age of 65 years, that is (according to Monsieur de Parcieux's table,) in the proportion of 48 to 395, or something more than the proportion of 1 to 8. At the end of the said second term of 20 years this benefit of survivorship should cease, and the few life-annuitants that should live beyond that term should continue to receive the same annuities during the remaining years of their lives as they had received at

the end of the last year of the said term of 20 years, or when they were of the age of 85 years: and the savings that would then arise every year by the deaths of some of the said life-annuitants, should accrue to the public, and be employed in the diminution of the national debt in either the first or the second method above-described.

And, if this method of diminishing the national debt were adopted, the annual sum of money that could be spared out of the publick revenue for this purpose, or a part of that sum, must be employed in paying the additions which it would be necessary for the Government to make to the perpetual annuities, now due to the publick creditors, which should be converted into life-annuities of the kind just now described, in compensation of the abridgement of the time of their continuance.

This seventh method of diminishing the national debt, if properly pursued, would be equally effectual for that purpose with either of the six preceding methods.

CCCXXII. This seventh method of applying a given sum of money every year to the diminution of the national debt would have exactly the same effect in the course of any given period of time as either of the first preceding methods; or the quantity of the interest of the national debt, or of the perpetual annuities now employed in paying it, which would be redeemed to the publick in any given number of years by employing a given sum of money every year for that purpose according to this seventh method, would be the very same that would be redeemed to the publick in the same period of time by employing the same sum of money every year for the same purpose according to either of the preceding methods. Of this I shall now proceed to give an example.

An example of the said seventh method of employing a given sum of money every year in the reduction of the national debt.

Preliminary
suppositions.

CCCXXIII. Let the sum of money that is to be employed every year for the purpose of diminishing the national debt according to this seventh method, be, (as in the foregoing examples,) a million of pounds, sterling, *per annum*; and the period during which it is to be so employed, be also (as before,) a term of 60 years. Let the life-annuitants for whose lives the new life-annuities, of the compound nature above-described, are to be granted, in lieu of certain portions of the perpetual annuities, now due to the publick creditors, (which are thereupon, with the consent of the proprietors of them, to cease,) be all of the age of 35 years: and let them be supposed to die off every year in the proportions set forth in Monsieur de Parcieux's table of the probabilities of the duration of human life; in consequence of which it must be supposed that some few of them

will live almost throughout the whole of the said period of 60 years, and die a few days before the end of it. Also let the number of life-annuitants for whose lives these life-annuities are granted, be (as before,) 69,400.

Further, let the first term, during which the life-annuities of the deceased life-annuitants are to be paid to their executors, or other personal representatives, be a term of 30 years; and the second term, during which the life-annuitants, who shall outlive the first term, shall be intitled to succeed by survivorship to the annuities of such of their companions as shall die in the course of the said second term, be 20 years. And, lastly, let the interest of money be supposed to be 4 per cent. and the perpetual annuities of 4 per cent. now due to the publick creditors, be supposed to sell at their *par*, or nominal value; and the perpetual annuities of 3 per cent. now due to the publick creditors, be supposed to sell at the price of $\frac{1}{2}$ per cent.

These things being premised, we must now proceed to inquire what quantity of the perpetual annuities, now due to the publick creditors, the government would be able to convert into life-annuities of the compound kind just now described, by means of the said sum of a million of pounds, sterling, *per annum*, allotted to that purpose during the space of 60 years. Now this quantity may be determined by proceeding in the manner following.

CCCXXIV. The number of life-annuitants of the age of 35 years, to whom these life-annuities are supposed to be granted, is 69,400. If therefore we suppose the annuities granted to them to be £10 a year a piece, the annual expence to the Government arising from these grants would be £694,000. And this expence would continue without any diminution during the whole of the said first term of 30 years, notwithstanding the numbers of the said life-annuitants who would be continually dying in every year of the said term; because during the whole of the said term the annuities of the deceased life-annuitants are to be paid to their respective executors, or other personal representatives.

An investigation of the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into these compound life-annuities, with a term of 30 years certain, and the benefit of survivorship during a second term of 20 years, by the help of a million of pounds, sterling, *per annum* allotted to that purpose during a period of 60 years.

CCCXXV. But at the end of the said term of 30 years a considerable sum would accrue every year to the publick from the said annuities of the life-annuitants who had died in the course of the said term; because when the said annuities are no longer to be paid to the representatives of the deceased life-annuitants. This saving will be equal to £10 multiplied into the number of life-annuitants who will have died in the course of the said

A a a

30 years.

30 years. Now it appears from Monsieur de Parcieux's table of the probabilities of the duration of human life, that the said original 69,400 life annuitants, of the age of 35 years, for whose lives the said annuities of £10 each were supposed to be granted, will, in the course of 30 years, be reduced to 39,500. Therefore the number of the said life-annuitants who will have died in the course of the said 30 years is (69,400 — 39,500 or) 29,900: and consequently the saving which will accrue every year to the publick, after the expiration of the said term of 30 years, in consequence of the deaths of the said life-annuitants, who will have died in the course of the said term, will be (29,900 x £10, or) £299,000. And therefore the sum which the Government will be obliged to pay, in consequence of the grants of these annuities, after the expiration of the said term of 30 years, will not, as before, be £694,000 a year, but only (£694,000 — £299,000, or) £395,000 a year. And this expence of £395,000 *per annum* will continue the same during the whole second term of 20 years, notwithstanding the deaths of several of the said life annuitants in every year of the said term; because it is supposed that the annuities of the several life-annuitants who shall die in the course of the said second term are to be divided amongst their surviving companions.

CCCCXXVI. But after the expiration of this second term of 20 years the annual expence of the Government, in consequence of the grants of these annuities, will grow less and less every year; because then the annuities of the life-annuitants who shall die in the remaining 10 years of the said period of 60 years are no longer to be enjoyed, by right of survivorship, by their few surviving companions, but are to accrue to the publick.

CCCCXXVII. In the course of the said second term of 20 years the life-annuity of annuities of the life-annuitants who shall have lived to the end of it, will have increased from £10 *per annum* to £82.291,666 *per annum*. For the number of the said life-annuitants will have decreased in the said term from 39,500 to 4,800: and consequently the sum of £395,000, which every year to be divided among them, will, at the end of the last year of the said term, be divided amongst only 4,800 persons; and therefore the annuity then paid to each of them will be = $\frac{£395,000}{4,800} = £82.291,666$. And this annuity, of £82.291,666 *per annum*, each of the said 4,800 surviving life-annuitants will be intitled to receive during the remainder of his life, but without any further increase of it by survivorship.

CCCXXVIII. In order to discover the value of one of these compound life-annuities of £10 *per annum*, it will be convenient to divide it into the three following parts, to wit, 1st, an annuity of £10 a year for a term of 30 years, to be certainly paid either to the life-annuitant, or, in case of his decease, to his personal representative; and, 2^{dly}, A remote annuity of £10 a year, depending upon the life of the life-annuitant, so as not to take place unless the said life-annuitant shall be then alive, and afterwards to cease immediately in case of his death; and which, even in the other event, of his living, shall continue only during 20 years; but which during the said 20 years shall be accompanied with the advantage of having the life-annuitant to succeed, by survivorship, to a share of the annuities of such of his fellow life-annuitants as shall die in the course of the said 20 years; and 3^{dly}, A remote life annuity of £82.291,666 *per annum*, that is to commence at the distance of 50 years, or whereof the first payment is to be received at the end of 51 years. The values of these three annuities may be determined in the manner following.

Division of a compound life-annuity of 10 pounds a year, for the life of a person of the age of 35 years, with a term certain of 30 years and the benefit of survivorship during another term consisting of 20 years, into three parts.

CCCXXIX. In the first place, it appears from Mr. Smart's fourth table of compound interest, page 78, that the value of an annuity of one pound a year for 30 years certain, when the interest of money is 4 per cent. is £17.292,033,30. And consequently the value of an annuity of £10 a year for 30 years certain is £172.920,333. Q.E.I.

A computation of the value of the first part of the said compound life-annuity of 10^{l.} *per annum*.

CCCXXX. Secondly, since during the whole second term of 20 years the Government is every year to pay to the surviving life-annuitants the sum of £395,000 *per annum*, it is evident that the price which the Government ought to receive from all the 69,400 original life-annuitants together or this annual payment of £395,000 during the said second term of 20 years, must be the same as if the same annual payment were to be made to them during the same term of 20 years in the more simple form of a remote annuity for a term certain of 20 years, that was to be paid either to them or their executors, or other personal representatives, during the said term, and was to commence at the end of 30 years, or whereof the first payment was to be received at the end of 31 years. Now the value of remote annuity of £395,000 *per annum* for a term of 20 years, that is to commence at the end of 30 years, is equal to the excess of the value of an immediate annuity of the same magnitude, or £395,000 *per annum*, for a term of 50 years above the value of an immediate annuity of the same magnitude for a term of 30 years. Now it appears from Mr. Smart's fourth table of compound interest, page 78, that the value of an immediate annuity of £395,000 *per annum* for a term of 50 years, when the interest

A computation of the value of the second part of the said compound life-annuity of 10^{l.} *per annum*.

of money is 4 per cent. is £395,000 $\times$ 21.482,184.62, or £8,485,462.924, and that the value of an immediate annuity of £395,000 *per annum* for term of 30 years is £395,000 $\times$ 17.292,033.30, or £6,830,353.153. Therefore the value of a remote annuity of £395,000 *per annum* for a term of 20 years, that is to commence at the end of 30 years, is equal to £8,485,462.9249 — £6,830,353.1535, or £1,655,109.7714. Therefore the sum which all the 69,400 life-annuitants, taken together, ought to pay to the Government for the said second annuity of £395,000 *per annum* which they are to receive amongst them during the said second term of 20 years, which is to commence at the end of the first term of 30 years, is £1,655,109.7714; and consequently the sum which each of the said 69,400 life-annuitants ought to pay for his contingent share of the said annuity of £395,000 *per annum* during the said second term of 20 years, accompanied with the benefit of survivorship during the said term, in the manner above-described, is $\frac{£1,655,109.7714}{69,400}$, or £23,848,843. Q.EI.

A computation of the value of the third part of the said compound life annuity of 10l. *per annum*.

CCCXXXI. Lastly, the value of a remote life-annuity of £82,291.66 *per annum*, for the life of a person of the age of 35 years, that is to commence at the end of 50 years, or whereof the first payment is to be received at the end of 51 years, may be determined by the help of Art. CCXLI in the manner following.

The number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life as living at the age 35 years, is 694; and the number of persons represented in the same table as living at the age of 85 years, or at the end of 50 years, is 48. Therefore, if $\frac{£}{N}$ be put for the value of an immediate annuity of one pound a year for the life of a person of the age of 85 years, it follows from Art. CCXLI that the value of a remote annuity of one pound a year for the life of a person of the age of 35 years, that is to commence at the end of 50 years, or when the said person shall be 85 years old, will be equal to $\frac{48}{694} \times \frac{1}{r^{50}} \times \frac{£}{N}$. Now $\frac{£}{N}$, or the value of an immediate annuity of one pound a year for the life of a person of the age 85 years, when the interest of money is 4 per cent. appears above Table xvi, page 225, to be = £2.424,216. And it appears by Mr. Smart's second table of compound interest, pages 60 and 62, that, when the interest of money is 4 per cent. or r is = 1.04, the fraction $\frac{1}{r^{50}}$

$\frac{1}{104}$, is = .140,712,62. Therefore $\frac{48}{694} \times \frac{1}{104} \times \frac{1}{2}$ is =

$\frac{48}{694} \times .140,712,62 \times \frac{1}{2} = \frac{48}{694} \times 34,117,784,805,92, \frac{1}{2} =$

$\frac{16,373,653,670,684,16}{694} = \pounds 0.023,593,160$; that is, the value of a

remote life-annuity of one pound a year for the life of a person of the age of 35 years, that is to commence at the end of 50 years, or when the said person shall have attained the age of 85 years, or so that he shall receive the first payment thereof at the age of 86 years, is = $\pounds 0.023,593,160$. Therefore the value of a remote life-annuity of $\pounds 82,291,666$ *per annum* for the life of a person of the age of 35 years, that is to commence at the end of 50 years, or when the said person shall have attained the age of 85 years, is = $82,291,666 \times \pounds 0.023,593,160 = \pounds 1,941,520,442$. Q.E.I.

CCCXXXII. Therefore the sum of the values of these three annuities, The value of the value of the whole life-annuity of $\pounds 10$ a year supposed to be granted to each of the aforesaid 69,400 life-annuitants of the age of 35 years, is = $\pounds 172,920,333 + \pounds 23,848,843 + 1,941,520 = \pounds 198,710,696$. Q.E.I.

CCCXXXIII. The annual interest of $\pounds 198,710,696$, at 4 per cent. is $\left(\frac{4}{100} \times \pounds 198,710,696, \text{ or } \frac{\pounds 794,842,784}{100}, \text{ or } \right) \pounds 7,948,427,84$.

Therefore a perpetual annuity of $\pounds 7,948,427,84$ *per annum* is equal in value to a life-annuity of $\pounds 10$ *per annum*, of the compound kind above-described, for the life of a person of the age of 35 years. Therefore, if each of the above-mentioned 69,400 life-annuitants of the age of 35 years, to whom we have supposed life-annuities of $\pounds 10$ each, of the compound kind above-mentioned, to have been granted, had been possessed of a perpetual annuity in the public funds, of $\pounds 7,948,427,84$ *per annum*, and were to resign, or relinquish, to the publick his right to such perpetual annuity, in exchange for, and as the price of, the said life-annuity of $\pounds 10$ a year, of the compound kind above-described, such a bargain would be fair and equal on both sides. Therefore the sum of all the said 69,400 life-annuities of $\pounds 10$ each, added together, would be equal in value to 69,400 perpetual annuities of $\pounds 7,948,427,84$ *per annum*, or to a portion of the perpetual annuities, now due to the publick creditors, of 69,400 $\times \pounds 7,948,427,84$, or $\pounds 551,620,892,096$ *per annum*. In order therefore to convert a portion of the perpetual annuities, now due to the publick creditors,

creditors, amounting to £551,620.892,096 *per annum*, into life-annuities of £10 each, of the compound kind above-described, it would be necessary for the Government to apply to this purpose an additional sum of money every year, during the space of 30 years, that is equal to the difference between £551,620.892,096, and £694,000, or that is equal to £142,379.107,904. And consequently, since an additional sum of £142,379.107,904 *per annum* would be sufficient to enable the Government to convert into life-annuities, of the compound kind above-mentioned, for the lives of persons of the age of 35 years, a portion of the perpetual annuities, now due to the public creditors, amounting to £551,620.892,096 *per annum*; we may conclude that an additional sum of a million of pounds every year would be sufficient to enable the Government to convert into life-annuities of the same kind, for the lives of persons of the same age of 35 years, a portion of the said perpetual annuities that is greater than £551,620.892,096 *per annum* in the same proportion in which £1,000,000 is greater than £142,379.107,904, that is, a portion of the said perpetual annuities amounting to $\frac{1,000,000}{142,379.107,904} \times £551,620.892,096$, or to $\frac{551,620,892,096.000,000}{142,379.107,904}$, or) £3,874,310.635,995, or £3,874,310.636 *per annum*. This therefore is the quantity which was proposed in Art. CCCXXII as necessary to be determined in order to ascertain the effect of the application of a million of pounds, sterling, *per annum* to the diminution of the national debt, during a period of 60 years, according to this seventh method.

Of the original quantity of each man's life-annuity, upon a supposition that the whole sum of £3,874,310 *per annum* of the perpetual annuities is converted into these compound life-annuities, and that the number of the life-annuitants be supposed to be equal to each other.

CCCXXV. The annuitants continues, as before, to be 69,400.

CCCXXV. The

CCCCXXV. Then, since the sum that is to be paid every year by the Government to the said 69,400 life-annuitants, or their several executors, or other personal representatives, during a term of 30 years, is $\pounds 3,874,310 + \pounds 1,000,000$, or $\pounds 4,874,310$; and this sum is to be equally divided between all the said 69,400 life-annuitants, or their representatives; it follows that the annuity which each of the said life-annuitants, or his representative, will be intitled to receive at the end of every year during the said term of 30 years, will be $\frac{\pounds 4,874,310}{69,400}$; or $\pounds 70.235,014$ *per annum*.

CCCCXXVI. At the end of the said term of 30 years the said 69,400 life-annuitants will be reduced to 39,500; and in the course of 20 years more the said 39,500 life-annuitants will be further reduced to 4,800. But it is supposed that during this latter term of 20 years the benefit of survivorship is to take place between the said life-annuitants. Therefore the same sum of money which was divided, at the end of the first year of the first term of 30 years, amongst the 39,500 life annuitants who were then living, will, at the end of the last year of the said second term of 20 years, be divided amongst only 4,800 persons. That sum $\pounds 39,500$ times $\pounds 70.235,014$, or $\pounds 2,774,283.053,000$. Therefore $\pounds 2,774,283.053,000$, or $\pounds 577.975,636$, will be the sum which each of 4,800 the said 4,800 surviving life-annuitants will be intitled to receive at the end of the said second term of 20 years, or when he shall have attained the age of 85 years. And this sum of $\pounds 577.975,636$ each of the said 4,800 surviving life-annuitants will be intitled to receive likewise at the end of every following year during his life; but without any further augmentation.

CCCCXXVII. We must now inquire what are the savings which will accrue to the publick by the deaths of any of the said life-annuitants any part of the said period of 60 years.

Of the savings that will accrue to the publick in the course of the whole period of 60 years.

Now it is evident that during the whole of the first term of 30 years the deaths of the several life-annuitants who will die in the course of the said term, will produce no saving to the publick; because the executors, or other personal representatives, of the deceased life-annuitants will be intitled to their annuities. But at the end of the said term of 30 years a great saving will accrue to the publick all at once. For then the annuities of the deceased life-annuitants are no longer to be paid to their representatives, The first and greatest saving will be at the end of the first term above-mentioned, consisting of 30 years.

This saving
will be
£2,100,026.
918,600 per
annum.

representatives, but are to become the property of the publick and to be employed in the reduction of the national debt in either the first or the second method above-described. The number of life-annuitants that will have died in the course of the said term of 30 years is the difference between 69,400, the original number of the said life-annuitants, and 39,500 the number of them who will be living at the end of the said term of 30 years; which difference is 29,900. Therefore the saving that will accrue to the publick after the expiration of the said term of 30 years will be 29,900 annuities of £70.235,014, or the sum of £2,100,026,918,600. This saving will accrue to the publick at the end of the 31st year; and the like saving will accrue to the publick at the ends of the 32d year, the 33d year, the 34th year, the 35th year, and of every following year, during the whole period of 60 years, to the last year of it inclusively; that there will be thirty such savings of £2,100,026,918,600, in the said time. Now all these savings are to be employed, as fast as they arise, for the purpose of diminishing the national debt, in either the first, or the second, method above-mentioned: and consequently the quantity of the capital of the national debt that will have been extinguished by them in the course of the said latter thirty years of the said period of 60 years, will be equal in value to the sum that would be produced by means of an annual revenue of £2,100,026,918,600 that should be received for 30 years together, and should, at the end of every year, or as soon as it was received, be immediately lent out at interest at the rate of 4 per cent together with the interest that had been already produced by it; which sum, according to Mr. Smart's third table of compound interest, page 70, is 56,084,937,76 $\times$ £2,100,026,918,600, or £117,779,879,024,005,586,336 or (neglecting all the decimal fraction .024,005,586,336, except the four first places of figures,) £117,779,879,0240. Therefore the quantity of the capital of the national debt which will be extinguished at the end of the said period of 60 years by means of these 30 annual savings of £2,100,026,918,600 each, which will accrue to the publick at the ends of the latter 30 years of the said period, will be equal in value to the sum of £117,779,879,0240; and consequently the quantity of the interest of the said debt, or of the perpetual annuities that are now employed in paying the whole period of it, which will be redeemed to the publick at the end of the said period of 60 years, will be equal in value to

60 years by means of the said savings, will be $\left(\frac{4}{100} \times £117,779,879,0240\right.$
 $£117,779,$
 $879,0240.$
 And the quantity of the interest of the said debt that will then be redeemed to the publick by means of the said saving, will be £4,711,195,1609.

or $\frac{£471,119,516,0960}{100},$ or) £4,711,195,1609.

CCCXXXVIII. Besides this very large saving of £2,100,026.918,600 Of the savings that will accrue to the publick at the end of the first 30 years in consequence of the deaths of the 29,900 life-annuitants who will have died in the course of the said 30 years,) there will be other savings of the publick after the expiration of the said second term of 20 years, by the deaths of the 4,800 then surviving life-annuitants in the course of the remaining 10 years of the said period of 60 years. We must therefore now inquire what these savings will amount to in each of the said 10 years.

CCCXXXIX. Now, in order to discover what these savings will amount to in each of the said 10 years, it will be necessary to determine, by the help of Monsieur de Parcieux's table of the probabilities of the duration of human life, what numbers of the said 4,800 life-annuitants who are supposed to be living at the end of the said second term of 20 years,) will die in each of the said 10 years. Now it appears by Monsieur de Parcieux's table that, out of 4,800 persons of the age of 85 years, all living at the same time, the numbers living at the subsequent ages of 86 years, 87 years, 88 years, 89 years, &c. will be as follows; to wit,

3,800,	700,
2,900,	400,
2,200,	200,
1,600,	100,
1,100,	and 000,

Therefore the numbers of the said 4,800 life-annuitants (who will have lived to the end of the said second term of 20 years, or to the age of 85 years,) that will die in the 10 succeeding years, will be as follows; to wit,

1,000,	400,
900,	300,
700,	200,
600,	100,
500,	and 100.

The savings accruing to the publick at the ends of each of the said ten years, by the deaths of the said life-annuitants.

CCCXL. It has been shewn above in Art. cccxxxvi, that each of the said 4,800 life-annuitants (who will be living at the end of the second term of 20 years, or at the age of 85 years,) will be intitled to receive at the end of every following year of his life the sum of £577.975,636. Therefore the saving that will accrue to the publick at the end of the first of the said 10 remaining years, by the deaths of the 1000 life-annuitants who will have died in the course of the said first year will be 1000 payments of £577.975,636 each, or the sum of £577.975,636 and the saving that will accrue to the publick at the end of the second year and the saving remaining 10 years by the deaths of the said 1000 persons who will have died in the said first year and likewise of the 900 persons who will have died in the said second year, will be 1000 + 900, or 1900, payments of £577.975,636 each, or the sum of (1900 x £577.975,636, or 1,098,153.708,400. And in like manner it may be shewn that the saving that will accrue to the publick at the ends of the 3d, 4th, 5th, 6th, and other following years of the said remaining 10 years, will be as follows; to wit,

At the end of the third year,

$$1900 + 700 \mid \times £577.975,636,$$

$$\text{or } 2,600 \times £577.975,636, = £1,502,736.6536.$$

At the end of the fourth year,

$$2,600 + 600 \mid \times £577.975,636,$$

$$\text{or } 3,200 \times £577.975,636, = £1,849,522.0352.$$

At the end of the fifth year,

$$3,200 + 500 \mid \times £577.975,636,$$

$$\text{or } 3,700 \times £577.975,636, = £2,138,509.8532.$$

At the end of the sixth year,

$$3,700 + 400 \mid \times £577.975,636,$$

$$\text{or } 4,100 \times £577.975,636, = £2,369,700.1076.$$

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At the end of the seventh year,

$$\frac{4,100}{100} + \frac{300}{100} \times £577.975,636,$$

$$\text{or } 4,400 \times £577.975,636, = £2,543,092.7984.$$

At the end of the eighth year,

$$\frac{4,400}{100} + \frac{200}{100} \times £577.975,636,$$

$$\text{or } 4,600 \times £577.975,636, = £2,658,687.9256.$$

At the end of the ninth year,

$$\frac{4,600}{100} + \frac{100}{100} \times £577.975,636,$$

$$\text{or } 4,700 \times £577.975,636, = £2,716,485.4892.$$

And at the end of the tenth year,

$$\frac{4,700}{100} + \frac{100}{100} \times £577.975,636,$$

$$\text{or } 4,800 \times £577.975,636, = £2,774,283.0528.$$

CCCXI. These savings are to be employed in diminishing the Of the quantity of the national debt in either the first, or the second, method above-mentioned: tional debt which means the portion of the national debt that will have been that will have extinguished at the end of the said period of 60 years, in consequence of been extin- th of these savings, except the last, to wit, £2,774,283.0528, (which guished at the accrue at the end of the last year of the said period,) will be equal in end of the whole amount to a sum that is greater than such saving, namely, to the sum which said period of the amount to which such saving will have increased at the end of the 60 years, by means of the period of 60 years, if we suppose it to have been improved in the said savings. an time at compound interest at the rate of 4 per cent. We must therefore, in the next place, compute the amounts of these savings at the of the said period of 60 years, if so improved at compound interest in mean time, or during the remaining years of the said term after the at which they will have respectively accrued, that is, during 9 years, 7 years, 6 years, and so on, to the last year of the said period. these amounts will be as follows.

Bbb 2

CCCXLII. The

The amounts of all the said savings, except the last, at the end of the said period of 60 years, if improved in the mean time by compound interest at the rate of 4 per cent. = £822,639.5486.

The amount of the second saving, £1,098,153.7084, improved the same manner during 8 years, is = £1,098,153.7084 × 1.368,569 = £1,502,899.1774.

The amount of the third saving, £1,502,736.6536, improved in the same manner during 7 years, is = £1,502,736.6536 × 1.315,931 = £1,977,498.9194.

And the amounts of the six following savings, improved in the same manner during the remaining years of the said period of 60 years after times at which they will have respectively accrued to the publick, be as follows.

£1,849,522.0352	×	1.265,319.02	=	£2,340,235.4090.
£2,138,509.8532	×	1.216,652.90	=	£2,601,824.2145.
£2,369,700.1076	×	1.169,858.56	=	£2,772,213.9555.
£2,543,092.7984	×	1.124,864.00	=	£2,860,633.5375.
£2,658,687.9256	×	1.081,600.00	=	£2,875,636.8603.
£2,716,485.4892	×	1.040,000.00	=	£2,825,144.9087.

The addition of the said nine amounts, together with the saving that will accrue at the end of the last, or 60th, year of the said period, into one sum.

CCCXLIII. Having thus found the amounts to which the yearly savings made at the ends of the fifty-first year and every following year of the said period of 60 years, except the last, will have increased the end of the said term by being improved in the mean time at compound interest at the rate of 4 per cent. we must now add up all these amounts together with the saving of £2,774,283.0528, (which will accrue at the end of the last year,) into one sum: which may be done as follows.

£822,639.5486

£	822,639.5486,
	1,502,899.1774,
	1,977,498.9194,
	2,340,235.4090,
	2,601,824.2145,
	2,772,213.9555,
	2,860,633.5375,
	2,875,636.8603,
	2,825,144.9087,
	2,774,283.0528.
	23,353,009.5837.

Therefore the sum of the amounts of all these ten savings at the end of the said period of 60 years, is £23,353,009.5837.

CCCXLIV. This sum of the amounts of these last ten savings made in the last ten years of the said period of 60 years, must now be added to the amount of the former great savings of £2,100,026.918,600 *per annum*, which began to accrue to the publick at the end of the first term of 30 years; which amount was shewn above (in Art. cccxxxi.) to be £117,779,879.0240. Therefore the sum total of the amounts of all the savings at the end of the whole period of 60 years will be £117,779,879.0240 + £23,353,009.5837, or £141,132,888.6077; the interest of which at per cent. is $\left(\frac{4}{100} \times £141,132,888.6077, \text{ or } £564,531,554.4308, \text{ or}\right)$

£5,645,315.5443 *per annum*. Therefore the quantity of the capital of the national debt which will have been extinguished, at the end of the said period of 60 years, by means of all the said savings, will be equal in value to the sum of £141,132,888.6077; and the quantity of the interest of the said debt, or of the perpetual annuities now employed in paying the said interest, which would be thereby redeemed to the publick at the end of the said period of 60 years, (so as to be free, at the end of the next, 61st, year, to be disposed of in any way the Parliament should think fit,) would be the sum of £5,645,315.5443 *per annum*, or (neglecting the fraction .5443) the sum of £5,645,315 *per annum*. QEI.

CCCXLV. To

The addition of the last sum total to the amount of the former great savings which began to accrue to the publick at the end of the first term above-mentioned, consisting of 30 years.

The quantity of the capital of the national debt that will have been extinguished at the end of the whole period of 60 years by means of all the savings above-mentioned.

The quantity of the interest of the said debt that will have been then redeemed to the publick by means of all the said savings.

The whole quantity of the interest of the national debt that will have been redeemed to the publick at the end of the said period of 60 years by the faithful application of a million of pounds, sterling, *per annum* according to this seventh method.

CCCXLV. To this sum of £5,645,315 *per annum*, which will thus have been redeemed to the publick at the end of the said period of 60 years by means of all the foregoing savings, we must add the sum of £3,874,310, *per annum*, which was the quantity of the perpetual annuities due to the publick creditors, which were supposed to be converted into life-annuities of the compound kind above-described; and the sum thereby arising, to wit, the sum of £9,519,625 *per annum*, will be the whole quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of the said interest, which will be redeemed to the publick at the end of the said term of 60 years, (so as to be free, at the end of the next, or 61st, year, to be disposed of by the Parliament in any other way,) by the faithful application of a million of pounds sterling, a year to that useful purpose during that period according to this seventh method.

This sum is the same with the quantity of the interest of the national debt that would be redeemed to the publick in the same time by the application of the same annual sum of a million according to either of the six foregoing methods.

CCCXLVI. This sum of £9,519,625 *per annum* of the perpetual annuities, (now due to the publick creditors,) which will be thus redeemed to the publick in the course of 60 years by means of an annual sum of a million of pounds, sterling, employed for this purpose according to this seventh method, is equal to the quantity of the said perpetual annuities which would have been redeemed to the publick in the same space of 60 years by employing the same annual sum of a million of pounds, sterling for the same purpose in either of the six preceding methods. For we have seen above that the said quantity is £9,520,000 *per annum*, or, more accurately, £9,519,627 *per annum* in the first, second, and third methods, and £9,519,599 *per annum* in the fourth method, and £9,519,852 *per annum* in the fifth method, and £9,519,590 *per annum* in the sixth method, all which quantities differ so very little from each other, that they may be considered as equal.

A comparison of this seventh method of applying a given annual sum of money to the diminution of the national debt, with the former methods of doing the same thing, with respect to the advantage mentioned in Art. 254.

CCCXLVII. This seventh method of employing a given sum of money every year in the diminution of the national debt, "by converting some of the perpetual annuities that now pay the interest of it, into life-annuities of the foregoing compound kind, or that shall continue at all events for a certain number of years (so as to be payable during the said term either to the life-annuitants themselves, or their executors, or other personal representatives,) and afterwards shall be accompanied, during the second term of years, with the benefit of survivorship, and then, after the expiration of the said second term, shall continue at their last, or greatest magnitude during the remaining years of the lives of the said life-annuitants," partakes, in a considerable degree, of the advantage mentioned in Art. CCXIV as belonging to the third method, to wit, the advantage of

turning to the publick the faithful application of such annual sum to the purposes of its destination, without suspension or interruption. For it would then be impossible to withhold any part of the life-annuities so created from the persons to whom they would be due, without an absolute breach of the publick faith; which (as we before observed,) may be considered as a moral impossibility. And this advantage will relate not only to so much of the publick revenue as will at any time be due to the life-annuitants themselves who will be still alive, (as was the case in the fourth method above-mentioned, by which some of the perpetual annuities were converted into simple life-annuities, without a concomitant term of years, or any benefit of survivorship;) but it will relate also, during the foresaid first term of years, to that part of the publick revenue which is allotted to the payment of the annuities of the deceased life-annuitants; because the annuities of such deceased life-annuitants will be due, during the said first term of years, to their respective executors, or other personal representatives: and it will relate also, during the second term of years above-mentioned, to the annuities of all the life-annuitants who are living at the beginning of the said second term, or at the end of the foresaid first term; because, during the whole of the said second term, the annuities of the life-annuitants who shall die in the course of the said second term, are to accrue to their surviving companions. But it will not relate to the great annual saving which will accrue to the publick at the end of the said first term, nor to the latter annual savings which will accrue to the publick at the end of the said second term by the deaths of the life-annuitants who shall die after the end of the said second term. For this part of the publick revenue, which will be thus saved every year the publick, will be as liable to be diverted, by the ministers of state and the Parliament, from its original destination of diminishing the national debt, as the money that should be allotted to the diminution of the national debt in either the first or the second method above-described: whereas in the third method of applying a given sum of money every year the diminution of the national debt, to wit, "by converting some of the perpetual annuities, that now pay the interest of it, into greater temporary annuities, that should continue for a limited number of years, and setting the said annual sum to the payment of the additions thereupon due to the perpetual annuities so converted," the whole of the said annual sum would be effectually secured from being diverted from its original and proper destination, and applied to any other purpose whatever, during the whole of the said term.

A remark on the seven foregoing methods of applying a given annual sum of money to the diminution of the national debt.

CCCXLVIII. These are the methods that, I imagine, the Government would be most likely to adopt, if they were earnestly to set about this good work of reducing the national debt to a more moderate quantity by setting apart and employing a given sum of money every year, out of the publick revenue, for that purpose. And I have therefore thought it might be useful to set them forth in a very full and plain manner, and illustrate the effects of each of them by an example. And, in order the better to compare these effects with one another, I have thought it best in the said examples, to suppose the same annual sum of money, to wit, a million of pounds, sterling, *per annum*, to be employed for the same number of years, to wit, a period of 60 years, in each of the said methods. And the result is, that all these methods are equally efficacious towards the great purpose of diminishing the national debt, if they are pursued with equal steadiness and fidelity.

Some of the foregoing methods could not, probably, be carried into execution to any great extent.

CCCXLIX. But, though all these methods would be equally efficacious towards the reduction of the national debt, they are not all equally fit to be adopted, or capable of being carried into execution to any great extent. For it is probable that some of them would be thought convenient by only a small number of the present publick creditors, and consequently could be adopted only to a small extent; the consent of the said creditors being necessary to all the foregoing methods, except the first. Thus, for example, the fourth method, "by which some of the perpetual annuities now due to the publick creditors, would, with the consent of the said creditors, be converted into mere life-annuities, without a term of years certain annexed to them, or any benefit of survivorship," would probably suit only a small number of the present owners of the perpetual annuities as, for instance, the owners of £180,393.5 *per annum* of the said perpetual annuities, instead of the owners of £1,803,935, *per annum* of the said annuities, as was supposed above in Art. cclix; and consequently an annual sum of only £100,000 *per annum*, and not of £1,000,000 *per annum*, could be employed in this method. And, indeed it does not seem probable that any one of the five latter methods of employing a given sum of money in the diminution of the national debt could be carried into execution to the extent supposed in the foregoing examples, or so as to employ so large a fund as a million of pounds, sterling, *per annum*; because it is not likely that any one of the said methods should suit the convenience of a sufficient number of the publick creditors, or proprietors of the perpetual annuities now due from the Government, to make such a measure practicable. But, perhaps, if the Government were to adopt all the said methods, and to propose to the people, in the first place, that a part of the said annual sum of a million of pounds, sterling, as, for instance, half of

But it seems likely that each of the five latter methods might be found practicable and convenient to a certain moderate extent.

or £500,000 *per annum*, should be employed in converting some of the perpetual annuities, now due to the publick creditors, into greater temporary annuities, that should continue only during a certain limited number of years, as, for example, 60 years, according to the third method above-described; — and, in the second place, that another part of the said annual million, as, for instance, £100,000 *per annum*, should be employed in converting a second portion of the said perpetual annuities into simple life-annuities, according to the fourth method above-described; — and, thirdly, that another part of the said annual million, as, for instance, £100,000 *per annum*, should be employed in converting a third portion of the said perpetual annuities into mixt annuities, or life-annuities, with a term of years certain annexed to them, or that should continue during the lives of certain persons of known ages, and likewise, in case those life-annuitants should die within a certain moderate number of years, (as, for instance, 30 years,) should continue during the remainder of the said term of years, and be payable to the executors, or other personal representatives, of the said deceased life-annuitants, according to the fifth method above-mentioned; — and, fourthly, that another part of the said annual million, as, for instance, £100,000 *per annum*, should be employed in converting a fourth portion of the said perpetual annuities into life-annuities, with the benefit of survivorship during a certain moderate number of years, as, for example, 30 years, according to the sixth method above-mentioned; — and, fifthly and lastly, that the remaining part of the said annual million should be employed in converting a fifth portion of the said perpetual annuities into life-annuities accompanied with both the foregoing advantages, that is, with a permanent interest in the said annuities to be enjoyed at all events by either the said life-annuitants or their representatives, during a certain moderate term of years, (as, for instance, 30 years,) and likewise, after the expiration of the said term, with the benefit of succeeding by survivorship to each other's annuities during a second term of years, as, for example, 20 years; —; I say, if the Government were to make these proposals to the people, it seems not unlikely that they might meet with the concurrence of a sufficient number of the publick creditors to enable the Government to carry them into execution to a sufficient extent to exhaust the whole of the said supposed fund of a million of pounds *per annum*. But, if they did not meet with that concurrence to so great an extent, the remaining part of the said annual million, which could not be employed in either of those five methods, might be employed, for the same good purpose of diminishing the national debt, in either the first or the second method above-described; and with equal advantage to the publick, if employed in either of those methods with equal steadiness and fidelity.

Of the disadvantage, that may arise to the public by granting life-annuities to persons of different ages for the same price.

CCCL. When life-annuities have, on former occasions, been established in England by the Government, it has, I believe, been customary to take the same price for annuities of the same yearly amount from persons of all ages, or, at least, of several different ages within certain, pretty distant, limits. Now this seems to be a very injudicious practice, and would be likely to prove disadvantageous to the nation, even though the price taken for the several annuities so granted should be a just medium between the true values of the same annuities for the oldest and the youngest lives for which they were made grantable: because it is probable that the greater part of the persons who would purchase such annuities would be nearer to the youngest age for which they would be grantable than to the oldest. Thus, for example, if a thousand annuities of 10 pounds a year each, were to be offered to sale by the Government for the lives of any persons between the ages of 20 years and 40 years at the price which is the true value of a like annuity of £10 a year for the life of a person of the age of 30 years, it is probable that the greater part of these annuities would be purchased for the lives of persons under 30 years of age, and consequently for less than their true value: and therefore the Government, or Nation, would lose more by the sale of these annuities to persons under the age of 30 years, than they would gain by the few annuities, of the same price, or class, which would be sold to persons who were above the age of 30 years, but under that of 40; and consequently they would, upon the whole, be losers by such a bargain. It would therefore, I apprehend, be more prudent in the Government, when annuities are to be granted to persons of different ages, to sell them at their true and proper prices according to the several ages of the persons for whose lives they are granted, having recourse, for the determination of the said prices, to the tables above-computed and set forth in pages 221, 222, &c.—232, and considering every such life-annuitant as being of the age to which he had attained on his last preceding birth-day.

This disadvantage would be still greater in the case of life-annuities with the benefit of survivorship.

CCCL. The disadvantage to the publick, mentioned in the foregoing article as resulting from the practice of granting life-annuities to persons of different ages at the same price, would, it is evident, be considerably greater in the case of life-annuities accompanied with the benefit of survivorship, than in the case of simple life-annuities: because in the case of survivorship those life-annuitants whose ages would have but little exceeded the youngest age of the class, and who would consequently have paid too little for their annuities, would, by means of this benefit of survivorship, if they happened to live a good number of years, enjoy much larger annuities than in the other case of simple life-annuities; and consequently the loss of the publick would be proportionably greater in the former case than in the latter. It would therefore be very highly expedient,

expedient, if the Government were to resolve to establish any life-annuities with the benefit of survivorship, to take care that there should be as many distinct classes of life-annuitants, intitled to the said benefit of survivorship with respect to each other, as there were different ages, differing from each other by a year, amongst the said life-annuitants, so that those persons only who were of the same age, or within a year of the same age, should belong to the same class, and pay the same price for their annuities. This might be done, as I imagine, without occasioning much difficulty or confusion, by dividing the whole annual sum which it was proposed to apply to the conversion of some of the perpetual annuities into life-annuities with the benefit of survivorship, into as many parts as there were different years in the ages of the life-annuitants for whose lives the said annuities were intended to be granted. Thus, for example, if it were the intention of Government that persons of any age from the age of 10 years to the age of 59 years, inclusively, should be admitted to become purchasers of these life-annuities with benefit of survivorship, and the whole annual sum intended to be employed in converting some of the perpetual annuities into these life-annuities with benefit of survivorship, was £100,000 *per annum*; it would be expedient to divide the said sum of £100,000 *per annum* into fifty equal parts of £2000 *per annum* each, and to apply one of these parts, or the annual sum of £2000, to the conversion of some of the perpetual annuities into these life-annuities with benefit of survivorship, for the lives of persons of the age of 10 years; and to apply another sum of £2000 *per annum* to the conversion of some of the perpetual annuities into these life-annuities, with benefit of survivorship, for the lives of persons of the age of 11 years; and to employ the like annual sums of £2000 each, in converting some of the perpetual annuities into these life-annuities, with the benefit of survivorship, for the lives of persons of the several ages of 12 years, 13 years, 14 years, 15 years, and every following number of years to 59 years inclusively. And the only difficulty that would attend this division of the said life-annuitants into the said fifty classes according to their ages, would be to determine what would be the proper prices of the life-annuities that should be granted to each class of life-annuitants, or what would be the quantity of the perpetual annuities, now due to the publick creditors, which the additional sum of £2000 *per annum* would enable the Government to convert into life-annuities of the kind intended, accompanied with the benefit of survivorship, for the lives of persons of each of the said fifty ages: which might be done by making a computation for each of the said ages similar to the computations entered into above in the examples of the fifth and seventh methods above-mentioned. And these computations the Government might easily procure to be made for the purpose by persons that were but moderately conversant with this doctrine of life-annuities, if they should be of opinion that these methods of diminishing the national debt by establishing life-annuities with benefit of survivorship, were worth adopting.

Of the manner in which such life annuities might be established, so as to avoid that disadvantage.

Of the frequent alienations of the Sinking Fund, that have been made for many years past.

A mistaken opinion that has sometimes been advanced in justification of the said alienations.

It has been observed above, in Article iv, that our ministers of state and our parliaments have, ever since the year 1733, (and we might have said ever since the year 1727,) gone into a most pernicious practice, of diverting the Sinking Fund (which had been appropriated in the year 1717 to the gradual discharge of the national debt,) from the original purposes of its destination, and applying it to the current services of the year. The consequence of this practice has been the increase of the national debt to a much greater quantity than it would otherwise have arrived at, notwithstanding the three wars of 1739, 1755, and 177, in which we have been unfortunately, and, perhaps, unnecessarily, engaged. Yet it has been often alledged in justification of this practice, "That it is a matter of indifference to the nation, and to the increase or diminution of its debts, whether, when any given sum of money is wanted for any particular service, the said sum be borrowed at interest, and a new tax be laid for the payment of the said interest, or whether the said sum be taken at once out of the sinking fund and no new tax be laid to supply its place." "Thus, for example, say these gentlemen, suppose it should be necessary at any time to apply the sum of a million of pounds, sterling, to some particular purpose not included in the constant and ordinary expences of the state, as, for instance, to the building some new ships of war for the Royal Navy, or purchasing naval stores; and suppose that the sinking fund produces an overplus of something more than a million of pounds, one year with another, above the necessary expences of the ordinary government in time of peace; why should we not take the said million, (which is wanted for the said purpose,) out of the sinking fund, and thereby only suspend, or rather lessen, for one year the operation of the said fund in diminishing the national debt, instead of borrowing the said sum of a million at interest of new creditors, and leaving the whole of the sinking fund to be applied to the purposes of its destination? By the former method of proceeding the national debt will not, it is true, have been diminished in the course of the year by the sum of £1,000,000, as it would otherwise have been; but then, on the other hand, it will not have been increased by a new loan of £1,000,000, as it will be in the latter case: and therefore, upon the whole, the quantity of the said debt at the end of the year will be the same in both cases." This, I believe, is the manner in which the said remark is usually stated. But it is, as Dr. Price has observed, a very great and dangerous sophism. For it is tantamount to saying, that it comes to the same thing, with respect to the diminution of the national debt, whether the Government does, or does not, raise new taxes towards defraying the extraordinary expences of the state; which, it is evident, cannot be true. If, indeed, the aforesaid million were to be borrowed at interest, and no new tax were to be laid to pay the said interest, but the said interest were to be taken

Remarks on the said mistaken opinion.

every year out of the sinking fund, the above-mentioned reasoning would be just, and such a proceeding would have exactly the same effect in regarding the discharge of the national debt, as if the Government had taken the whole million itself out of the sinking fund at once. For a million of pounds, sterling, paid at once, is equal in value to the interest of a million of pounds, sterling, to be paid every year for ever. But, when a new tax is provided to pay the interest of the new loan, the said two methods of procuring the said million, or other sum that is wanted, can be no longer equivalent to each other, but the former method, to wit, that of borrowing the money, and at the same time laying a new tax to pay the interest of it, must tend less than the other method, to wit, that of taking the whole sum at once out of the sinking fund, to regard the discharge of the national debt, and therefore will be preferable to it with respect to that important operation. This is so evident that, I confess, I am surprized it can ever have been doubted of. But, since it has been doubted of, and since it is a truth of very considerable importance, I will endeavour to illustrate it by an example: which may be done as follows.

An example, to illustrate the falshood of the opinion stated and confuted in the preceding article.

CCCLIII. Let it be supposed (as in the several examples above-mentioned,) that the interest of money is 4 per cent. and that the 4 per cent. annuities, now due to the publick creditors, sell at the publick market at their *par*, or nominal value. And let it further be supposed that the Parliament had set apart the sum of a million of pounds, sterling, year out of the sinking fund, for the diminution of the national debt; and that the said £1,000,000 *per annum* were to be faithfully applied to that purpose during a period of 20 years, in the first of the seven methods above-described, that is, in paying off some of the 4 per cent. annuities, now due to the publick creditors, at their *par*, or nominal value: by which means it is evident that a very considerable portion of the national debt will be extinguished at the end of the said 20 years. And let it be supposed that during every one of the said 20 years the Parliament shall find it necessary to employ the same sum of £1,000,000 in some other important services, and that, being unwilling to suspend, or impede, the operation of the sinking fund, they shall forbear taking this million out of it, and shall, instead thereof, borrow a million every year of new creditors, at 4 per cent. and at the same time impose every year a new perpetual tax of £40,000 *per annum* to pay the interest of the million so borrowed. And then let us inquire what will be the effect of this method of proceeding at the end of the said 20 years, with respect to the increase or diminution of the national debt. This may be done in the manner following.

CCCLIV. The

CCCLIV. The quantity of the capital of the 4 per cent. annuity which will be extinguished at the end of the said 20 years by employing the said £1,000,000 every year for that purpose in the first of the several methods above-described, will be equal to the sum to which an annuity revenue of £1,000,000 will amount at the end of 20 years, if every payment of the said revenue, together with the interest arising from the said payment, should, immediately after it is received, be lent out at interest at the rate of 4 per cent. Now it appears from Mr. Smart's table of compound interest, page 68, that this sum is = £1,000,000 $\times$ 29.778,078,58, or £29,778,078.58. Therefore the sum of £29,778,078.58 of the capital of the 4 per cent. annuities, now due to the public creditors, would be extinguished by means of the said £1,000,000 *per annum* in the course of the said 20 years; or the national debt would be diminished, by means of the said £1,000,000 *per annum*, at the end of the said 20 years, by the sum of £29,778,078.58. But on the other hand it will be increased in the course of the said 20 years by means of the aforesaid loans of a million each, (which we have supposed to be made every year with new funds to pay the interest of them,) by only the sum of £20,000,000. Therefore it will have been more diminished than increased, and consequently will, upon the whole, have been diminished in the course of the said 20 years, by the excess of £9,778,078.58 above £20,000,000, that is, by the sum of £9,778,078.58; which is not very much less than ten millions. Such will be the good effect, with respect to the diminution of the national debt, of the said first method of proceeding, "by borrowing the money that is wanted every year, and laying new taxes to pay the interest of it."

CCCLV. But, if the other way of proceeding be adopted, or if a million of pounds, that is wanted every year for the supposed service be taken out of the sinking fund, it is evident, the operation of the said million, in diminishing the national debt, will be thereby totally suspended and the said debt will be exactly the same at the end of the said 20 years as at the beginning of them.

CCCLVI. Therefore the difference between the quantity of the national debt at the end of 20 years, if the former method be taken and the quantity of it at the same time, if the latter method be adopted is no less a sum than £9,778,078.58, or not much less than ten millions of pounds. Q.E.I.

CCCLVII. If we had taken a longer period than 20 years, the difference of the two quantities of the national debt at the end of it, arising from these different methods of raising the said annual sum of a million of pounds, sterling, would have been much greater than £9,778,078.58. In a period of 30 years it would have been (£56,084,937.76 — £30,000,000, or) £26,084,937.76; in a period of 40 years it would have been (£50,025,515.72 — £40,000,000, or) £10,025,515.72; in a period of 50 years it would have been (£52,667,083.68 — £50,000,000, or) £2,667,083.68; and in a period of 60 years it would have been (£57,990,685.24 — £60,000,000, or) £177,990,685.24; as will appear from Mr. Smart's third table of compound interest, pages 68, 70, and 72.

Other examples for the same purpose.

CCCLVIII. The difference between these two methods of raising the money that may from time to time become necessary to defray the extraordinary expences of the state, is a matter of such importance that it can hardly be too much insisted on. And it seems to have been well understood, and properly attended to, by our ministers of state in the reign of king George the 1st, when the sinking fund was first established. At that time it was customary for the Parliament to borrow money to supply the occasional exigencies of the state, and lay new taxes to pay the interest of the money so borrowed, instead of taking the said money out of the sinking fund and thereby interrupting the operation of that useful establishment. Of this salutary practice Dr. Price in his excellent *Appeal to the Publick on the Subject of the National Debt*, 2d edition, published in the year 1772, page 29, note b, has given us several examples. He there informs us, that

The fallshood of the said mistaken opinion seems to have been well understood in the reign of king George the 1st.

In 1718 was borrowed towards the supplies	—	—	—	£505,995;
In 1719	—	—	—	312,737;
In 1720	—	—	—	500,000;
In 1721	—	—	—	1,000,000;
In 1725	—	—	—	500,000;
In 1726	—	—	—	370,000;
In 1727	—	—	—	1,750,000.

The Sinking Fund was faithfully applied to the discharge of the national debt to the end of that reign.

In all these instances (if I understand the doctor rightly,) the Parliament not only borrowed these several sums, rather than take them out of the sinking fund, (as they might have done,) but laid new taxes to pay the interest of them. And in the year 1726, (the nation being then under the apprehension of a war,) the land-tax was raised from two shillings in the pound to four shillings in the pound, rather than take any money from the sinking fund. And in the following year, 1727, (in which the nation still continued

continued

continued under the apprehension of a war,) king George the 1st in his speech at the opening of the parliament, (on the 17th of January, 1721, after congratulating the gentlemen of the House of Commons on the great addition that would be made that year to the *Sinking Fund*, warns the against being led by the NECESSITIES OF THE NATION to diversion of it. "Let all that with well (are the words of the speech) to the peace and quiet of my government, have the satisfaction to see that our PRESENT NECESSITIES shall make no interruption in the progress of that desirable work of gradually discharging the national debt. I hope therefore you will make a provision for the immediate application of the produce of the Sinking Fund to the uses for which it was so wisely contrived, and to which it stands now appropriated. To which the House of Commons in their address to the king in answer to his said speech, (which was carried, upon a debate and division, 251 voices against 81, and was presented to the king, on the 19th of January, 1722¹;) reply in the words following, to wit, "And that all will wish well to the peace and quiet of your Majesty's government, may have the satisfaction to see, that our PRESENT NECESSITIES shall make no interruption in the progress of that desirable work of gradually discharging the national debt, we will consider of the most proper methods for immediately applying the produce of the Sinking Fund to the uses for which it was so wisely contrived, and to which it stands now appropriated." By this speech of the King and address of the House of Commons it seems clear that the advantages attending a faithful application of the Sinking Fund to the purposes of its destination, were that time well understood and generally allowed.

A change of conduct in the ministers of state and the Parliament, with respect to the Sinking Fund, took place, in the year 1728.

CCCLIX. But soon after the death of king George the 1st a different opinion seems to have prevailed with many people, and, most unhappy for the kingdom, a different practice began to be adopted by the Government. Dr. Price informs us, in the passage above-mentioned, that in the years 1728 and 1729 the Sinking Fund was charged with the interest of the money borrowed in those years; which was just as pernicious a measure, with respect to the interruption of the great object of diminishing the national debt, as if the Parliament had taken the whole sums themselves at once from the Sinking Fund, instead of borrowing them: as we have already observed above in Art. cccclii, page 380, line 4 from the bottom &c. It was, however, a less open and manifest violation of the Sinking Fund than the other measure of taking the gross sums themselves out of it; and therefore probably it gave less offence to that part of the nation which had most approved of the institution of the Sinking Fund, and it contributed less to expose Sir Robert Walpole (who was charged

the Exchequer at the institution of the Sinking Fund in the month of March, 1717, and was the principal adviser and promoter of that useful measure, and who was likewise in the same great office at the time of the aforesaid misapplications of the said fund in the years 1728 and 1729,) to the charge of *intemperancy* and of becoming the destroyer of his own favourite and justly-applauded measure. And accordingly we find that, while this appearance of delicacy with respect to the violation of the Sinking Fund continued, the King and Parliament, both, spoke of it with respect, and pretended to be unwilling to break into it. For in the year 1729, when (the nation being still threatened with a war,) extra-ordinary supplies were wanted, and the Sinking Fund would have afforded all that was necessary, king George the second, in his speech to the Parliament the opening of the session, on the 21st of January, 1729, expressed himself to the members of the House of Commons in these words; And, as the produce of the Sinking Fund has exceeded our expectations, I must recommend it to your care to make a farther application of it to its proper uses. To which the Commons, in their address to the King upon this speech, make the following answer, to wit, "That they will not fail to make the proper disposition of the growing produce of the Sinking Fund." And accordingly a million of the South-Sea annuities was ordered to be paid off that year out of the Sinking Fund; and at the same time the interest of the new money that was borrowed this year, was charged upon the said fund, and became a perpetual incumbrance on it. And the same thing had been done in the preceding year, 1728, with respect to the interest of the money borrowed in that year.

The Sinking Fund was established by Stat. 3 George I, Cap. vii, Sect. 37, which is in these words. "And be it enacted and declared by the authority aforesaid, That all the monies to arise from time to time, as well of, or for, the said excises, or surpluses, by virtue of the said act made for redeeming the Funds of the Government and Company of the Bank of England; and of, or for, the said excises, or surpluses, by virtue of the said act made for redeeming the Funds of the said Government and Company of Merchants of Great-Britain trading to the South-Sea, and other parts of America, and for encouraging the Fishery; as also of and for the said excises, or surpluses, of the said duties and revenues by this act appropriated, as aforesaid; and the said overplus monies of the said general yearly fund by this act established, or intended to be established, as aforesaid; shall be appropriated, reserved, and employed, to and for the discharging the principal and interest of such national debts and incumbrances as were incurred before the five and twentieth day of December, one thousand, seven hundred, and sixteen, and are declared to be national debts, and are provided for by act of Parliament, in such manner and form as shall be directed or appointed by any future act or acts of Parliament to be discharged, with or out of the same, and to, or for, none other use or purpose whatsoever."

In the year 1733 the Parliament took a gross sum of half a million of pounds sterling, out of the Sinking Fund, in order to keep the land-tax at only one shilling in the Pound.

Dr. Price's account of the debates upon this occasion.

CCCLX. But in a few years time this appearance of delicacy in respect to the misapplication of the Sinking Fund was laid aside, and the ministers of state and parliament openly took out of it whatever gross sum of money they thought necessary for the publick exigencies. The manner in which this unfortunate change of conduct happened, is related by Dr. Price in his tract above-mentioned, intitled *An Appeal to the Publick on the Subject of the National Debt*, in these words. "In the year 1732 the land-tax had been reduced to one shilling in the pound; and, in order to supply the deficiency arising from hence, half a million had been procured for the current service by the revival of the salt-dues, which but two years before, had been repealed, because reckoned too burthensome to the poor.—In the year 1733, in order to keep the land-tax low as it had been the year before, it was necessary either to borrow another half-million, or to take it from the Sinking Fund. The latter method was chosen; and proposed by Sir Robert Walpole to the House of Commons.—Long and warm debates ensued.—A proposal to alienate in a time of profound peace, a fund which the law had made sacred and the alienation of which no possible exigence of publick affairs could justify, only for the sake of keeping the land-tax for one year at one shilling in the pound, justly kindled the indignation of the parious party. They urged, in opposition to it, the prohibition of the law, the faith of Parliament, and the security of the kingdom. The proposer of the alienation was reminded of his inconsistency and treachery in endeavouring to beat down that very monument of glory which he had boasted of having erected for himself; and Sir John Barnard warned him that he was drawing upon himself the curses of posterity.—But arguments were vain.—The ministry pleaded that the landed interest wanted ease; that there was no occasion for being in a hurry to pay the national debt; and that the circumstances of the kingdom had altered so much since the establishment of the Sinking Fund, that the competition among the publick creditors was, not who should be *first*, but who should be *last*, paid. Thus argued, among others, Sir Walpole. His reasons prevailed; and the House of Commons, not used to refuse in any thing, consented.

And the same pernicious practice has continued ever since.

"The practice of alienating the Sinking Fund having been thus begun it went on of course. In the next year, or 1734, £.200,000 was taken from it. In 1735 it was even anticipated and mortgaged.

"Thus then expired, after an existence of about eleven years, the *Sinking Fund*,—that sacred blessing—once the nation's only hope; prematurely and cruelly destroyed by its own parent.—Could it have escaped the hands of violence, it would have made us the envy and the terror of the world, by leaving us at this time [in the year 1772]

not only TAX-FREE, but in possession of a treasure greater, perhaps, than was ever enjoyed by any kingdom. — But let me not dwell on a recollection so grievous!

CCCLXI. Dr. Price informs us further in the excellent tract above-mentioned, that about the year 1726 an opinion had been propagated, notwithstanding the establishment of the Sinking Fund, the publick debts had been for some years increasing rather than decreasing. “ And this, says he, occasioned the publication of a very curious and important pamphlet in defence of the Sinking Fund and the ministry, intitled, *An Essay on the Publick Debts of this Kingdom*. I have now by me the fourth edition of this pamphlet; and I wish I could put it into every hand in the kingdom. It contains an excellent account of the importance of discharging the publick debts, and of the provisions made for that purpose by the institution of the Sinking Fund. It proves particularly, in opposition to the opinion I have mentioned, that the publick debts *had decreased*; and that, of the 52 millions then due, 50 millions would in 28 years be extinguished by the *Sinking Fund*. The same explanation is given of the nature of this fund with that which I have given: the same representation is made of its powers; and the same arguments are used to demonstrate the evil of alienating it, in order to avoid making new loans charged on new funds. — And he concludes with this general inference from all his observations, “ That the provision which had been made of the *Sinking Fund*, was an expedient from which the full and effectual payment of the principal of the national debt, in a few years, might, with great assurance, be expected.”

Dr Price's account of an excellent pamphlet on the publick debts of this kingdom, that was published in the year 1726.

CCCLXII. This pamphlet, Dr. Price further informs us, was annexed in a pamphlet intitled, *A State of the National Debt*; and this produced another pamphlet by the author of the first, (or the *Essay on the Publick Debts*,) which was intitled, *A Defence of an Essay on the Publick Debts of this Kingdom, in answer to a pamphlet, intitled, A State of the National Debt*. And in this last pamphlet, intitled, *A Defence of an Essay*,

3. there are these words. “ The success of my *Essay on the Publick Debts*, and the satisfaction of hearing from the throne, that my zeal for the preservation of the Sinking Fund cannot have been disagreeable to his Majesty, gives me a pleasure that, alone, is an ample reward for my attempts to serve my country.” From these words it seems probable that this pamphlet (of which I have now in my hands the second edition, which was published in the year 1726;) had been very much read and approved of, and had tended very much to confirm the ministers of state in their resolution

tion to continue, for that year at least, to apply the Sinking Fund to its purpose of its destination, and to induce them to advise the king to make use of the words above mentioned in his speech to the Parliament on the 17th of January, 1722, to wit, "Let all that with well to the peace and quiet of my government have the satisfaction to see, that our present necessities shall make no interruption in the progress of that desirable work of gradually discharging the national debt. I hope, therefore, we will make a provision for the immediate application of the produce of the *Sinking Fund* to the uses for which it was so wisely contrived, and which it stands now appropriated."

A conjecture concerning the author of the said pamphlet.

CCCLXIII. The aforesaid excellent pamphlet, intitled, *An Essay on the Public Debts of the Kingdom*, was published without a name. But seems probable that it was written by Sir Nathaniel Gould, an eminent merchant, who was one of the directors of the Bank in the year 1726 when the second edition (and, perhaps, also the first edition of it,) was published. And the other pamphlet above-mentioned, which was written in answer to it and was intitled, *A State of the National Debt*, seems to have been written by the famous Mr. William Pulteney, who was afterwards created Earl of Bath. This may be conjectured with a considerable degree of probability from the following extract from Chandler's Collection of the Proceedings of the House of Commons, vol. vii. page 23. "On the 23d of February, 1727, the Commons being in a committee of supply, Mr. William Pulteney [who was then in opposition to the ministry,] observed, "That, notwithstanding the great merit that some persons had built upon the establishment of the *Sinking Fund*, it appeared that the national debt had been increased since the setting up of the *Pompous Project*." Upon which Sir Nathaniel Gould, an eminent merchant, said, "That he apprehended that gentleman had his notion out of a treatise, intitled, *A State of the National Debt*, &c. supposed to be written by that very gentleman: but that, if he [Sir Nathaniel Gould] understood any thing, it was *Numbers*; and he durst pawn his credit and reputation to prove that author's calculations and inference to be false and erroneous." To this Mr. Pulteney replied, "That he took them to be right; and he would likewise pawn his credit and reputation to make good his assertion." Upon this Sir Robert Walpole took up the cudgels, and said, "He would maintain what Sir Nathaniel Gould had advanced." Several warm expressions having passed on both sides, Mr. Hungerford interposed, in a jocular speech that put the House in good humour; and so the dispute ended."

CCCLXIV. As this pamphlet, *intituled, An Essay on the Public Debt of the Kingdom*,^b (which, for the reason just now given, I conjecture to have been written by Sir Nathaniel Gould,) is, in Dr. Price's opinion, of such importance as to deserve to be put into every hand in the kingdom; and, as it tends strongly to illustrate and enforce the utility of the measure I have ventured to recommend above in Art. ccclix and cccliv, (pages 195 and 298, 299,) "of appropriating, in the strictest manner possible, some part, at least, of the Sinking Fund to the purposes of its original destination, or the gradual discharge of the national debt;" I am persuaded my readers will not be sorry to have an opportunity of perusing it; and I therefore have determined to reprint it, word for word, as a part of the present Treatise. It is as follows.

Всего в 1811 году, по числу подписавшихся, 174 человека, из коих 100 человек были из числа дворян, 74 человека из числа чиновников, 10 человек из числа купцов, 10 человек из числа мещан, 10 человек из числа крестьян, 10 человек из числа солдат, 10 человек из числа казаков, 10 человек из числа других сословий.

МОСКОВСКИЕ

Всего в 1811 году, по числу подписавшихся, 174 человека, из коих 100 человек были из числа дворян, 74 человека из числа чиновников, 10 человек из числа купцов, 10 человек из числа мещан, 10 человек из числа крестьян, 10 человек из числа солдат, 10 человек из числа казаков, 10 человек из числа других сословий.

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AN
ESSAY
ON THE
PUBLIC DEBTS
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THIS KINGDOM.

WHEREIN

the Importance of discharging them is considered; the Provisions for that Purpose by the SINKING FUND, and the Progress therein hitherto made, are stated and explained; the Sufficiency of those Provisions is demonstrated; some general Mistakes about the Nature and Efficacy of this Expedient examined and removed; and the Progress of the SINKING FUND described and computed from *Midsummer, 1727.*

To which is subjoined,

An Enquiry into the General Convenience of reducing farther the Interest of our Publick Debts below 4 *per Cent. per Annum.*

In a Letter to a Member of the House of Commons.

Printed from the Second Edition, which was published by J. PEELE, in Paternoster-Row, in the Year 1726.

This Pamphlet is supposed to have been written by Sir NATHANIEL GOULD, an eminent Merchant and a Director of the Bank.

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Итак, если вы хотите узнать, как правильно писать буквы, то вам нужно знать, что каждая буква имеет свой собственный способ написания. Например, буква "А" пишется так: сначала ставим точку, затем проводим две диагонали, соединяющие точку с нижними углами. Буква "Б" пишется так: сначала ставим точку, затем проводим вертикальную линию, а потом полукруг, который начинается с середины этой линии.

Буква "В" пишется так: сначала ставим точку, затем проводим две диагонали, соединяющие точку с нижними углами, а потом вертикальную линию, которая соединяет эти два угла.

Буква "Г" пишется так: сначала ставим точку, затем проводим вертикальную линию, а потом полукруг, который начинается с середины этой линии.

Буква "Д" пишется так: сначала ставим точку, затем проводим вертикальную линию, а потом полукруг, который начинается с середины этой линии.

Буква "Е" пишется так: сначала ставим точку, затем проводим вертикальную линию, а потом полукруг, который начинается с середины этой линии.

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PREFACE.

HERE may perhaps appear something too assuming in the attempts of an author, to inform the publick, or to direct their sentiments about matters of equal importance, to admit of a better reception or entertainment from the more than what performances of this kind have of late years generally met with. I have thought therefore, that it may not be improper to mention my reason for the publication of the following sheets, as an apology for it with the reader; to whom I can with great truth and sincerity represent, that I should not have thought any knowledge which I had, or any discoveries in my way, of our circumstances with regard to our present debts, worth the publick notice, if I had not frequently met with some mistakes on this subject, which would tend to me very generally to prevail, and firmly to be insisted on and believed to be the disadvantage of our publick credit, and which at the same time I have flattered myself, might be consulted and removed, from such informations as I should be able, on this occasion, to collect. I have so often heard it said, that our publick debts have increased upon us since the provisions made for the discharge of them, that it has sometimes seemed to me to be the more common opinion even of those persons who are most interested to be rightly informed in this particular; and have almost as often heard it from hence insisted, that those provisions are therefore insufficient to answer the expectations we supposed to have from them. And from the bad influence that the truth of this assertion, and the inference from it, must have on our publick credit, especially when it falls in with any general apprehensions for the publick or welfare on any other account, I have been induced to think, that as such a fact is not true, nor the inference from it rightly made, it would be of great convenience that they were publicly contradicted, and proved to be otherwise; and that this were better done from that less exact and partial information which I have been able to come at upon this subject, than not done at all, or only put off in expectation of its being some time done by such persons who have the exactest knowledge of our circumstances in this respect, or the best way for improving it for this purpose.

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Nor

THE PREFACE.

Nor should I have been diverted from communicating the few or part discoveries that the following sheets may be thought to contain, by being that such misrepresentations of our circumstances were made with design and by persons who better than myself knew the state of our affairs in respect; because in this case in particular, it appears to be the publick interest that the truth should be as generally known as may be, and that every person who is or may be an adventurer in our publick funds, should, as distinctly as be, understand the provisions that have been made for supporting credit of them.

What I have farther added beyond the general design, by which I was first engaged to write upon this subject, the reader will judge of on the perusal of it. I am not insensible that there are several parts of this performance to exceptions; but I have more hopes of the reader's indulgence to these faults as they shall occur to him, than I have that he will forgive the recital of them, or my detaining him by endeavouring in this place to explain away or obviate any exceptions of this kind. I shall therefore mention but two particulars, in which I may be thought more than once to have offended. One is, that I have not every where used the utmost exactness in supposing, stating, or describing the publick debts, or the variations in them. To this fault, as far as I have been guilty of it from any other cause than my want of materials that purpose, I have been chiefly induced by the views of being thereby intelligible; having presumed that it would be better to omit any such degree of exactness in this respect, as were more than sufficient to answer the general design of this essay, which would at the same time render it more tedious and perplexing. I have also, for much the same reason, been induced to mix myself with the use of some words in what has seemed to me to have been more ordinary acceptance, when applied to this subject; which in a longer and more elaborate enquiry, I should have thought myself obliged to define and explain distinctly before I ventured upon the use of them.

ESSAYS

ON THE

PUBLIC DEBTS

THIS KINGDOM.

In a LETTER to a Member of the House of Commons.

SIR,

UPON recollecting the conversation that was the occasion of your desiring my thoughts, in writing on the subject of our Public Debts; I have concluded, that I should best answer your expectations from me in this affair, by confining my thoughts,

1. To the consideration of what advantage to the publick may be reasonably expected from the discharge of those debts, and the redemption of the duties provided for the payment of their interest.

2. To an enquiry into the reasons we have at present to expect or hope that these debts, or any considerable part of them, will within any reasonable compass of time be discharged and paid off. And,

Ecc 2

3. To

3. To such reflections as have occurred to me upon those measures that may for the future be entered upon, for the more speedy and effectual discharge of our present debts, from the income of the Sinking Fund already provided for that purpose; or for still farther increasing the annual income of that fund by such reductions as may yet be made in the interest or annuities payable for the principal sums of which the present debt consists.

Of the advantages that will arise to the publick from the discharge of the national debts.

As to the first of these, or the advantage arising to the publick by the discharge of the present debts; there seems to be but little room to enlarge, after the consideration of that great annual revenue at present levied and applied to the payment of our debts, which, after the total discharge of them, will, without any loss or injury to private persons, be redeemed, and become the property of, the publick. The present yearly expence to the Government, on account of our publick debts, computing the annual income of the Sinking Fund and the yearly interest of those debts together, will be found to amount to little less than, if not to exceed, the sum of £3,000,000. A revenue exceeding the whole farther annual expence of our civil and military government in a time of peace; and which, together with the ordinary supplies which our Government require in a time of peace, may perhaps be a fund sufficient to answer our utmost probable expences during the most expensive war.

I do not think myself at liberty to suppose, or promise it as an advantage arising to the publick from the discharge of the present debt, that the several duties appropriated to the payment of them will, as far as they are redeemed, be immediately removed or determined; for reasons which in the following sheets I shall have a further occasion to mention when I shall recommend it to be considered, whether the revenues arising from those duties, or the greatest part of them, are not raised with more ease, greater equality, and more to the common benefit of the subject of Great-Britain, than some part of the supplies that are annually voted for the current service of the year; and consequently, how far it may be reasonable to substitute a great part of the revenues arising from the duties, after the redemption of them, in the place of our annual taxes. But it will, I presume, appear no small convenience to the publick, arising from the redemption of the aforesaid duties, that, when they shall be longer appropriated to the payment of our debts, the principal difficulty will be removed, which has at any time obstructed the removal or lessening of these duties, though the convenience of the publick may, upon other accounts, have persuaded to it; either as such duties may have appeared to give too great perplexity to persons employed in trade, or prevent or obstruct any profitable branch of our commerce with foreign countries; as they may have been thought to require too strict an enquiry

or too great severity or expence in the collecting them; as by being laid on any commodities universally necessary, they may have seemed too great a burthen on the poorest of our inhabitants; or as by bearing too great a proportion to the bulk of the commodities on which they have been laid, they may have made the gain arising from defrauding the publick, or the temptation to attempt it, bear too great a proportion to the hazard of being discovered; or, as in any other respect they may be found to be attended with general inconvenience, or unreasonable hardship on particular persons, employments or conditions of life amongst us.

And, however it shall be determined, after the discharge of our present debts, as to the continuance or removal of the whole or any part of the duties appropriated for the payment of them; the revenues arising from them, being redeemed, will become the property of the publick, and, if not from thenceforth removed, will be employed in the room of, and take away the occasion for, such other taxes as shall then appear a greater burthen to, or to be more unequally levied upon, the subjects of this kingdom.

Having mentioned the quantity of annual expence to the Government, occasioned by our publick debts, it seems unnecessary to proceed further in proving the importance of discharging them, or to descend to or enumerate any further inconveniencies, that upon this account we labour under. An uncomfortable employment! and which, I hope, I shall be excused from, for this further reason; that the inconvenience of our present debts, and the importance of discharging them, are so universally believed and felt, and so unanimously agreed to, that I know none of my fellow-subjects who want to be convinced of them. I shall proceed therefore to what I proposed in the

Second place, to make out the probability, and represent the reasons we have to hope, that the present publick debts will, within the compass of a few years, be effectually and honourably discharged.

Of the probability that the said debts will be discharged in a few years by means of the Sinking Fund.

What I have chiefly proposed under this head, is to describe and explain, as far as my materials for that purpose will carry me, those measures which have been already taken for the discharge of our publick debts by the provision of the Sinking Fund. To which attempt, though this provision has already been made as publick as our acts of parliament, and though the operation and progress of it, in the discharge of our debts, is without any difficulty to be computed, I find myself induced, from that general suspicion of the inefficacy of this provision to answer the ends proposed by it; and which seems to have prevailed amongst some people, who have either not had leisure for that purpose, or who have declined the trouble of collecting the materials for, or making those computations from

from them which are requisite, in order to their satisfaction about the use and efficacy of the Sinking Fund.

Of the establishment of the Sinking Fund in the month of March, 1717.

The first material provision that was made for discharging the principal of our present debts, was enacted in the third year of his present Majesty's reign, by three several acts of parliament at that time made; the first of which (in the order that they should have been printed amongst the statutes published for that sessions) is intitled, *An act for redeeming several funds of the Governour and Company of the Bank of England, pursuant to former provisos of redemption; and for securing to them several new funds and allowances redeemable by Parliament; and for obliging them to advance sums, not exceeding £2,500,000 at five per cent. as shall be found necessary to be employed in lessening the national debts and incumbrances; and for continuing certain provisions formerly made for the expenses of his Majesty's civil government, and for the payment of annuities formerly purchased at the rate of five per cent. and for other purposes in this act mentioned, page 331. The second intitled, *An act for redeeming the yearly fund of the South-Sea Company (being after the rate of six pound per cent. per annum, and settling on the said Company a yearly fund after the rate of 5 per cent. per annum, and to raise for an annuity or annuities, at 5 per cent. per annum, any sum not exceeding £2,000,000, to be employed in lessening the national debts and incumbrances and for making the said new yearly fund and annuities to be hereafter redeemable in the time and manner thereby prescribed, page 375. And the third intitled, *An act for redeeming the duties and revenues which were settled to pay off principal and interest on the orders made for the four lottery acts passed in the 9th and 10th years of her late Majesty's reign; and for redeeming certain annuities payable on orders, according to a former act in that behalf, and for establishing a general yearly fund, &c. page 291.***

Of the several sums of which the Sinking Fund is composed.

The Sinking Fund of late years, (called so from its being understood to be appropriated to the sinking and discharging, as far as it will go, the principal sums of the present publick debt,) is made up of money arising yearly into the Exchequer, as the surplus of the produce of three several funds established by the three aforesaid acts of parliament, by the names of the Aggregate Fund, the South-Sea Fund, and the General Fund; the surplusses of which three funds, or what they annually produce more than the yearly sums to the payment of which they are first appropriated, and by the last of the aforesaid acts of parliament reserved for, and made applicable only to, the discharge of the principal and interest of such debts as had been before the year 1716 contracted and provided for by Parliament. The yearly sums to the payment of which those funds are first appropriated (except the sum of £700,000 *per annum* to his Majesty's civil government) are generally the interest, or annuities payable for several principal sums, of which our publick debts consist

As often therefore as any of those principal sums are paid off, or the proprietors of any part of the publick debt are induced to accept of a less interest or annuity for the same principal sums, the Sinking Fund is understood to increase by the yearly addition of the interest of such sums as are paid off, or the abatement of the annuity for such part of the publick debt as is agreed to be continued at a lower rate of interest. But the duties, the surplus of which the Sinking Fund consists of, not bringing in every year an equal sum of money, the surplus likewise is not every year alike; and therefore, in order to compute in what time the present publick debt may be discharged by the Sinking Fund, it is necessary to enquire from what different annual sums have of late years been produced by it, what yearly sum it is reasonable to suppose may for the future be produced by it at a medium, or one year with another. To which yearly sum we are from time to time to add the yearly interest or annuity of such principal sums, part of the present publick debt, as may be paid off by it; and all such abatements of interest of all or any part of the publick debts, as are already agreed hereafter to take place, or may for the future be agreed to by any of the creditors to the publick.

There are a great many particulars which you will see, Sir, I must want the knowledge of, in order to make this supposition with any great exactness. I could wish here to be able to state the produce of the several particular duties, the excesses of which constitute and supply the Sinking Fund; the different sums produced by them in different years, from the times they were severally granted; to assign the most probable causes of their variation, and from thence infer the probability of their producing more or less for the future: but however unprovided I am with materials for an inquiry of this kind, it may be yet worth while to proceed in describing the proportions in which any determined yearly sum (though by mistake) supposed to be the present yearly produce at a medium of the Sinking Fund, will increase, when applied to the payment of the publick debts; as those proportions will be the same with those in which any other sum, with more truth or probability supposed to be produced one year with another by the Sinking Fund, will increase when applied to the same purpose.

The best account I have been able to get of the produce of the Sinking Fund for some years last past lies now before me, and states the produce of the surplusses of the several funds, commonly called the Aggregate Fund, the General Fund, and the South-Sea Fund, (the sum of which surplusses our acts of parliament call the *Sinking Fund*) to be from the 31st of December, 1722, to the same time in the year 1723, £619,000 and upward; and the produce of the same surplusses from thence to the 31st of December, 1724, to amount to upwards of £653,000. This amount

Of the annual amount of the Sinking Fund,

amount of the produce of the Sinking Fund for the two years above mentioned, exceeds the produce of the same fund for some years before by a greater sum than can be accounted for by the discharge or reduction of the interest of any part of the publick debt before that time; and which therefore I am inclined to attribute to several provisions about that time made by the Legislature, for preventing frauds in the payment, and for the more fully and effectually collecting of several duties which in part supply the revenues appropriated to the payment of our publick debts, and of which provisions I would hope we may long enjoy the benefit in the increase of the Sinking Fund. And from hence, I should think, we might venture to expect an annual produce from the Sinking Fund for the future, equal to the produce of the same fund at a medium for the two years above-mentioned, ending in December 1724; and increasing by the yearly addition of the interest of such principal sums as may be henceforth paid off, and of the abatements of the interest, or annuities, of any of the publick debts when the same shall take place, that already are or may hereafter be agreed for.

Of the savings of interest that will accrue to the Sinking Fund in the year 1727.

The abatements of interest in the year 1727 are so considerable, and the time when they are to take place so near, that I believe it will be thought reasonable to step forwards to the time when the Sinking Fund will be increased by the addition of those abatements; and from that time to consider the progress that may be made in discharging the present publick debt by the Sinking Fund.

From Midsummer in the year 1727, it is already provided, that the Sinking Fund be increased by the reduction of the interest from 5 to 4 *per cent.* or an abatement of 1 *per cent. per annum* on the principal sum following.

On £13,061,878, being the amount of the publick debt to the South-Sea Company, excluding £3,839,363, } 130,618 15 7
part of it, for which an annuity at 4 *per cent.* only is at present payable,

On 16,901,241*l.* 17*s.* of the South-Sea annuities, - - 169,012 8 4

On 3,775,027*l.* 17*s.* 10*½*, part of the debt to the Bank of England, - - - 37,750 5 6

On £4,000,000, farther part of the debt to the Bank of England, purchased by them of the South-Sea Company, - - - 40,000 0 0

To

To this, if the annual income of the Sinking Fund on }
the 31st of December, 1724, be added, supposed to be } 600,000 0 0

And the increase of it by the discharge of £600,000 }
per annum of such Exchequer bills as remained uncan- }
celled on the 31st of December, 1724, and are made } 45,000 0 0
payable out of the Sinking Fund from the said 31st of }
December, to the 24th of June, 1727, viz. the interest }
and charge of circulating £1,500,000 Exchequer bills }
at 3 per cent. }

1,022,381 9 5

The amount of the said several annual sums will be upward of £1,022,000, the produce of the Sinking Fund from the 24th of June, 1727.

The publick debts on the 31st of December, 1724, are stated to amount to £52,363,471, or thereabouts: from which, if it be allowed me to deduct £1,500,000 Exchequer bills above supposed to be paid off by the Sinking Fund on the 24th of June, 1727, and such further principal sums as provision is made for the discharge of otherwise than by the Sinking Fund, the remainder to be paid off on the 24th of June, 1727, will be considerably less than 50 millions. Which sum however, (that I may not be thought to strain matters in favour of this scheme of discharging the publick debts by the Sinking Fund) I will suppose to be the principal debt to be paid off on the 24th of June, 1727, and the annual produce of the Sinking Fund to be from the same time one million only. I will likewise suppose, (as is most generally true) that the above-mentioned principal sum of 50 millions, will from the same time carry interest after the rate of 4 per cent. And, because there are some persons so sanguine as to imagine, that by force of our Sinking Fund, or some schemes formed upon it, the same debt may be still further reduced to a lower rate of interest, and the Sinking Fund increased further by such reduction; I will likewise suppose such a scheme to have taken effect, and the above-mentioned principal sum to carry 3 per cent. interest only, and the annual produce of the Sinking Fund to be increased, by an abatement of 1 per cent, interest on 50 millions, to £1,500,000. Upon both which suppositions, I shall subjoin a computation, describing in what number years, from Midsummer 1727, the above-mentioned principal sum of 50 millions, or any particular part of it, may be discharged and paid off; in which, when I had not time to correct them, I discovered a small mistake too; which I hope the reader will excuse, when I have assured him, that by no where misrepresent the time in which the aforesaid debt, or any part of it, may be paid off, by so much as two days.

The produce of the Sinking Fund from the 24th of June, 1727, will be upwards of £1,022,000 per annum; and the whole of the national debts at that time will be less than 50 millions.

FFf Computation

Computations of the numbers of years in which the aforesaid national debt of 50 millions may be paid off by means of the Sinking Fund, upon suppositions of its bearing the two different rates of interest of 4 per cent. and 3 per cent.

The Principles of the Doctrine of

Computation at 4l. per Cent.

	Payments made at Mid- summer every year.			Total of all the payments from the beginning in every year.		
	£.	§.	D.	£.	§.	D.
1728	1,000,000	0	0	1,000,000	0	0
	40,000	0	0			
29	1,040,000	0	0	2,040,000	0	0
	41,600	0	0			
30	1,081,600	0	0	3,121,600	0	0
	43,264	0	0			
31	1,124,864	0	0	4,246,464	0	0
	44,994	11	2 $\frac{1}{2}$			
32	1,169,858	11	2 $\frac{1}{2}$	5,416,322	11	2 $\frac{1}{2}$
	46,794	6	10 $\frac{1}{2}$			
33	1,216,652	18	0 $\frac{1}{2}$	6,632,975	9	0 $\frac{1}{2}$
	48,666	2	3 $\frac{1}{2}$			
34	1,265,319	0	4 $\frac{1}{2}$	7,898,294	9	5 $\frac{1}{2}$
	50,612	15	2 $\frac{1}{2}$			
35	1,315,931	15	6 $\frac{1}{2}$	9,214,226	4	11 $\frac{1}{2}$
	52,637	5	5			
36	1,368,569	0	11 $\frac{3}{4}$	10,582,795	5	10 $\frac{1}{4}$
	54,742	15	2 $\frac{1}{2}$			
37	1,423,311	16	2 $\frac{1}{2}$	12,006,107	2	0 $\frac{3}{4}$
	56,932	9	5			
38	1,480,244	5	7 $\frac{1}{2}$	13,486,351	7	8 $\frac{1}{4}$
	59,209	15	2 $\frac{1}{2}$			
39	1,539,454	0	10	15,025,805	8	6 $\frac{1}{4}$
	61,578	3	2 $\frac{1}{2}$			
40	1,601,032	4	0 $\frac{1}{2}$	16,626,837	12	7
	64,041	5	9 $\frac{1}{4}$			
41	1,665,073	9	10	18,291,911	2	5
	66,602	18	9 $\frac{1}{2}$			
42	1,731,676	8	7 $\frac{1}{2}$	20,023,587	11	0 $\frac{1}{2}$
	69,267	1	1 $\frac{1}{2}$			

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Computation at 3l. per Cent.

	Payments made at Mid- summer every Year.			Total of all the payments from the beginning in every year.		
	£.	s.	D.	£.	s.	D.
1728	1,500,000	0	0	1,500,000	0	0
	45,000	0	0			
29	1,545,000	0	0	3,045,000	0	0
	46,350	0	0			
30	1,591,350	0	0	4,636,350	0	0
	47,740	10	0			
31	1,639,090	10	0	6,275,440	10	0
	49,272	14	0			
32	1,688,363	4	0	7,963,803	14	0
	50,650	17	4½			
33	1,739,014	1	4½	9,702,817	15	4½
	52,170	8	5½			
34	1,791,184	9	9½	11,494,002	5	1½
	53,735	10	8½			
35	1,844,920	0	6	13,338,922	5	7½
	55,347	12	0			
36	1,900,267	12	6	15,239,189	18	1½
	57,008	0	6½			
37	1,957,275	13	0½	17,196,465	11	2
	58,718	0	6½			
38	2,015,993	13	6½	19,212,459	4	8½
	60,479	16	2½			
39	2,076,473	9	8½	21,289,032	14	5
	62,544	4	1			
40	2,139,017	13	9½	23,428,050	8	2½
	64,170	10	7½			
41	2,203,188	4	5½	25,631,238	12	7½
	66,095	12	11½			
42	2,269,283	17	4½	27,900,522	10	0
	68,078	10	3½			

Fff 2

Computation at 4l. per Cent.

	Payments made at Mid-summer every year.			Total of all the payments from the beginning in every year.		
	£.	s.	D.	£.	s.	D.
1743	1,800,943	9	9 $\frac{1}{2}$	21,824,531	0	9 $\frac{1}{2}$
	72,037	14	9 $\frac{1}{2}$			
44	1,872,981	4	6 $\frac{1}{2}$	23,697,512	5	3 $\frac{1}{2}$
	74,919	4	11 $\frac{1}{2}$			
45	1,947,900	9	6 $\frac{1}{2}$	25,645,412	14	10
	77,916	0	4 $\frac{1}{2}$			
46	2,025,816	9	11	27,671,229	4	9
	81,032	13	2 $\frac{1}{2}$			
47	2,106,849	3	1 $\frac{1}{2}$	29,778,078	7	10 $\frac{1}{2}$
	84,273	19	3 $\frac{1}{2}$			
48	2,191,123	2	5	31,969,201	10	3 $\frac{1}{2}$
	87,644	18	6			
49	2,278,768	0	11	34,247,969	11	2 $\frac{1}{2}$
	91,150	14	1			
50	2,369,918	15	0	36,617,988	6	2 $\frac{1}{2}$
	94,796	15	0			
51	2,464,715	10	0	39,082,703	16	2 $\frac{1}{2}$
	98,588	12	4 $\frac{1}{2}$			
52	2,563,304	2	4 $\frac{1}{2}$	41,646,007	18	7
	102,532	3	3 $\frac{1}{2}$			
53	2,665,836	5	8 $\frac{1}{2}$	44,311,844	4	3 $\frac{1}{2}$
	106,633	9	0 $\frac{1}{2}$			
54	2,772,469	14	8 $\frac{1}{2}$	47,094,313	19	0
	110,898	15	9 $\frac{1}{2}$			
55	2,883,368	10	6 $\frac{1}{2}$	49,977,682	9	6 $\frac{1}{2}$
	115,334	14	9 $\frac{1}{2}$	22,317	10	5 $\frac{1}{2}$
56	2,998,703	5	4	50,000,000	0	0
	1296	14	8			
	3,000,000	0	0			

Computation at 3l. per Cent.

	Payments made at Mid- summer every year.			Total of all the payments from the beginning in every year.		
	£.	s.	D.	£.	s.	D.
1743	2,337,362	7	8½	30,237,884	17	8½
	70,120	17	5			
44	2,407,483	5	1½	32,645,368	2	10
	72,224	9	11½			
45	2,479,707	15	1	35,125,075	17	11
	74,391	4	7½			
46	2,554,098	19	8½	37,679,174	17	7½
	76,622	19	4½			
47	2,630,721	19	1	40,309,896	16	8½
	78,921	13	2			
48	2,709,643	12	3	43,019,540	8	11½
	81,289	6	2			
49	2,790,932	18	5	45,810,473	7	4½
	83,727	19	9			
50	2,874,660	18	2	48,685,134	5	6½
	86,239	16	8½	1,314,865	14	5½
51	2,960,900	14	10½	50,000,000	0	0
½	39,099	5	1½			
	3,000,000	0	0			

You

Observations on
the preceding
computations.

You will be pleased to observe, Sir, that the annual income of the Sinking Fund, in this manner applied to the discharge of the principal of the publick debts, increases yearly in the same manner and proportion as principal sum put out and continued at compound interest, or interest upon interest, at such a rate of interest as the principal sum to be paid off is supposed to carry: that the increase of it in every year, is by the interest of that principal sum which was paid off in the year next before it; and that the whole of the increase of it in any one year, from the beginning to apply it in discharge of the principal debt, is the sum of the interest of all the principal sums that have been in the year before paid off by it, and that the whole of the debt proposed to be paid off by a Sinking Fund in this manner applied, will be compleatly discharged the year before the Sinking Fund itself is increased, by the addition of the whole interest of the debt to be paid off.

From which observations, it will be easy to compute the progress of any other annual sum, greater or less, than what I have supposed to be the produce of the Sinking Fund in the year 1727, in the payment of principal sum of 50 millions, at 4 per cent. or any other rate of interest, or any other principal sum which you may think it more reasonable (our affairs now stand) to provide for the payment of, by the common rule for calculating the increase of principal sums continued at compound interest.

In the use of which rules, you will find, Sir, if you should think more reasonable to set the income of the Sinking Fund, from the year 1727, at £800,000, or (as some persons have represented it) at £1200,000 *per annum*, that a debt of 50 millions, carrying 4 per cent. interest, would in the first of these cases be paid off in about 32 years, and in the other 25 years and one month; or if in either case the Sinking Fund should be supposed to be increased by £500,000 *per annum* added to it, from the interest of the same debt, reduced to 3 per cent. from the same time, it would appear that it might be fully discharged and paid off, by a Sinking Fund of £1,300,000 *per annum* in about 25 years, and by a Sinking Fund of £1,700,000 in 21 years and 8 months, or thereabouts.

A mistaken opinion has of late been propagated, that the publick debts have increased since the establishment of the Sinking Fund.

But to whatever may in this manner be observed or proved, relating to the efficacy or progress of the Sinking Fund increasing annually by addition of the interest of such debts as are discharged by it, I have heard of no objection and strongly insisted on to be true, that our publick debts have been far from decreasing or made less since the contrivance and application of this expedient for that purpose; but, on the contrary, have been growing upon us, and are now considerably greater than they were about the time when the surplusses of several funds were first appropriated to the discharge

discharge of those debts. And this melancholy circumstance the same persons aggravate, with observing, that the increase of our debts has been a time of almost uninterrupted peace; and infer, that our debts must increase still faster upon us, in case of any publick troubles.

I have often wondered how so uncomfortable a mistake could so generally prevail, against the testimony that the memory of every person at all acquainted with publick transactions of this kind must bear, that our publick loans of late years (except such as have been made on funds provided to discharge the monies advanced upon them within the year,) have not been equal to the sums that have within the same time been paid off; still upon further enquiry upon this subject, I have had put into my hands copies of accounts, supposed to be made up at the Exchequer, stating the totals of the publick debts for different years to be greater considerably from the year 1720, than in that year, and in that year to be more than in any year before it. From which accounts I cannot but think this mistake must arise and prevail with persons who satisfied themselves with observing the totals only, and have not attended to the particular articles of which they were made up; but in examining the particular articles of which those totals are made up, they will find that the great increase of figures in the description of our present incumbrances, is not owing to any real increase of their true quantity.

In an account now before me, of the amount of the publick debts on the 31st of December in several years, beginning in 1717, and ending in the year 1724, the amount of the publick debts in the first of those years is described to be £47,894,950, and in the last to be £52,363,471. Of which great increase in the description of our debts, the chief reasons are; first, the subscription of several irredeemable annuities for different terms of years into the South-Sea Company's stock, in the years 1719 and 1720; by which those annuities were converted into a redeemable debt from the Government, and purchased back from the proprietors at higher rates, or a greater number of years purchase, than were paid by the proprietors for the same annuities when they were first purchased from the Government. Before these subscriptions made, this part of our publick incumbrances is described in the aforesaid account, by the principal sums originally advanced by the proprietors on the purchase of them; and afterwards by the quantity of redeemable debt, for which by virtue of the aforesaid subscriptions they were exchanged; which generally exceeds by four years and one half's purchase the sum originally contributed by the proprietors of those annuities, and which upon the whole of the said annuities at those different times subscribed, amounts to about £3,155,858. This in the present view must, I think, be admitted to be no real increase of the publick incumbrances, or at least not properly brought into the

account

An account of the circumstances that have given rise to the said mistaken opinion.

1st, The purchase of certain irredeemable annuities for different terms of years.

account of those years in which the aforesaid subscriptions were made those subscriptions being well enough known and understood to have been of great advantage to the publick, and very much to have facilitated the discharge of the whole of our present debts; and it being very obvious that whatever real incumbrance has been growing upon us on account of those annuities, it is to be attributed only to the increasing value of the annuities, and to be computed from the times of their being valued higher prices, and not from the times of the subscriptions above-mentioned, by which the further increase of their value was most fortunately prevented; and about which, all that we have to wish is, that it has been done sooner.

2dly, Army-debentures.

Another article increasing in the aforesaid accounts of the public debts from the year 1717 to the year 1724, is of army-debentures, annuities charged and made payable out of the fund commonly called the General Fund, after the rate of 4 per cent. for such principal sums, as pursuance of several acts of parliament for appointing commissioners state the debt due to the army, have been certified to have been due for services in the late war, and before the year 1717. This, Sir, from 40,157*l.* 8*s.* 5*d.* which on the 31st of December, 1717, is only stated to be due from the publick under this article, is on the 31st of December 1724, by the aforesaid accounts described to amount to upwards of £2,140,157. But as this debt was due before the year 1717, in the present inquiry, whether the publick debts are since that time increased or no, this sum is (now that the quantity of it is determined,) to be reckoned the amount of the publick debt, as well in the year 1717 as in the year 1724; or, in other words, to be considered as due from the publick from the time it was contracted, and not from the time only when it was certified to be due.

3dly, Exchequer notes.

Another article increasing the total amount of the publick debts in the year 1724 beyond that of the year 1717, in the aforesaid accounts, that of £1,000,000 of Exchequer notes made out and lent to the South-Sea Company in the year 1720, and in that year added to the amount of the publick debts. This sum, on the re-payment of it by the South-Sea Company, would have been deducted from the amount of the publick debts in that year in which it was repaid, if it had not been provided by a subsequent act of parliament, that the aforesaid Exchequer notes should be cancelled and paid out of the Sinking Fund; and that the sum of £1,000,000, due from the South-Sea Company, should be applied, with the sum paid, to the discharge of a farther million of Exchequer notes made out in the year 1722, and upon which money was raised for the discharge of a like sum in arrears to the navy; which said sum of £1,000,000 being in this manner ultimately supplied out of the Sinking Fund, it is necessary

supp

suppose it to have been due from the publick before the year 1716, the Sinking Fund being, as I have above observed, about that time appropriated to the discharge of such debts only as were due before that year; and consequently this sum of £1,000,000 being in the year 1717 owing, and in arrear from the Government, should also in our present inquiry about the increase of the publick debts, be in that year added to the amount of them.

As should also, for much the same kind of reasons, the following material articles, viz.

l. s. d.

Navy-annuities, a debt, though before due, not brought into the publick accounts till the year 1718, } 110,312 0 0

A further provision for the sufferers at Nevis and St. Christophers, about — — — — — 41,000 0 0

The increase of a deficiency on the East-India Company's fund stated in the publick accounts, to be from the year 1717 to the year 1720, about — — — — — 67,500 0 0

A sum in the year 1723, raised for immediate service on the credit of Exchequer notes, the payment of which was at the same time provided for by a tax on the estates of Roman Catholics, — — — — — 100,000 0 0

To these articles are to be added the three last above-mentioned, viz.

The increase computed on the subscription of indemnables, — — — — — 3,155,858 0 0

Of army debentures, — — — — — 2,100,000 0 0

And the sum raised for discharging arrears to the navy, 1,000,000 0 0

The amount of which sums together is, — — — — — 6,574,670 0 0

And this sum, Sir, must be added to the above-mentioned total of our publick debts in the year 1717, before the comparing it with the total of the same debts in the year 1724 will truly determine how far our debts are increased or grown less from one time to another.

47,894,950 0 0
6,574,670 0 0

And the aforesaid amount of our debts in 1724, 54,469,620 0 0
52,363,471 0 0
2,106,149 0 0

G B B

And

The national debts have really been diminished by the sum of £2,106,149 from the year 1717 to the year 1724.

Another method of estimating the diminution of the national debts in the same period.

And it will appear, that our debts are not in reality increased from the year 1717, to the year 1724; but, on the contrary, are diminished by the sum of £2,106,149, or thereabouts.

The same thing will appear from enumerating the particulars of the real increase or decrease of our debts from one time to the other; of which, Sir, the following is very nearly a true account, viz.

Money at different times borrowed on the duty on coals for building churches, more than in the mean time has been paid off by the particular provision made for that purpose, — — — — — 92,778 2 0

Money borrowed for the service of the year 1719, more than paid off by the provision made for that purpose on the 31st of December, 1724, — — — — — 439,300 0 0

Money borrowed on the plate-act for the service of the year 1720, — — — — — 312,000 0 0

Total, £844,078 2 10

And this sum of 844,078*l.* 2*s.* is the whole sum that our debts can with any propriety, be said to be increased by from the year 1717. Such other sums as have been since that time borrowed having been employed in aid of the Sinking Fund, and applied in the discharge of some other debts at a higher interest; of which the following (except what of this kind has been already mentioned) is likewise a true account, viz.

Borrowed in the year 1719 by lottery, — — — — — 500,000 0 0

Advanced in the same year by the South-Sea Company on the increase of their stock and funds, about, } 544,142 0 0

Advanced in the year 1723, towards the discharge of the lottery annuities subscribed to the South-Sea Company, about — — — — — 1,000,000 0 0

Total, 2,044,142 0 0

By which sum, together with the Sinking Fund, have been paid off from 1717 to 1724, viz.

Of Exchequer notes, — — — — — 2,924,612 0 0

Of lottery annuities unsubscribed, — — — — — 1,204,786 0 0

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Bank annuities unsubscribed, — — — — — £. 235,297 0 0
 Deficiency of the East-India Company's fund, — — — — — 191,028 0 0

Besides, there has been in the same time paid in part
 of a principal debt contracted by two lotteries in the } 429,490 0 0
 years 1713 and 1714, by provision for that purpose at }
 the same time made, about — — — — —

From whence the total of the last above-mentioned } Total, 4,985,213 0 0
 loans being deducted, viz. — — — — — } 2,044,142 0 0

The remainder — — — — — 2,941,071 0 0
 will be the sum of what has been paid off from the year
 1717 to the year 1724 by the Sinking Fund, or other-
 wise without the assistance of those loans.

And from thence — — — — — 2,941,071 0 0
 Let us farther deduct the total of the aforesaid }
 articles by which our debts have really, in the mean } 844,078 0 0
 time, been increased, viz. — — — — —

And that remainder — — — — — 2,096,993 0 0
 will be the sum by which our debts, within the aforesaid seven years,
 appear by this computation really to have been diminished; differing
 indeed from that sum which I have from the first computation stated to
 be the decrease of the publick debts in the same time by near £10,000.
 But which difference, if I pretended to the utmost exactness, might be
 removed, by either adding to the last remainder, or reckoning amongst
 the particulars by which our debts have decreased within the time aforesaid,
 the value of such annuities for lives as within that time have reverted
 to the Crown.

This sum, perhaps, especially if it be farther reduced by the deduction
 of one million at two different times borrowed, to supply the deficiencies
 of the provision for the expence of his Majesty's civil government, con-
 sidered as part of, and an addition to, our publick debts, may be thought
 too inconsiderable a diminution of our debts to be boasted of as the effects
 of this expedient for so great a length of time. But, as it is no real
 objection to the truth of those computations which I have made, of the
 progress of the Sinking Fund from the year 1727, I presume it will like-
 wise be no discouragement to our dependance on this provision for the
 payment of our debts; especially after we have considered the great
 addition that will be made to the Sinking Fund in the year 1727, and
 have

G g g 2

have farther observed the much greater dispatch which a yearly sum applied to the payment of any determined debt at interest, and increasing annually in the manner above-described, will make in the discharge of such a debt in a few years after the first application of it to that purpose, than it will do when it first begins in that manner to be applied.

Another mistaken opinion which has lately been advanced concerning the Sinking Fund,

The little progress, however, hitherto made in the diminution of our debts, leads me to the examination of another opinion, which I think have observed to prevail with the same persons, who affirm our public debts to have increased upon us; which is, that upon the supposition that such debts are really increasing upon us by new loans equal to or exceeding the discharges made in the same time by the Sinking Fund, the Sinking Fund is in such case making no effectual progress at all in the diminution of our debts. It is perhaps the more material to consider here, however, far this opinion is true, for this reason, that though this supposition or which it is founded has not been true hitherto, it must be, however, admitted to be not improbable, that some future exigencies of the Government may make such new loans necessary, as may exceed any sums in the same compass of time produced by or applied to the discharge of our debt from the Sinking Fund. And in this case, upon the supposition that such new loans are made upon further funds found out for payment of the interest of the money so to be advanced upon them, this opinion, that the Sinking Fund, applied as aforesaid, would be making no effectual advance to the complete discharge of the whole of our public debts, would not be true. This will be best explained, if during the time that the above supposed Sinking Funds are employed in the discharge of the aforesaid debt of 50 millions, the whole of our debts should be supposed, by new loans upon further funds borrowed at 4 or 3 per cent. interest, to be increased by a further sum of 50 millions, and that sum to be discharged in the same manner, and by the same Sinking Funds, after the discharge of the first 50 millions; or if the account of the progress of the above supposed Sinking Funds be carried on, till instead of 50 millions they shall have discharged a principal debt of 100 millions.

Computations of the times in which another debt of 50 millions of pounds, (contracted upon new funds during the time of discharging the first debt of 50 millions by means of the Sinking Fund,) might be paid off by means of the said fund, if the interest of money is 4 and 3 per cent.

LIFE-ANNUITIES.

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Computation at 4 per Cent.

Payments made at Mid-Summer every year.				Total of all the payments from the beginning in every year.			
£.	s.	d.		£.	s.	d.	
1755							
56	2,998,703	5	4	49,977,682	9	6 $\frac{1}{2}$	
	119,948	2	7 $\frac{1}{2}$	52,976,385	14	10 $\frac{1}{2}$	
57	3,118,651	7	11 $\frac{1}{2}$	56,095,037	2	9 $\frac{1}{2}$	
	124,746	1	1 $\frac{1}{2}$				
58	3,243,397	9	0 $\frac{1}{2}$	59,338,434	11	10	
	129,735	17	11 $\frac{1}{2}$				
59	3,373,113	7	0	62,711,547	18	10	
	134,924	10	8				
60	3,508,037	17	8 $\frac{1}{2}$	66,219,585	16	6	
	140,321	10	3 $\frac{1}{2}$				
61	3,648,359	7	11 $\frac{1}{2}$	69,867,945	4	5 $\frac{1}{2}$	
	145,934	7	6				
62	3,794,293	15	5 $\frac{1}{2}$	73,662,238	19	11	
	151,771	15	0				
63	3,946,065	10	5 $\frac{1}{2}$	77,608,304	9	4 $\frac{1}{2}$	
	157,842	12	5 $\frac{1}{2}$				
64	4,103,908	2	10 $\frac{1}{2}$	81,712,212	12	3 $\frac{1}{2}$	
	164,156	6	6				
65	4,268,064	9	4 $\frac{3}{4}$	85,980,277	1	8	
	170,722	11	6 $\frac{1}{2}$				
66	4,438,787	0	11 $\frac{1}{2}$	90,419,064	2	7 $\frac{1}{2}$	
	177,551	9	6 $\frac{1}{2}$				
67	4,616,338	10	6	95,035,402	13	1 $\frac{1}{2}$	
	184,653	10	9 $\frac{1}{2}$				
68	4,800,992	1	3 $\frac{3}{4}$	99,836,394	14	5	
	192,039	13	7 $\frac{1}{2}$	163,605	5	7	
69	4,993,031	14	11 $\frac{1}{2}$	100,000,000	0	0	
	6,968	5	0 $\frac{1}{2}$				
	5,000,000	0	0				

The Principles of the Doctrine of

Computation at 31. per Cent.

	Payments made at Mid-Summer every Year.			Total of all the payments from the beginning in every year.		
	£.	s.	d.	£.	s.	d.
1750						
51	2,960,900	14	10½	48,685,134	5	6½
	88,827	0	5½	51,646,034	0	5½
52	3,049,727	15	4½	54,695,761	15	9½
	91,491	16	1½			
53	3,141,219	11	5½	57,836,981	7	3
	94,236	11	7½			
54	3,235,456	3	1½	61,072,437	10	4½
	97,063	13	8½			
55	3,332,519	16	9½	64,404,957	7	2½
	99,975	11	10½			
56	3,432,495	8	8½	67,837,452	15	10½
	102,974	17	3			
57	3,535,470	5	11½	71,372,923	1	10½
	106,064	2	2			
58	3,641,534	8	1½	75,014,457	9	11½
	109,246	0	7½			
59	3,750,780	8	9½	78,765,237	18	9
	112,523	8	3½			
60	3,863,303	17	0½	82,628,541	15	9½
	115,899	2	3½			
61	3,979,202	19	4½	86,607,744	15	1½
	119,376	1	10			
62	4,098,579	1	2½	90,706,323	16	4
	122,957	7	4½			
63	4,221,536	8	6½	94,927,860	4	10½
	126,646	1	9½			
64	4,348,182	10	4	99,276,042	15	2½
	130,745	9	6	723,957	4	9½
65	4,478,927	19	10	100,000,000	0	0
	21,072	0	2			
	4,500,000	0	0			

Observations on the great efficacy of the Sinking Fund in diminishing the national debts, notwithstanding new sums of money should in the mean time be borrowed by the publick upon new funds to pay the interest of them.

From hence, Sir, it presently appears that the above supposed Sinking Funds, in this manner increasing by the addition of the interest of the principal sums in every year paid off, and consequently by additions in every year greater than those made to it in the year before, will be sufficient not only to discharge our present debts, but any probable addition in any mean time to be made to them by further loans on new-invented funds, a few years after the present debts shall be discharged: and that the sum required for the discharge of our debts, increased by any addition in the manner made, will by no means be lengthened out, or the payment of the whole of our debts by the Sinking Fund retarded or delayed in proportion to the addition to or increase of the debt itself: the total payment of our publick debts becoming by no means desperate from any sinking Fund, however less than those above supposed, upon account of the determined increase of or additions made to them; unless those additions are supposed to be continued increasing in every year in the same or a greater proportion to one another than that in which the additions are made to the Sinking Fund increase. This is so true, that suppositions about the increase of the publick debt might be carried to the most extravagance, and still appear to be provided for by the above-mentioned Sinking Fund of £1,000,000, increasing at the rate of 4 per cent. compound interest; which, if it were worth while, might be shewed to be sufficient, in about 105 years, to pay off a debt of 1575 millions, growing for the increase of the present debt of 50 millions, by an addition of 15 millions in every year in which that Sinking Fund should be supposed. Nor will this at all surprize persons who have been accustomed to attend to the increase of money put out at compound interest, or to see annuities continued in geometrical progression; an enquiry into which will remove all doubts about the truth of what I have here advanced. It would however be true, that if at any time, on the discharge of any part of the principal of the present debt, the interest were not added to, and applied in the further discharge of, the remaining debt, but another sum or greater principal sum should be borrowed on the same annuity; or greater principal sum should be borrowed on the same sum as the sum borrowed, be stopped; and, if a greater, be put backwards: but as these measures are not taken, or the Sinking Fund diverted or applied to any other purpose than the discharge of our debts; the full and actual payment of all our debts by this expedient, is by no means to be supposed of from the increase of them by new loans on further duties.

But this is upon a supposition that the Sinking Fund is never charged with the interest of the new loans;

And that the Sinking Fund will, from time to time, be applied to the discharge of the publick debts; and not be diverted or applied to any other purpose whatsoever, is what, I think, we may securely promise ourselves; from considering that the aforesaid fund has been appropriated to the purpose by the Legislature, and our publick faith in the same manner engaged

The publick faith is engaged to the proprietors of the national debt, that the Sinking Fund shall never be applied to any other purpose but that of discharging the said national debt.

engaged to the creditors of the Government, that the surplus of the aforesaid duties should be applied to the discharge of the principal of the debts, as the funds themselves to the payment of the interest or annuity contracted for: which faith of the publick in this manner engaged, think we have all the reason in the world to believe will be as inviolably observed in this as in any other part of their contract with the proprietors of the publick debts.

The clause of the act of parliament, (3 Geo. 1, cap. 7, sect 37,) by which the Sinking Fund is so appropriated.

This appropriation of the Sinking Fund to the purpose aforesaid, will find, Sir, to have been made by the aforesaid acts of parliament. In the last of which, taking them in that order in which I have referred them, page 320, after reciting that by the two other acts of parliament the surplusses of the Aggregate and South-Sea Funds are provided to be reserved to the disposition of Parliament only; it is enacted, That the surplusses of the General Fund thereby created, should in like manner be accounted for and reserved for the disposition of Parliament. And then is further enacted in the words following, "That all the monies to be from time to time, as well of or for the said excess or surplus, by virtue of the said act made for redeeming the funds of the Government, as of the said Company of the Bank of England [viz. the Aggregate Fund] and of or for the said excess or surplus, by virtue of the said act for redeeming the funds of the said Governor and Company of Merchants trading to the South-Seas, &c. and of or for the said excess or surplus of the said duties and revenues by this act appropriated as aforesaid, [viz. the General Fund] and the said overplus monies of the said general reserve fund by this act established or intended to be established as aforesaid, shall be appropriated, reserved, and employed to and for the discharge of the principal and interest of such national debts and incumbrances as were incurred before the 25th of December, 1716, and are declared to be national debts, and are provided for by act of parliament in the manner and form as shall be directed and appointed by any future act or acts of parliament to be discharged therewith or out of the same, to and for none other use, intent or purpose whatsoever."

The said clause ought to be considered as a solemn contract entered into by the Government with the publick creditors.

By these words, I think, the surplusses therein mentioned, of which the annual income of the Sinking Fund is made up, sufficiently appear to have been appropriated by the legislative power to the payment of the publick debts, till they shall be jointly discharged and paid off. Nor is this provision well be understood as made by the Government for what then appeared for publick convenience only, and consequently to be altered by subsequent acts whenever it shall appear, or be pretended to be otherwise; but must, I think, be considered as a contract by the Government with the publick creditors, if the occasion of these acts of parliament be attended to. In which case it will appear, that the several provisions

by these acts made, were enacted and proposed to the creditors aforesaid, as inducements to them to accept of an interest, or annuity, for their debts by one sixth part less than that which till that time they had received; of which the most obvious inducement was, that what was thus deducted from the yearly interest of their debts, should be applied for the better securing and gradual discharge of the principal of the said debts. To which security, amongst the other benefits by the same acts of parliament proposed to them, they must, I think, be considered to have intitled themselves by their subscriptions afterwards made, subsequent to, and in consideration of, such proposals made to them by the Legislature. And whoever will be at the trouble of turning over the several subsequent acts of parliament relating to the publick debts, will find this provision for the application of the Sinking Fund frequently repeated and confirmed: and in cases where by act of parliament application of monies in the Sinking Fund to the discharge of debts that were less obviously, or less generally, known to have been within the description of the debts intended by the provision above-recited, such debts have been, by the recitals, declared and explained to have been debts incurred before the 25th of December, 1716, and provided for by Parliament in a manner that has plainly intimated it to be understood by the Legislature, that the above-recited provision was an engagement, or contract, of the Government with the publick creditors, about the punctual observation of which from time to time they were intitled to have all possible satisfaction; or at least, that the punctual application of the above-mentioned surplusses to the discharge of our present debts, was regarded by them as a matter of the highest consequence to the publick welfare. And as long as the publick welfare shall be in the least regarded, and this continues to be the only expedient for removing such any incumbrances on our affairs, and redeeming so considerable a revenue to the use of the publick, I think we may confidently expect, that no persons whatsoever, whose hands the administration of our affairs may at any time for the future be committed to, can ever be induced to approve of, or recommend, the application of the produce of the Sinking Fund, in any possible exigence of our affairs, to any other uses than those to which it stands now appropriated, though there were no other considerations to enforce it.

For let us inquire a little, what publick exigencies can be supposed to happen, that can make it at any time advisable to divert or apply the produce of the Sinking Fund to any other purpose till after the entire payment of our publick debts. Let the expence that the circumstances of our affairs may at any time make necessary, be, or be supposed to be, ever much more than what can be conveniently raised within the year; it must, I think, always appear more eligible in regard to the publick interest, as well as more easy to those persons in the administration, to whom the

H h h

care than to break in upon the Sinking Fund.

care of providing the necessary supplies that at any time be allotted, to raise what shall be further wanted by increasing the publick debt with further loans upon interest provided for by new duties, than to supply the same sums in any way from the produce of the Sinking Fund.

The computation that I have last made was to shew, that the time in which the above supposed Sinking Fund of £1,000,000 will be sufficient to complete the discharge of the publick debts, will by no means increase equally to the increase of the principal sum of those debts by further loans of new funds: but it may be of further use to shew, how much less an increase of the publick debts, by borrowing further sums at interest provided for by new funds, will retard the discharge of the whole of the publick debts, than the supplying the same sums in any way from the produce of the Sinking Fund would do. Let us suppose, for instance that the Government were obliged for 25 years together to increase the present debt, by a million borrowed in every year at an interest of 4 per cent. provided for by further funds, the above made computation will shew that that additional debt of 25 millions would be paid off by a Sinking Fund of one million, applied as is therein supposed, in little more than 7 years after the discharge of the present 50 millions. But if the sum were to be supplied out of the produce of the Sinking Fund, it is obvious that the payment of the publick debts must stand still for 25 years, and be by more than two thirds of that time retarded beyond the time in which they would otherwise be discharged, though increased as aforesaid; and the greater the sum is supposed to be, that in these direct ways is to be supplied, the greater will be the proportion in which the payment of our debts will be delayed, by supplying such expenses from the Sinking Fund, more than by the other way: or if the sums in these direct ways supplied should be supposed less, the difference of the delay in the two cases will be indeed less: but on supposition of the smallest sum and these two different ways supplied, the delay arising to the discharge of the publick debt by this misapplication of the Sinking Fund, will be at least three times as great as that which will be occasioned by increasing the publick debt in the other method.

The borrowing money, and charging the interest of it on the Sinking Fund, instead of laying new taxes for that purpose, would be as pernicious an alienation of the Sinking Fund as the taking gross sums out of it that were equal to the money so borrowed.

not only to delay the payment of the public debts, but the taking away
 solely the only security that provided, that they shall ever be paid off.
 for which reason I shall not trouble you, Sir, with any computation of
 the different degrees in which different steps in pursuing these measures
 will affect us; but at once suppose it impossible that any persons can pro-
 vide so borrow money, (or much less to succeed in it) on the credit of
 schemes that themselves destroy all probability of the re-payment of it;
 which, such measures as these, must evidently appear to do, to those that
 consider, that we have already had the greatest advantage from the re-
 duction of interest that can, with reason be hoped for in the provision of
 the present Sinking Fund, which if we once part with, in exchange for an
 increased principal debt at a lower rate of interest only, it will be made
 expect that either such a lower rate of interest, or any alteration in our
 circumstances for the better, will admit of the same kind of provision to
 be made again for the payment of our debts increased by such measures
 these are, to which or without you have said I think will not hurt
 I cannot therefore Sir, amongst the ordinary vicissitudes of the affairs
 of any nation, not even amongst any long and expensive wars, that it may
 be necessary for the defence and safety of these kingdoms, to carry on with
 neighbours, and our that exigence of our affairs that can make the
 application of the Sinking Fund appear necessary, or probable to be
 in practice, while it is so certain, that the lands, estates, expence, or
 income of Great-Britain, will yet easily admit of farther duties, sufficient
 furnish new funds to answer the interest of such sums as any public
 actions that I can represent to myself can call for. Nor can I fear, that
 such duties will not be cheerfully voted and submitted to, when they shall
 appear necessary to prevent the misapplication of an annual sum employed
 to useful and necessary a service to the publick, as the reduction of our
 jobs; while that appears to be retarded so much more by discontinuing
 the payment of those debts, than by the increase of them.

There is another objection to the probability of the payment of our
 public debts, which, if I did not frequently meet with it, I should chuse not
 mention, from my apprehensions, that in stating of it as I have met
 with it, I should be obliged to mention my superiors with less decency,
 in that grateful sense of the happiness we enjoy under the present reign
 and on all other occasions lead me to, or than you, Sir, from the same
 views would expect from me. But as you are pleased to admit you have
 met with it from others, you will give me leave to mention it, in my
 answer to it. The objection I mean is, That the continuance of our
 public debts is, and always must be, the interest of persons in the admi-
 nistration; that the greatest profit of their employments arises from hence;
 that the necessary power and influence to support themselves in those
 employments,

Examination of
 another reason
 which has lately
 been advanced for
 suspecting that the
 ministers of state
 will not long con-
 tinue to apply the
 whole of the Sink-
 ing Fund to the
 discharge of the
 national debt.

employments, depend greatly on their having reserved to themselves the disposition of the various offices and employments in collecting and applying the revenues appropriated to the payment of the publick debts; when those debts shall be discharged, can subsist no longer.

It must be observed, in answer to this reason, that both the king and the ministers of state have hitherto been anxiously careful to apply the Sinking fund to its proper use, the discharge of the national debt.

It may further be observed, in answer to the said reason, that it is not necessary, or probable, that, when the national debt shall be all paid off, the taxes which now pay the interest of it will be abolished.

Whatever truth we should admit to be in this objection, we have pleasure of observing, that it appears to be equally true, from the frequent and earnest recommendations from his Majesty of the necessary measures discharging the publick debts to the care and endeavours of the Legislature the several steps that have been taken by them; and the great and effectual provision that is already made for this purpose; that nothing can be better, or will be, more sincerely intended and endeavoured by his Majesty or the persons who have had, or shall have, the honour to be employed by him.

But from the sense I have just now professed to have of the distress we enjoy under the present government, I must confess, I should with pleasure look forwards on that period of time, when his present Majesty his successors should be deprived of the means of supporting it, or even rewarding and encouraging the fidelity and services of their best subjects. The chief use therefore that I have proposed to make of this objection is to take an occasion from it, of considering how far it is probable that such a reform as is above supposed, of the various employments in collecting and receiving the present revenues, will take place on the discharge the publick debts; or how far it is reasonable that it should do so. At this supposition being founded on a presumption, that the particular debt now appropriated to the payment of the publick debts will, after the payment of them, be immediately removed, the reasonableness of this presumption will be the matter in question.

The Government is now supported by other taxes, that are granted anew every year. To wit, the land-tax and the malt-tax.

These taxes might, when the national debt was all paid off, be dropped, and their

For the purpose of this inquiry, Sir, I should propose it to be considered, that the support of our government necessarily requires a considerable annual expence, that is at present ordinarily supplied by other taxes than those which have been provided to answer the payment of the publick debts; that the present ordinary provision for that annual expence has been hitherto determined, rather by the necessities of the publick, than by choice; and that it yet remains to be debated, how far the duties presently appropriated to the payment of our debts, or part of them, may after the discharge of those debts, be continued and made to answer ordinary annual expence of our government, more to the advantage of the publick, with less burden and expence to the particular estates of Majesty's subjects in this kingdom, and consistently with a more equal and reasonable proportion of the burthen or expence by every subject submitted to, than at present. And that it yet remains to be debated, how far the duties presently appropriated to the payment of our debts, or part of them, may after the discharge of those debts, be continued and made to answer ordinary annual expence of our government, more to the advantage of the publick, with less burden and expence to the particular estates of Majesty's subjects in this kingdom, and consistently with a more equal and reasonable proportion of the burthen or expence by every subject submitted to, than at present. And that it yet remains to be debated, how far the duties presently appropriated to the payment of our debts, or part of them, may after the discharge of those debts, be continued and made to answer ordinary annual expence of our government, more to the advantage of the publick, with less burden and expence to the particular estates of Majesty's subjects in this kingdom, and consistently with a more equal and reasonable proportion of the burthen or expence by every subject submitted to, than at present.

to the benefit, he receives, from the support of our government, than is done by the present provision made for the aforesaid ordinary annual expence.

It is in vain to suppose, that the necessary expences of a government are to be supplied by any taxes that are no ways burthensome to the whole or some part of the community, and consequently to which some objections may not be dressed up by persons interested in avoiding them; which objections, however, when such taxes appear necessary, it is unreasonable to oppose or aggravate. I shall not therefore point out any inequality or hardship that I may apprehend to be in the ordinary annual provision made amongst us by a land-tax; but content myself with making some observations, tending to recommend the greatest part of the duties now appropriated to the payment of our debts, as the most convenient and reasonable way to supply the ordinary expence of our government, when redeemed by the payment of those debts.

This would be found to be very equitable and convenient.

Upon enumerating the several duties which at different times have been provided to answer the demands of the publick creditors, it will appear that the greatest part of them (whether collected by custom or excise) have been laid upon commodities in general use and consumption amongst a part of the inhabitants of this country, whose circumstances will admit the expence.

The latter duties (which now pay the interest of the national debt,) are, for the most part, duties on the consumption of commodities.

About these duties it will appear upon reflection to be generally true that they have been added to the price which those commodities had before imposition of such duties, and from thenceforth to be ultimately paid the last price of such commodities by the consumer.

Upon which supposition, if the aforesaid duties are either, by way of custom or excise, generally collected throughout the country where such commodities are consumed; it is plain that the said duties will generally be paid by every person residing in such a country, nearly in proportion to his ordinary annual expence.

And this, Sir, is the share or proportion which, of all others, I think most eligible to be taken from every person residing in a country where a part of the inhabitants subsist by commerce, towards the publick expences of the government of that country, when it can in this manner be done, without enquiring exactly into the expence of every particular inhabitant.

These are the most convenient taxes that can be laid in a commercial country.

An advantage
belonging to these
taxes.

For, first, in this way the publick expence is least sensibly felt by the who really contribute towards it; every person being voluntary in his expence, and gratifying himself while he is contributing from his estate the expence of the government.

A second advantage
belonging to
them.

2. Contributions in this manner generally made by the inhabitants a country in proportion to their expence, will be likewise made in a proportion to the real value of the property of the same inhabitants; perhaps, a better than it would be done by a law made, directing the public expences to be levied in that proportion, from the great difficulty of finding out, and plain inconvenience of exactly inquiring into the real value every man's property for a purpose of this kind, in a country so much engaged in traffick as our's is. Nor will taxes upon our expences be much from taxes proportioned to the value of our property, (if long continued) from what may at first sight appear a reason for that conclusion; mean, the different choice of the thrifty and extravagant in the proportion of their expences; the first of which, by contributing little himself to the publick expence, is providing for larger contributions by his successor and the other, by contributing too largely in haste, is incapacitating himself for contributing at all.

Advantages of
these taxes above
direct taxes on
property.

I think also, that in those particulars in which a tax proportioned to our expences, either does, or may be contrived to, vary from one intended to levy the same sum in proportion to the value of property in Great-Britain such a tax on our expences appears the more eligible.

1. A tax proportioned to the expences of persons residing in Great-Britain, will collect a proportion of the income of the various profitable professions and employments amongst us, and of the annual gains foreign and inland commerce; all which being received and enjoyed by virtue of the laws, and under the protection of this government, should together with the annual income of our property, contribute towards it.

2. It will likewise collect and take in a proportion of the annual income of such estates or employments as supply the expence of foreign on different accounts residing in Great-Britain, as well as of such of Majesty's subjects who chuse to reside here and support their expences in the income of estates in Ireland, or any of our colonies or plantations America or elsewhere; from whom, in return for the protection the estates receive from the arms or influence of Great-Britain, supported by our expence, no contributions in common with the inhabitants of the kingdom can be thought unreasonable.

Contributions

Contributions thus made by persons residing in Great-Britain, in proportion to their expences, will likewise include a proportion of the annual income of such estates as may be brought hither by foreigners chusing to reside amongst us, or by any of our own countrymen returning with their families from other countries.

In short, it will include a proportion of all estates whatsoever, whether within or without the kingdom of Great-Britain, and whether discovered or not discovered, that any way supply the expences of our inhabitants, in a manner (as is above observed,) not grievous to, or liable to be complained of by the contributors themselves, and with the further good economy of sparing on ordinary occasions, and increasing, that publick stock, that unmovable part of our property within this kingdom, to which in times of extraordinary danger and expence we must necessarily have recourse.

Duties of this kind may also be made instrumental to the judicious regulation of our trade.

It may likewise be considered, in recommendation of this manner of applying the ordinary expences of our government by duties in the manner above supposed, levied in proportion to our expences, what farther conveniences to the publick may be procured by such duties, over and above such a supply to its ordinary expences; such as discouraging the consumption of such foreign commodities as may, in a manner plainly convenient to the publick, interfere with, or hinder the consumption of, or produce or manufactures of our own country; abating the extraordinary price of foreign commodities, or the exorbitant gains of foreigners by the importation of them; the diminishing a trade carried on with any of our neighbours, the balance of which is too evidently in their favour; or encouraging any other more profitable branch of the British commerce; or the preventing the increase of any particular article of expence, that may too plainly tend to debauch the manners, or abate the industry, of his Majesty's subjects. Of this kind many are the conveniences that may be secured to a country, by the same measures that supply the ordinary expences of its government. And when it shall be considered to how many publick uses of this sort several of the duties appropriated to the payment of our debts are subservient, besides the annual income produced by them; believe it will appear by no means eligible, and much less necessary, that a whole of those duties should, immediately after the payment of the publick debts, be removed and determined; when the same conveniences may be still preserved to us by the continuance of them, and the income of those duties be made to supply such of our expences as are now provided for by less equal, or less beneficial, taxes.

Such considerations as these, I think, are sufficient to remove the above-mentioned supposition, that the payment of our publick debts is consistent with the interest of a British ministry; in which, however, I could

It may further be observed, in answer to the reason advanced above in page 419, that the views mentioned in that reason are too remote to influence the ministers of state for many years to come.

could still advance farther, by remarking how remote the views of interest of this kind are placed, by the length of time that will be necessarily required for the discharge of our present debts from a Sinking Fund and by observing, that the removal of any part of the present duties which are any ways inconvenient to the publick, and are continued only because appropriated to the payment of some part of our debts, by no means imply or even admit of a reduction of officers employed in the collection of those kind of duties, either by way of custom or tax in the several ports or districts in Great-Britain, in proportion to the increase of such abolished duties; and from several other reflections that have occurred to me on this subject, if I did not think it unnecessary any farther to follow so groundless and indecent a jealousy of the integrity and public spirit of such of my countrymen, who shall for the future deserve to attain to the favour and confidence of his Majesty or his successors.

Thus far I have been endeavouring to make out, that the project already made of the present Sinking Fund is an expedient, from which may with great confidence expect the full and effectual payment of principal of our present debts within a few years. Upon which, Sir, I have dwelt longer than you may have thought necessary, I hope you will be pleased to consider in excuse of it, how far I must have been led to do, by attending to the happy influence that a general confidence in the efficacy of this expedient would have on the credit of our publick funds especially in case that the measures lately taken by some neighbouring princes should make a rupture with them necessary to us; and how far a general opinion of the efficacy of this scheme has a tendency to fortify and increase the success of it.

An inquiry whether the Government ought, in prudence, to endeavour to procure a further reduction of the interest of the national debt.

I am now brought, Sir, to the last task that, in obedience to your commands, I have assigned myself; and am to inquire what measure may be most for the interest of the publick to take in the application of the Sinking Fund from the year 1727. About which the question that can, as I think, be put is, Whether it shall be from the first advisable for us to endeavour after a greater increase of the Sinking Fund, by a farther reduction of the interest of the publick debts? Or it may not be then on the whole more for the publick interest, so end only after such an increase of the aforesaid fund, as will be produced the application of it from time to time to the discharge of the publick debts, and the addition of the yearly interest of such of the said debts shall be from time to time paid off.

Before I proceed to any other consideration which it may be thought material to attend to in determining this question, I shall take leave to state the greater effect the first of these different measures would have in ac-

ing the payment of the publick debt than the other of them. And I shall first to do, because in an affair of this publick concern, and as we are not to be supposed to give ourselves the trouble of the same things in computation, that we should use in our own private affairs. I little apprehensive that people, when they turn their thoughts to this, are apt, upon any increase of the Sinking Fund, to promise themselves a farther degree of dispatch in the payment of the publick debts in proportion to such increase. For an instance, to explain my meaning: let, that upon stating from the above-mentioned supposition, that the Sinking Fund of £1,000,000 was increased to £1,500,000 *per annum*, by a payment of 1 per cent. interest on 50 millions, the debt supposed to be paid off by it, on stating such a case, I say, I fear it would be inferred, from the Sinking Fund's being increased to half as much as it was before, that the publick debts would be likewise paid off by the Sinking Fund so increased half as soon again, or that the publick debts would be paid off by a Sinking Fund of one million and a half *per annum* in two third parts of the time that would be taken up in discharging by a Sinking Fund of one million *per annum* only. But this inference would not be true, by whatever means the Sinking Fund were supposed to be increased; and least true, when the increase of the Sinking Fund is by a reduction of the interest of the debt to be paid off by it.

Of the effect which an increase of the Sinking Fund from £1,000,000 *per annum* to £1,500,000 *per annum*, by a reduction of the interest upon a debt of 50 millions from 4 per cent. to 3 per cent. would have in accelerating the time in which the said debt would be completely discharged.

If the aforesaid fund of £1,000,000 *per annum* were increased to 500,000 by an addition made to it of £500,000 *per annum* provided a new tax, or any otherwise than by an abatement of the interest of the millions to be paid off, which should continue to carry 4 per cent. interest, it would be true, that while the said increased Sinking Fund is supposed to be applied to the discharge of that debt, it would pay off in any year half as much again as the Sinking Fund of one million only, running at the same time to be applied to the same purpose, would do the same year; and at the end of any number of years, in which both debts are supposed to continue so applied, will have paid off a principal exceeding the principal sum paid off by the Sinking Fund of one million only, by one half part of the latter; or in other words, the principal sum paid off by the aforesaid greater fund will be so that paid off by itself, either in an equal number of years from the time they begin to be applied, or in any one year equally distant from that time, in the proportion of three to two. And in this sense the aforesaid greater fund may be said to pay off the publick debt half as fast again, as in the same time will pay off half as much again. But from hence it is not to be inferred, that the less fund will be half as long again as the greater in discharging the same principal sum; or that the same principal sum would be paid off by the greater fund in two thirds of the time that would be taken in discharging it by the smaller fund: and of this the plain reason will

soon appear on inspecting the above-made computations, from which may be observed, That the Sinking Fund applied, as we have all along supposed it, is increasing by an addition in every year made to it of the interest of that principal sum which was paid off by it in the year before from whence both the income of the fund itself, and the principal sum annually paid off by it, are in every year greater than in the year before, and increasing in every year by an addition greater than the addition made to it in the year before: from whence it necessarily follows, that in a series of payments made by the Sinking Fund for any number of years come on, the payments towards the latter end of such series must be considerably greater than those before, and that the amount of the payments for any number of years separated at the latter end from the rest of the series must greatly exceed the amount of the payments for any equal number of years in any other part of the same series. And from hence it must appear that the excess of the payments made by the greater Sinking Fund above those made by the less in the same number of years, will not be a rule for determining the time in which they must severally be employed in discharging the same principal sums.

And it will be further from the truth, in the case of the Sinking Fund increased from an abatement of the interest of the debt to be paid off, to an addition of an annual income equal to one half part of its income before such increase, to suppose, that from thenceforth the debt will be discharged in two third parts of the time which would have been otherwise required, because the additions from time to time made to a Sinking Fund employed in the payment of a debt carrying 3 per cent. interest only, do not increase in the same or so great a proportion, as those made annually to a Sinking Fund in the discharge of a debt at 4 per cent. From which circumstance the less Sinking Fund increasing by this greater ratio or proportion, would in a longer series than I hope we have any thing to do with in the present case, have so considerable an advantage, as to overtake the greater Sinking Fund in its payments, and from thence to be every year discharging greater debt.

But in the case we have supposed, of a debt of fifty millions, the time in which we have before computed that that debt, carrying 3 per cent. interest, may be paid off by a Sinking Fund of £1,500,000, is 23 years and one half nearly; and by the Sinking Fund of £1,000,000, the debt continuing at 4 per cent. interest, it may be paid off in about 28 years; that the time saved in the discharge of our debts by the reduction of them to 3 per cent. interest, appears, on the aforesaid suppositions, to be 4 years and a half, or thereabouts; which is something less than one fifth part of the time in which the same debt might be discharged without a further reduction of the interest.

The time of such complete discharge would not be thereby diminished to two thirds of what it was before.

But it would only be less than before by about one sixth part of its former quantity.

Another way of stating the advantage to the publick in this contraction the time which our debts may take up in the discharge of them, from 23 to 23 years and a half, would be to find out and assign that annual sum, which, added to the above-supposed Sinking Fund of £1,000,000 at the publick expence, and without any further reduction of the interest of the debt to be paid off, would answer the same purpose as the addition of £500,000 to that fund taken from the income of the publick creditors, and contract the time in which the payment of 50 millions would be completed, from 28 years to 23 and a half. And this, Sir, will be found to be almost £322,000; which yearly expence to the Government for 23 years and a half, would answer the same purpose as the above-supposed deduction of £500,000 *per annum* from the income of the publick debts. And this advantage I chuse to state distinctly as it is, before I proceed farther, because I think in all the discourse I have met with on this publick affair, I have seldom heard any distinction made about the convenience of the several reductions of interest from 6 to 5 per cent. and from thence to the rate of interest, at 4 per cent. which is shortly to take place; or relating to the further reduction to 3 per cent. which we seem to intend and be providing for; but on the contrary, they seem all to be considered and expected alike, as of equal advantage in dispatching the discharge of the publick debts; though it be at the same time true, that by the first of these reductions we came only to have any Sinking Fund at all; and to the second of these reductions, together with the provisions at the same time made about the unredeemable annuities, we owe it, that the total payment of our debts by this expedient begins to appear practicable. But in those circumstances in which we now are, and with those views which we at present have of the payment of our debts within no great length of time, from the provisions already made for that purpose, by the reduction of interest hitherto effected or contracted for; I think we are at liberty, before any further steps of this kind, to consider of some probable consequences that may follow upon them; which to have produced as objections to any former reductions of publick interest, while they appeared so necessary, might have been thought impertinent or unaimely.

Another way of estimating the advantage to the publick which would arise from such a diminution of the time in which the national debt would be discharged,

It is by no means a just conclusion, that, because the former reductions of the interest of the publick debts from 6 per cent. to 5 per cent. and from 5 per cent. to 4 per cent. have been beneficial to the nation, therefore a further reduction of the said interest from 4 per cent. to 3 per cent. will also be beneficial to it.

It seems to me to have been an opinion of late years pretty generally agreed to, (perhaps as long since as the celebrated Mr. Locke's performance on that subject) That all attempts to reduce interest by compulsive methods, or by force of any laws made for that purpose, are not only unlikely to succeed, but on other accounts inconvenient to the publick; but I know not if the interest of the publick in the reduction of it by any other means effected, has been much considered; or if such a reduction of interest is not usually expected by us with general satisfaction, arising from our regarding it as the effect of the common and natural causes of a lower interest in every country, and such alterations in our circumstances as are truly enumerated amongst the instances of publick prosperity.

A reduction of the interest of money, if it is brought about without compulsion, is commonly esteemed a sign of general prosperity.

Of the *causes of the variations of the rate of interest for money.*

When the interest of money is lowered in a country by the causes here mentioned, it may be considered as a sign of general prosperity.

But when the interest of money is lowered by the operation of other causes than those above-mentioned, it is not certain that it ought to be so considered.

Mr. Locke, in his *stated treatise on this subject*, mentions, as the natural causes of the variations of the rate of interest in any country, the variations of the proportion that the quantity of money bears in such country to the demand for it, arising either from the quantity of debt contracted amongst the inhabitants, or of the trade carried on by them. To which occasions of a demand for money, I think should be generally added all other circumstances in the affairs of such a country, as may be there supposed ordinarily to contribute so, or be the occasion of, a greater or less demand of that kind. And as a further natural and ordinary cause of a variation any where in the rate of interest, I should chuse to add such alterations in the circumstances or situation of the affairs of the country where such a variation happens, as may make it more or less dangerous or secure to advance money upon loans in all or any of the different kinds of business, where negotiations of this kind are usually carried on; by which last cause I am apt to think, that the more sudden and sensible variations in the rate of interest have been chiefly and most frequently every where occasioned. And when a lower rate of interest is supposed to be produced amongst us, by such causes as these are, it is perhaps most reasonable that it should be regarded with general satisfaction; as it is a proof of such a situation of our affairs as is of itself, and independently on this consequence from it, an instance and part of the description of our general welfare and prosperity; and as the monied man himself has in this case an equivalent for what he may be supposed to lose by the abatement of his income, in the greater safety, with which, on such an occasion, he lends his money, or the less hazard which he runs of the repayment of it, as well as in the greater frequency of opportunities which such a situation in our affairs produces, of putting out and improving money with greater safety. But as far as a lower rate of interest may be produced amongst us, either without such compulsive methods, or the concurrence of such natural and ordinary causes for it as are above mentioned, I apprehend that it yet remains to be inquired into, if it be the interest of the publick that it should be so? and, as far as the success of any measures entered upon for this purpose may be uncertain, if it be, with regard to the publick, as visible that such a reduction of our interest should be attempted or endeavoured after? And in this case it will be allowed me, that a lower rate of interest thus produced, or supposed to be so, is no longer to be considered as a proof of, or attended with the above-mentioned instances of, or publick happiness; such a rate of interest having no tendency in itself to increase our money, or the lenders security for the repayment of it, nor consequently, being of any effect to produce a real increase of our negotiations in advancing money; which, while no provision is made for increasing either, our capacity or disposition to lend money can by no means become greater, or more frequent, from the greater application to borrow money only.

I shall therefore endeavour, for, to describe such transactions in a manner with respect to our publick debts, as I apprehend may have been, first, referred to have had a great share, and influence, in producing amongst us, a very great and general variation in the rate of interest, and from which a still further reduction of the rate of interest, may be yet expected. I mean those great adventures in the publick funds, of late years so, apparently undertaken with a view to such gains, as might be quickly made by the different prices of them, and which have so much contributed to the late great and sudden variations in the market prices of these securities. In the infancy of these adventures, the chief or only motives to them, probably were those pieces of intelligence about the situation of our publick affairs, from the publication of which the adventurer might reasonably infer the general satisfaction or dissidence of the proprietors of the publick debts in their several securities. And as far as intelligence of this kind was true, and the general sense of the proprietors upon the publication of it rightly conjectured or inferred, the rise or fall of stocks produced by these adventures might be regarded as an event, which in a longer time, or in a less proportion would have happened, if these adventures had not been made; and in this view may not improperly have been called, the growth or declension of our publick credit. But as this practice grew upon us, it is not to be wondered at, if from the general industry of great numbers to be first acquainted with every material occurrence, to the publick, and to be earliest in the improvement of their information in adventures of this nature, several variations in the prices of our funds have been produced by transactions in them, undertaken upon false or uncertain intelligence, and groundless inferences and conjectures from it, which variations have not been afterwards to be accounted for from any real alteration in the posture of our affairs, or the general sentiments of the proprietors of the publick debts; and from which therefore the real state of publick credit at such a time would be uncertainly, if not falsely, inferred or determined. The later variations in the prices of our stocks would be still more improperly described to be the growth or declension of our publick credit; which credit, since the restoration of our tranquillity, and during the absence of our apprehensions for the publick safety, can only with propriety be said not to have been disputed or called in question, and which cannot, I think, be supposed to have been of late at all attended to by the purchasers of our publick securities, at premiums and advanced prices far beyond those sums for the re-payment of which the credit of the Government is any ways depended on. In short, by whatever names we have been accustomed, or may chuse, to describe the rise or fall of our stocks, I submit it to such persons who have made any observations on the late transactions in Exchange-Alley, if they have not (and especially the rise of them) been generally occasioned by such adventures made in them, as persons have been induced to from the hopes of gain, from a further

Of the effects of
stock-jobbing on
the prices of the
publick stocks.

first, in the nature
of the publick
debts, and in the
state of the publick
affairs, and in the
general sense of the
proprietors of the
publick debts, and in
the growth or declension
of our publick credit.

And as this practice
grew upon us, it is not
to be wondered at, if
from the general industry
of great numbers to be
first acquainted with
every material occurrence,
to the publick, and to be
earliest in the improvement
of their information in
adventures of this nature,
several variations in the
prices of our funds have
been produced by transactions
in them, undertaken upon
false or uncertain intelligence,
and groundless inferences
and conjectures from it,
which variations have not
been afterwards to be
accounted for from any
real alteration in the
posture of our affairs,
or the general sentiments
of the proprietors of the
publick debts; and from
which therefore the real
state of publick credit at
such a time would be
uncertainly, if not
falsely, inferred or
determined.

further speedy variation in the price of them, without any regard to the continuance of it; and if these variations are not of late come to be expected from any the most inconsiderable occasions, or perhaps for no reason at all, but what is to be inquired for in the market, and amongst the accounts and contracts depending there,

While this disposition continues amongst numbers to be constantly adventuring in the publick funds, and consequently upon expectations that must be generally supported by the most inconsiderable reasons, it is hardly to be doubted but that at any time in the absence of our apprehensions of any general danger, the intelligence being spread amongst them that any scheme or proposals were to be set on foot, by which the rise of stocks was either intended or supposed, would generally determine the adventurers to expect and provide for such a rise of stocks, and by the contracts founded on these expectations in a great measure to produce it to effect which purpose, I hardly think it material, that any further reasonable provision should be made in the proposals or schemes themselves or that any thing would be further necessary for this purpose, than declaring the rise of stocks to be intended by them. Such a rise of stocks I am almost inclined to believe might be the first effect of any intelligence communicated in Exchange-Alley at such a time as I have above supposed that some proposals were shortly to be made to all or great part of the publick creditors, to agree to the further reduction of their interest to 3, or 3½, per cent. such a proposal would, probably, occasion a rise in the price of stocks.

Reasons in support of this opinion.

First, Such a proposal must suppose and lead our expectations to a rise of stocks in general; without which, or at least if the contrary should happen, such a proposal could by no means be executed or complied with it being necessary to the success of this proposal, that the market price should, at the time of making it, offer the proprietor as much, or more, as, if he declined to comply with it, would be payable to him by the Government. And as the greatest part of the proprietors of the publick debts have been at different times incorporated for the purpose of carrying on certain trades, from the profits of which (as I would willingly suppose 1 per cent, or more, has been annually divided over and above the interest of their interest in the publick debts; if their annuity from the Government, when reduced and diminished, continues to be valued as before, price of that part which is not liable to any diminution from these proposals, may well enough be expected to rise in some proportion to the reduction of their annuity. Thus, if to the proprietors of South-Sea stock for instance, it were proposed that their annuities in the year 1727, should

When thence reduced to 9 pence upon the supposition that their shares in the publick debt should, after such a reduction, continue to be valued as before, at par, it might be as reasonable to expect that the 10 per cent. continuing to be divided on every hundred pound stock, should be from thenceforth valued in the price of it at 33*l*. 6*s*. 8*d*. as it was before, so that it should be ever valued at 2*5*l. And from the rise of that part of our publick securities which fall under this consideration, some advance in our other securities may likewise be expected, as the money received on the sale of those stocks which shall first, and in the greatest proportion, rise in this occasion, is generally observed to be applied to the purchase of that part of our publick debts which is conceived to be less liable to variation in the prices of them.

Secondly, Such persons as are observed to be constantly adventuring in the stocks from expectations of gain, either from the rise or fall of them, will be generally supposed to be determined to these adventures by the lowest degree of probability, that they shall succeed in them; and it is hardly therefore to be doubted, but that the ordinary adventures in our stocks would be made upon expectations of the rise of them, upon the publication of any proposals from authority that supposed the rise of stocks, implied that it was expected by our superiours.

Thirdly, The rise of stocks upon this occasion would be further increased, by the disposition of those proprietors who are not ordinarily engaged in adventures of this kind, to wait for the utmost advantage to be made of the rise of stocks, whatever might be their sentiments about continuing proprietors of the publick debts when reduced to a lower price.

Fourthly, A rise of stocks on this, as well as former occasions, may be still further advanced, by the spreading of false computations of the value of our stocks, and idle opinions about credit and circulation, and by the use of a general example, assisted by the confidence of the proprietors of our publick debts in the authority by which these proposals may be recommended.

And when the stocks shall be sufficiently advanced to colour any proposals of this nature, it is perhaps not impossible that the concurrence of proprietors to such proposals should be obtained; though at the same time they may be generally dissatisfied with the lower rate of interest proposed to them, and severally determined on that account to quit their stock in the publick debts on the next convenient opportunity. For it is to be considered,

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That the reason for their objecting publicly or declaring their sentiments against any proposals of this kind, is removed by the price of the securities at market, where they are offered for the present as much, or more for them, as, if they thought proper to decline these proposals, would be payable to them by the Government; and that the general dissatisfaction of the proprietors should not determine them to take advantage of the then market price for the sale of their securities, and by that means occasion the fall of stocks, and prevent the success of these proposals, may in great measure be accounted for from a general inclination to have the utmost possible advantage from the rise of stocks, from the difficulty of finding on such an occasion any immediate employment for their money, and their impatience of its lying by them unemployed, joined with the dependance which men generally have on their own foresight and skill in the choice of the fittest opportunity for this purpose; from which motive while the bulk of our public securities may be supposed to be kept from market, that part of them which shall be brought there, by the more war or determined of the proprietors, will be found for a time provided for by those considerable sums which the estates and credit that persons engage in such adventures in the stocks as I have above described, will for a time supply the market with.

From the foregoing reasons it seems probable that the consent of the publick creditors might be obtained to such a reduction of the interest of their debts.

Nevertheless such a measure might not, upon the whole, be advisable.

I think, Sir, from hence it appears possible, that a proposal for reducing the interest of our publick debts, though without any reasonable foundation, may, as our affairs now stand, succeed even so far as to obtain the concurrence of the proprietors. And the inference which I would draw from thence make is, that it belongs to them, by whose influence or advice such a proposal shall at any time be made, first to consider if there be a solid foundation for it, or if it be likely still farther to succeed; and not to depend on the consent of the proprietors, in this manner obtained, as a sufficient proof that such a proposal was reasonable, or as a security for the still further success of it.

Whenever therefore the further reduction of the interest of our publick debts shall be attempted, it should be first enquired if the real proportion of our ordinary necessities for money to our capacity and disposition to supply them, have been so far altered as to admit of it; and if such an alteration has proceeded from those reasonable and general causes of which we are likely long to continue and support it. Nor will such an alteration be safely inferred from the market-prices of our stocks, any further than those prices are determined and produced by such purchases only as are made with a view to the improvement of the money laid out in these securities from the interest or income of them; by which purchases of many years the prices of our stocks have so seldom been determined, that perhaps it may be more reasonable in this case to conclude from an enquiry into the

are of interest ordinarily reserved on private loans, or into those other transactions in which we are usually directed by computations upon the ordinary rate of interest amongst us. These transactions, however, must, as well as the prices of stocks, in this case be considered and attended to in this account, that persons in the disposition of their money will, as often as they think themselves equally secure, be determined by the greater interest they are offered for it; from whence it must be expected, that when the income of our publick securities, compared with the prices they are sold for, offer a less improvement for our money than may with equal security be made of it by private loans or otherwise, the general industry of mankind to make the best improvement of their estates, will quickly reduce either the price of our publick securities, or the rate of interest in such private transactions as aforesaid.

I question therefore, if any attempts to reduce the interest of our publick debts below 4 per cent. at present will be of any lasting convenience to the publick, or ever can be so till such a lower rate of interest shall be preceded by its being customarily accepted of upon private loans on unquestionable securities. For let it be considered how such a lower rate of interest can otherwise appear to be founded on any real variation in the proportion of our necessities for money to our capacity or disposition to supply them; or if the contrary does not appear, from a higher rate of interest ordinarily paid at the same time upon private loans. And while this continues to be the case, how reasonable is it to apprehend, that when the money and credit of those adventurers, who first advanced the price of stock, shall be withdrawn, the same ordinary necessities for money, without any increase of the provision for supplying them, will bring the proprietors of the reduced securities to expect and look out for the former annual income for their money, and thereby occasion a declension in the price of these securities proportioned to the diminution of their interest?

How far the continuance of those adventures, by which the price of stocks is supposed first on such an occasion to be advanced, may be depended on for the support of it, may be collected from the motives by which the adventurers were first engaged in them; and is from thence to be expected but till the utmost probable rise of stocks from such proposals has been effected: after which that the former supplies from their credit and estates should be withdrawn from market, is not all that is in this case to be apprehended, it being further probable that they will be from thence employed in depreciating those securities which were at first advanced by them, with a view to the same kind of profits from the fall, as they before expected from the rise of stocks. From which, together with the fresh necessities which the more inconsiderate of these adventures will naturally produce, it would not be at all strange if the price of stocks should be carried lower,

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The reduction of the interest of the national debts, from 4 per cent. to $3\frac{1}{2}$ or 3 per cent. ought not to be attempted till a proportional reduction of the interest of money lent upon private mortgages shall have previously taken place.

beyond the declension of them in proportion to the diminution of their income, and the rate of interest for a time become higher than it would have been, if such an attempt for the reducing it had not been made. And this consequence of a rise of stock from adventures of this kind must some time or other be expected, as far as that rate of interest which our real necessities would produce is varied from or misrepresented by such adventures. It may not probably immediately succeed a rise of stocks by these means effected; these adventures in a time of general tranquillity may for a considerable time be protracted by further views, or the market supplied by a succession of them; and this has often been the case, till upon the arrival of some intelligence about the situation of our affairs which we call bad news, these views have been given over, and the declension of our stocks on that occasion attributed to, and accounted for only from, that intelligence: but if the real occasion of the great variation in our stocks at such a time were further enquired for, it would be found to be the precipitate sale of great quantities of stock, which, with such views as aforesaid, had been before bought up; and that this declension would be certainly, if not so suddenly, have happened from the same occasion, without the intervention of such intelligence, when these views should have been on any other account given over; or when (as, I think, I have heard some persons acquainted with these transactions express themselves) *the game had been played out.*

It is true, indeed, that the Government by those terms on which they borrow, I mean, by engaging only for the payment of the interest of annuities contracted for till the repayment of the principal, avoid all inconvenience from the interest of money advancing after their contract for the reduction of it, and leave the entire disappointment upon the proprietors. But I submit it, how far the publick can be considered to be unconcerned in a disappointment of the publick creditors, obviously owing to their concurrence with proposals recommended to them by authority; or in that general mutiny and discontent, which will be the necessary consequence of such a disappointment: which from a remarkable instance of this nature, after the execution of the late South-Sea scheme in the year 1720, we must have observed to have been once regarded by the Legislature, as of sufficient moment to induce them to release the most considerable advantage that the publick had agreed for from that scheme, though set on foot upon the proposals of the creditors themselves. And if ever the publick creditors should be generally disappointed by a considerable discount upon their securities, obviously owing to their concurrence to an abatement of interest recommended by publick authority, and proposed for the convenience of the publick; I doubt if their expectations of relief from the Government could be thought less reasonable. I cannot, for my own part, but think, that the general submission of the publick creditors in their contracts with

It is very much to be wished that the market-price of the publick debts may be always either equal to, or greater than, their *par*, or original, or nominal, value.

the Government, to wait for the re-payment of their principal till the publick convenience will admit of it, and waving any such agreement about a determined time for the re-payment of it, as in private contracts is ordinarily provided, would be far from removing their expectations of profits under a disappointment of this nature; and rather apprehend, that this submission would be urged on such an occasion, as a meritorious instance of their confidence in the care and protection of our Government, and as a reason for their expecting in return for it, that what may be then called Publick Credit should be kept up, and their securities by all possible means preserved at *par*, till the time when they could be discharged.

The success attending the reduction of so great a part of our debt from 6 to 5 per cent. and from thence afterwards to 4 per cent. cannot certainly be looked on as a foundation for expecting the same event of our endeavours to reduce interest still further. As to the first of these reductions, we shall find it, on looking back, to have been attempted quickly after such an alteration in the circumstances of our affairs, as furnished the best foundation for our hopes of succeeding in it: at the end of a long and expensive war, that threatened us with the loss of every thing valuable, but more particularly of that part of our property which had been advanced for the services of the publick; at a time when those necessities of the Government were removed, which had obliged us for several years before to be continually increasing the publick debts, and at the same time admitted of no provisions for the discharging of them; at a time when the lasting prosperity of Great-Britain was lately secured to us by his present Majesty's accession to the throne, and, soon after, by the entire defeat of the last attempt that was likely to be made to disturb or prevent the present happy establishment. From such a foundation as was then laid for the growing wealth of the kingdoms, from the increase of our people, our commerce and manufacture, and for the particular security and greater confidence of the publick creditors, it was most reasonable to expect, that the abatement of publick interest then proposed should take place; especially, when these proposals were attended with the provisions that were then first made, for securing and rendering practicable the discharging the principal of the publick debts.

Of the causes of the success of the first of the two late attempts to reduce the interest of the publick debt; by which it was reduced from 6 per cent. to 5 per cent.

or do I think it unreasonable to have expected, that by degrees, and from the fruits and sensible effects of this happy alteration in our circumstances, a further reduction of publick interest to 4 per cent. which has since been effected for, and which in the year 1727 is generally to take place, might likewise be effected. And, though it may be doubted, whether the effect of this reduction of publick interest has been yet fully tried, upon recollecting how little the interest of the proprietors of the publick debts in this reduction was attended to by themselves, at the time when it was agreed to; and how possible it is, that a far greater number of the proprietors of the publick funds may have proposed to quit their interest in them, when the

Of the further reduction of the said interest from 5 per cent. to 4 per cent. which is to take place in the year 1727.

reduction is actually to take place, than will be able to find customers for it unless at a considerable discount; yet when, on the other hand, it shall be considered, how far this reduction has been preceded by considerable loans amongst us at the same rate of interest, as well before the exigencies of our government during the late war with France, as since his Majesty's accession to the throne; I hope we may cheerfully conclude, that this reduction of the publick interest may well enough be supported by the regular application of the annual income of the Sinking Fund to the discharging of our present debts. But no difficulty of finding employment for money at 4 per cent. interest, nor any private loans at a lower rate of interest amongst us, can yet, I think, lead us to expect that a further reduction of publick interest to 3 per cent. will be for any length of time submitted to.

There is no reason to think that a further reduction of the said interest to 3 per cent. will, for many years to come, be practicable.

And while this continues to be the case, and from 4 to 5 per cent interest is every day offered upon unexceptionable securities, I should think it a more reasonable use made of recollecting the late reductions of our publick interest, to place them to the account of our present happy circumstances; and, before we proceed to expect from the late alteration in our affairs a further reduction of interest, to consider how far we are indebted it upon that account already.

I have indeed sometimes heard it said, that the last reduction of our annuities to 4 per cent. still wants to be taken care of; and that the price of our publick securities proportioned to that rate of interest, is only to be supported by such adventures as will be encouraged by keeping in view the prospect of a still further reduction of those annuities to be attempted from these persons I very much differ; and cannot but think that this last reduction (if no new troubles presently fall out) would be effectually supported by the future regular application of the Sinking Fund; from which in a number of years, I should rather expect that a further reduction of interest may be naturally and reasonably produced, if the effect of the provision be not before-hand too far presumed upon and anticipated. But whoever really thinks that the further reduction of publick interest must be kept in view, in order to support the reductions already made, evidently supposes us to be proceeding in measures with regard to our publick debt in which we must somewhere stop, and whenever we do so, repent of every step we have taken in advancing thither.

For want of examples amongst ourselves, as I suppose, to support our expectations of the further reduction of publick interest, I have sometimes heard the present low rate of interest in some of the trading towns of our neighbours the Hollanders, quoted to prove the probability, that the same, or something near the same, low rate of interest may be made to take place amongst us too. But I see not why the low rate of interest in that country should

Examination of an argument in favour of a farther reduction of the said interest, drawn from the low rate of interest in some of the towns of Holland.

would be more regarded as the standard for the rate of interest amongst us, than the higher rate of interest in other neighbouring countries, unless on account of our greater commerce and negotiation with the Hollanders. And after I have admitted that the rate of interest amongst them is on this account most likely to have some influence upon our's; I must expect it could be allowed me, that this lower rate of interest than our's having for several years prevailed amongst the Hollanders, has already had its effect in us in the reduction of our interest to that rate which we now consider to be brought to; and that the further effect or influence of their example in the reduction of our interest, is only to be expected from the further reduction of the rate of interest below what it is at present supposed to be amongst them.

That the circumstances of our affairs are the same in all those particulars as lead to a low interest in any country with those of our before-named neighbours, is by no means, in the present inquiry, to be presumed; if they were so, our rate of interest must now be pretty nearly the same with theirs. But if our rate of interest considerably exceeds theirs, and has which, as I have been informed, is true) for a long succession of years constantly done so; it must be inferred, that our circumstances, in some particulars that influence the rate of interest, differ much from theirs. And the constancy with which our rate of interest has been observed for a long time to have exceeded theirs, is enough to satisfy us that the occasion of it is to be enquired for in some difference in our circumstances which has continued with equal constancy, and for the same length of time; and not amongst any projects or contrivances at different times set on foot by either us, to answer any purpose of this nature.

The true and general reason of this difference between our rate of interest and theirs, has, perhaps, been long since assigned by the above-mentioned Mr. Locke, and seems most probably to be the very different proportion which the lands or property of any other kind producing a certain annual income amongst the Hollanders, taxed as that kind of property has been with them, bear to the great stocks and other personal estates of the inhabitants of that country, from that which the value of lands and other property of the same kind here bear to the personal estates of this kingdom. To this difference it seems owing, that while the Hollanders can find little other employment for the money he can spare from his adventures within his own country, than in supplying the necessities of their commerce from time to time produces, the monied inhabitant of this country, besides the opportunities offered him from the ordinary necessities of persons engaged in trade, is hardly ever without proposals for employment of his money in supplying the wants of the proprietors of lands, by either purchasing or advancing money upon their estates; and

The true reason of the difference between the rates of interest given for money in England and in Holland.

and from hence is in a condition to demand and obtain a greater reward for the use of money than the Hollander can do, where the demand for in his own country is so much less. This difference between us, as far as it will, be allowed to have been one cause why our interest has hitherto exceeded theirs, will be allowed also as a reason why it should continue so, till the inhabitants, wealth and commerce of Great-Britain, should have increased in the same proportion to the extent and value of our land as it may be observed they have done in the Seven Provinces.

The expectation of a further reduction of the interest, of the publick debt has contributed to increase the pernicious practice of *stock-jobbing*.

I cannot forbear thinking, that upon this occasion it deserves more seriously to be considered, how far our late expectations of continuing attempts to reduce the rate of interest, has contributed to promote and increase the aforesaid traffick in Exchange-Alley: a practice that, in the midst of those reproaches which it lies under by the name of *stock-jobbing* and the most serious complaints of its ill consequences to the kingdom upon every declension of our stocks, seems to me to be still growing upon us. Whenever it shall be seriously intended to prevent or restrain this practice, I believe it will appear, that whatever contrivances may be provided for prohibiting the contracts in Exchange-Alley in the manner they are now made, or altering the manner of conveying our interest in stock from one person to another, while they increase the difficulty of the more innocent and necessary transactions in the publick funds, will have little further effect on this practice, than to force it into some other channel, and perhaps increase the profit and employment of the banker only, by making his credit or assistance further necessary; and that the most reasonable method of preventing it, will be removing the encouragement and temptation to it. And though our complaints of these adventures are then on generally made, when they seem to contribute to the declension of our stocks, a little enquiry will convince us, that the foundation for such a fall of stocks was really laid by those adventures which seemed to contribute to, and attended, the *rise* of them. If during a time of general tranquillity from unlimited expectations of the perpetual advance of publick credit countenanced amongst us beyond any sufficient foundation for it, persons are induced to spread their estates upon the utmost price of our publick funds, in such a manner that a variation of 2 or 3 per cent. in the price of them threaten them with the loss of the greatest part of their estate: what can be expected, but from the earliest appearance of publick trouble an idle rumour, (though improbable to be true,) or the apprehension of an ill accident, (though most unlikely to fall out,) should determine them, in this situation of their affairs, to consult their safety with the utmost precipitation, and crowd the market with the stock of which before they continued proprietors upon such desperate terms. A fall of stocks by this means occasioned, with persons less exactly acquainted with the reason of it, serves as a confirmation of every false report at the same time published

the prejudice of our affairs: from hence still further quantities of stock brought to market, and a further declension in the price of them reasoned; from whence, to greater apprehensions of publick danger, from thence to the further fall of stocks, by turns producing and raising each other, we may have often been observed to proceed without possibility of putting a stop to either of them. Upon such an occasion as it has often, and perhaps constantly, happened, that several persons applying to themselves gain from the calamities of the publick, have, on presumption of the fall of stocks, contracted for the delivery of stock which they had not, and could propose to furnish only by the purchase of the growing apprehensions of others: should afterwards bring to market; and of these adventures it has been usual, on the fall of stocks, especially to complain. This is a practice, which has doubtless often contributed to the misfortunes of the publick on an occasion of this nature; in which, I doubt not, would in a great measure be prevented for the future, if the excessive adventures in the purchase of stocks, in expectations of gain from the rise of them, were first prevented; to which the contrary practice is chiefly owing. For it will, on enquiry, be found, that stock-broking begins from, and people have been usually initiated into this trade by, general expectations of the rise of stocks; in which when they are once habituated, and the expectations of gain from the variations in the price of stock are become the only end of their transactions, their view of advantage by the rise of stock is quickly changed for hopes of it from the fall of it.

Besides, the unreasonable prices to which the extravagant adventures for the rise of stock have carried them, must first have taught the more wary and credulous adventurers to expect the fall of them, and have been an occasion of that success that has encouraged the continuance of these ventures for the fall of stocks: nor would the performance of their contracts, for the delivery of stock, which they had not, at certain times, be really practicable, but from the contracts for the same time ordinarily made for purchasing stock without providing the money to be paid.

And if it be to these sanguine expectations of the rise of stocks, and the ventures founded on them, that the frequent and excessive variations in the price of them are really and ultimately owing; how dearly do the stock pay, in every instance of perplexity in our affairs, for any convenience to be reaped from, or use to be made of, this prevailing humour in the absence of our apprehensions of publick danger?

The

All consequences that result from the variations in the price of the publick funds occasioned by stock-jobbing contracts.

The rise of our stocks, produced by the assistance of such inconsiderable adventures in the purchase of them as have been above described, is at best of itself, and without attending to any consequence from it, a matter of absolute indifference to the publick in the absence of general danger; the consequence of it in the declension of our stocks, upon the approach of publick troubles, is by no means so; then it is that the general diffidence in our securities and wreck of publick credit is of the utmost service to us, by rendering difficult, if not impracticable, the raising such supplies as an occasion of this kind may necessarily call for; and the variation in our publick funds at such a time may be regarded by our neighbours as the measure of our apprehensions from their attempts upon us, and encourage them in their presumption on the unsettled circumstances of our affairs: all which difficulties in our affairs on such an occasion attended with false and groundless reports and apprehensions of our danger general mutiny and discontent, seditious exceptions to the conduct of superiours, and great distress and interruption to our commerce, I cannot but think we in a great measure owe to such inconsiderate purchases of our publick funds during the general tranquillity; and that they might a great degree for the future be prevented, if, by removing all encouragement to the extraordinary rise of stocks, the publick funds were suffered to fall generally into the hands of such persons, who, satisfied with their income, shall purchase them as a supply for their ordinary expences, and money which they are not soon likely to have any other occasion for.

The proprietors of the publick debts ought to be treated with lenity and tenderness, as persons who have deserved well of the publick.

The proprietors of our debts have, as such, not deserved severity from the publick; but, as subjects of this kingdom, are intitled to have their interest regarded by the Government, as far as the publick convenience admit of it. And in this view there may be some room to consider unequal hardship to the publick creditors, by the loss of a fourth part of the annual income of their estates, implied in the success of an attempt to reduce their annuities to 3 per cent. And while the convenience to the publick, to be obtained by such a reduction, is supposed to be the cause of the discharge of the publick debts; the hardship appears greater from the circumstance, that what shall be thus annually deducted and taken from their income, will not go so far in answering this purpose, as two thirds of the same yearly sum any other way supplied, and for this general convenience, more equally levied upon the subjects of this kingdom: £322, *per annum*, or thereabouts, raised at the general expence, and added to the Sinking Fund of £1,000,000, being, as I have before observed, sufficient on the above-made suppositions, to effect the total discharge of the publick debts, as soon as the addition of £500,000 *per annum* deducted from the interest of those debts when reduced to 3 per cent. For it should be attended to, that though the gain or convenience to the publick is to be computed upon such of our debts only as from time to time remain

unsatisfied

and as long as they remain so; yet supposing the continuance and general success of this reduction of our interest, the loss to the publick creditors is from the time of such a reduction to be computed on the whole of the annuities that shall be reduced. If this, as a hardship on the publick creditors, should not be proper in this case to be considered; it may be so, however, for the purpose of collecting what their sentiments on this affair must some time or other be. The loss and inconvenience to them by this reduction will be so sensibly felt, that no misrepresentations can possibly long mislead them: the continuance of their submission to former statements of their interest has been already accounted for, by the then alteration in our circumstances for the better, making the purchaser of our reduced annuities a large amends in his greater security, and being content to the mortgaged men in general in the frequency of opportunities of improving their estates with safety. But will the present happy situation of our affairs admit of a further equal alteration in our circumstances for the better, or that shall in the same proportion increase our security in advancing money upon the publick credit? The former reductions of our annuities may have been recommended to the publick creditors, as the only means that could render the discharge of our debts practicable to the Government; but, as far as they are interested in it, is not that end sufficiently obtained? or is the prospect that the payment of the publick debts may be thereby effected sooner, by 4 years and a half in 28, than it would be otherwise, of consequence enough to the proprietor of any part of them, to induce him for that purpose only to part with for the future one fourth part of the annual income of his estate? However the reduction of publick interest hitherto effected may have contributed to the security of the proprietors, from the next reduction it is perhaps not unreasonable to apprehend a contrary effect; and next to the great difficulty in the discharging of our debts, the most reasonable foundation of our apprehensions may be, is becoming, in the opinion of some persons, a matter of too much indifference to the publick whether they are ever discharged or no. When the publick debts, by the further reduction of their interest, shall sit so easy upon us, as to require but one moiety of the annual provision at first made for the payment of it, and leave the other at the service of the publick, the danger seems to me by no means inconsiderable, that it may soon after be determined to employ the annual income of the Sinking Fund in the room of, and to ease the publick of some other taxes by which our ordinary expences are supplied; and that it may be thought as reasonable to rest contented with the recovery of half the annual income of the publick funds without any expence to us, as to redeem the whole of them with the trouble and expence of really discharging so considerable a debt.

And if this reduction of interest be successfully carried on, the loss and inconvenience aforesaid cannot be confined to the proprietors of our debts

L 11

And it would also be highly inconvenient to many other persons besides the publick creditors.

only, or to their property in the publick funds; for if it were so, it plain, the price of them, after such a reduction, must be abated in proportion to it: it must therefore, if it succeeds, take place in the interest of all private loans, in the profits of all the different employments of money, and by degrees must affect the profits of our commerce, and spread itself throughout the kingdom: an effect, which when not produced by or attended with the increase of our wealth, the revival of commerce amongst us, the succession of general tranquillity to a dangerous or unsettled situation of our affairs, or other like instances of general prosperity, I know not how to regard otherwise, than as an uncomfortable and general inconvenience in a country where the personal estates are so considerable here they are; which, if it should be thought not material to attend to, as hardship or inconvenience merely to particular persons, should at least put us in mind of the opposition that must sooner or later be expected amongst us to measures from which a great reduction of interest is apprehended.

Against this great and general inconvenience to the proprietors of personal estates from a lower interest, I would willingly place any further publick or private advantage that may arise from it, besides hastening the payment of our publick debts. The chief, if not the only, advantages of this kind that I have met with by any persons proposed from a lower interest, have been the increase of our foreign commerce, and the advance of the value of our lands and irredeemable annuities of any kind.

The increase of
our foreign com-
merce.

As to the first of these, it must be admitted, that cases may be put about the particular circumstances of any country in which a lower rate of interest would have a tendency to increase their commerce; as it might be an inducement to such persons who could no longer support themselves, or were not contented with the income of their estates at such a lower rate of interest, to engage in trade; and as it might be the means of furnishing others with money for the purpose of undertaking any particular branch of trade, at such interest and upon such terms as the profit of such a trade would only answer. But all the advantage of this kind that in our circumstances, and in the present case, we have to expect, is to be collected only by an inquiry into the present state of our commerce; from whence, if it cannot be made to appear that there is at present any profitable branch of foreign commerce neglected by us, the profit of which will, over and above the hazard and other expences of adventuring, exactly bear 3 per cent interest for the money employed in it, but will not answer four; I should think we have more reason to apprehend some ill consequences from a sudden reduction of interest amongst us with relation to our foreign commerce, which are by no means inconsiderable: such as

An examination
of the advantages
which some peo-
ple imagine
would arise from
such a reduction.

It is rather to be
apprehended that
such a reduction
will be hurtful to
our foreign com-
merce, and pro-
duce the follow-
ing ill consequen-
ces.

The rashly engaging unexperienced persons in unprofitable adventures, it ill consequence,
 their own and the nation's prejudice.

The increasing our adventures in the several branches of our present ad ill consequence.
 commerce beyond the demand for our commodities, or the possibility of
 rendering them with advantage at foreign markets; and thereby rendering
 the whole of our foreign commerce for the future less profitable; and
 this means,

The furnishing a temptation to the more skilful and experienced persons, at present employed in our foreign commerce, to remove with their effects and settle in other countries, from whence the commerce they are at acquainted with may be carried on with more advantage. An inconvenience which we have the greatest reason to guard against at this particular juncture, when our neighbours in the different parts of Europe are generally attempting to rival us in our foreign commerce.

And if amongst these consequences of the sudden reduction of our interest, the money of foreigners, which either our Government or private persons amongst us at present have the use of, (to whom most certainly a higher rate of interest than they can have at home for it, must have been a general inducement for their trusting it here) should be called from us, and applied to other uses; a higher rate of interest than before may not be apprehended, but an absolute impossibility of supplying the ordinary demands of our commerce for some time at any rate at all.

As to the proprietors of our lands and irredeemable annuities, I am Of the increase of the value of lands, or the price they will sell for, by such a reduction.
 content to admit that they may reasonably expect a higher price to be paid for their estates, in some measure proportioned to, and regulated by, a lower rate of interest produced by, and in proportion to, any solid and reasonable causes for it. But I think it has been with truth observed by

Mr. Locke on this occasion, that in this higher price of their estates, those proprietors are only interested who have contracted, or want to contract, and upon their estates; it being of no consequence to the person who either sells nor mortgages his estate, or intends to do so, what price he

This is no real publick advantage.

may procure for it; and it being as plain, that the person who on this occasion receives a higher price on the sale of his estate, from thenceforth stands in the place of the monied man, possessed of a greater sum of money indeed than he could have had before, but which will produce no more annual income, nor, generally speaking, go farther in any provision he intended for himself or family, nor in any other use that he can apply it to (except the discharge of such debts as he may have contracted) a less sum would have done when the rate of interest was higher. The principal, if not the only, general advantage of a lower interest to

the proprietors of land, is therefore so far as they have contracted debts which advantage to them, and to all other persons who have contracted debts, is exactly balanced in the publick accounts by an equal loss and inconvenience to their creditors.

Such a reduction would likewise, probably, be attended with a diminution of the general expences of the people, and, consequently, of the publick revenues arising from taxes on the consumption of commodities.

I will desire your attention, Sir, but to one consequence more, which I think will naturally and necessarily follow a further reduction of interest, if it can by any means be effected, or for any length of time prevail amongst us, without the concurrence of what I have hitherto supposed to be the natural and only reasonable causes of it, viz. a considerable diminution of our expences, which the publick, as our affairs now stand, and the proprietors of land in particular, seem to me not a little interested to prevent. A fourth part of the income and usual profits of the proprietors in this kingdom, withdrawn and deducted from the whole of ordinary annual expence, must occasion a very considerable diminution, when not supplied by the increase of those personal estates, or the growing wealth of our inhabitants, and must from thence occasion a considerable diminution in the price and consumption of our commodities. And this I apprehend will be the sooner and more sensibly felt, as the interest of money and the profits of personal estates are more generally funds for, and supply the expence of, the inhabitants of this metropolis of the kingdom, than of any other part of it; and as a variation in the fashionable expences here is most likely to spread itself, by the force of example, throughout the other parts of this kingdom, where perhaps they may not be the same occasion for it; from hence it deserves well to be considered, if the publick may not lose as much, or more, in their revenues arising from different commodities consumed amongst us, as may be saved by the reduction of our interest; or if the proprietors of land may not at last find themselves obliged to furnish from their own revenues the supplies for the service of the Government, which have been hitherto furnished by our expences. And if it be possible that this diminution of our expences should proceed further, in reducing the price of labour, from thence of our necessary provisions and the produce of our lands, the proprietors of those estates must in their turn suffer from the reduction of their annual revenues.

And it might be necessary to increase the land-tax to make good the deficiencies of these taxes.

It might be of great publick benefit to determine immediately, by an act of parliament, the order in which the publick debts should be successively discharged by the application of the Sinking Fund.

From such reflections as these, Sir, it has seemed to me not unreasonable, that we should at least for some time rest contented with such reductions of publick interest as have been hitherto made: from whence I have been further induced to think, that it would be of considerable convenience to the publick, if the application of the present Sinking Fund which stands now appropriated to the discharge of the publick debts in general, were by act of parliament determined as to the course and order in which those debts should be for the future discharged by it. The

measures with regard to the publick debts may possibly have not been hitherto proposed, on account of that advantage which the publick may have been supposed for the future to be in a condition to make in the further reduction of publick interest, while they reserved to themselves the preference of one creditor to another in the order of discharging them; but I submit it, how far this advantage would be prudently exchanged for the following conveniencies to the publick, from determining the order in which the Sinking Fund should be applied in the discharge of our present debts.

Advantages that would arise from such a measure.

First, The annual income of the Sinking Fund will, by this means, be more fully appropriated to the payment of the publick debts, and the application of it to that purpose more effectually secured, by entitling every particular creditor to expect the application of it in the order that shall be so determined.

Secondly, It will be of considerable use in fixing the credit of the publick funds, and the confidence of the proprietors on such foundations, as will support them in any time of publick difficulty, by removing all grounds for those apprehensions, which, on such occasions, are observed (greatly to the disadvantage of the Government) to prevail amongst us, that the income of the Sinking Fund will be applied to some other purposes than the discharge of our debts; and by giving every particular creditor an opportunity of computing and satisfying himself in the value of his interest in the publick funds, from the knowledge of that time when his principal will be punctually paid off.

Thirdly, It will in a great measure prevent stock-jobbing, by removing the temptation to it from the great variations in the market-prices of our debts, from such extravagant premiums paid for them in a time of peace, as if the income of them was conceived to be an irredeemable annuity; and such discounts on the other hand allowed upon them, in a time of the least general apprehension, as if they were regarded as debts almost desperate.

Fourthly, It will lay a further foundation for a greater equality in the prices of our publick debts, by giving an opportunity to the proprietors to suit their own convenience in the purchase of such part of those debts as are determined to be payable, as near the time as possible when they expect any occasion for their money; and prevent in a great measure the necessities of the proprietors being brought to market, especially in the manner in which, when any declension in the price of stocks is apprehended, it may be observed often to be done long before they have any real occasion for their money.

Fifthly,

Fifthly, It will tend to the increafe of our credit, and the facilitating both of publick and private loans at the prefent, or as far as is reasonable to wifh for it, at a lower rate of intereft, by capacitating fuch of the creditors, whose debts fhall be in a lefs remote order of payment, to lend out fuch fums as they may have by them referved for diftant ufes, in expectation of being fupplied for fuch diftant occafions by the payment of their fhare in the publick debts in the order and at the time appointed for it.

And laftly, fuch a determination of the order in which the Sinking Fund fhould be applied in the difcharge of our publick debts, and the notice the creditors would thereby have when they fhould be paid off, would give them an opportunity of looking out for, and providing, the moft apt and convenient employment for their money againft the time of receiving it; a convenience to the creditors themfelves, which, as the publick is always interefted in the innocent improvement of our eftates, may, I think, be efteemed a general advantage.

A recapitulation
of the feveral propo-
fitions that have
been advanced
and recommended
in the courfe
of this Eflay.

I fhall conclude, by putting together what I have been endeavouring to reprefent about the reduction of our publick intereft, viz. That the general and ufual rate of intereft in every country is determined by the proportion that the ordinary neceffities or demands for money amongst the inhabitants bear to their capacity and difpofition to fupply them; That any other rate of intereft produced without a variation in the proportion aforefaid, or a foundation laid for it, is not likely to continue; That we feem here to have had the effect of the late happy alteration in our publick circumftances, in fuch reductions of our intereft as have been made already; That the profpects of a ftill further reduction of publick intereft are a continued encouragement to adventures, which, though they may be made to contribute to the producing fuch a reduction for a time, are not to be depended on for the fupport of it; That thefe adventures are themfelves at all times a general inconvenience, and particularly prejudicial to the publick on the approach of troubles. I have likewife endeavoured to reprefent, that the further reduction of publick intereft is neither equally neceffary, nor of equal advantage to the publick, as either of thofe that have been already made; nor does it want to be explained, that the fame addition to the Sinking Fund, to be now made by the next reduction of the publick intereft below 4 per cent. will diminifh the remaining income of the creditors in a greater proportion than thofe before made, and be a greater inconvenience to them. I have recommended it to be confidered, how far a reduction beginning with the publick intereft muft, if it fucceeds, neceffarily fpread itfelf, and affect the reft of our perfonal eftates; and from thence the oppofition that meafures for reducing intereft will fome time meet with, where fome real alteration in our circumftances does not perfuade to it. I have propofed it to be inquired, if there be any other general advantage to be obtained by a lower rate of intereft amongst us, than in

regard

regard to our publick debts only. And from such considerations, would admit it, if it might not be convenient, that not only our measures for further reducing interest, but our expectations of it were at least for some time suspended; till after the regular application of the Sinking Fund now provided for a few years, and the intermission of such extraordinary adventures as aforesaid in our publick stocks, we might with more certainty collect what lower of interest our real circumstances will admit of.

In what little I have said about the consequences of a lower interest on commerce and expences, I have referred myself, Sir, to sentiments in which I have had the honour to agree with you, and must not pretend to have made out any thing to general satisfaction: if I had attempted to do so, I should have been carried too far beyond the design of this essay; and I should have been obliged to examine some prevailing opinions on this subject, which seem to me so far from being reasonable or true, that I have sometimes thought that part of them which the private interest of particular persons have not introduced amongst us, to have been taken up merely on account of their resemblance to paradoxes, and for that reason affording the worst amusement in conversation.

I would not have it, from any thing I have said, inferred, that I am in general against any expedient for the much speedier discharge of our present debts; I should be glad if any reasonable method for this purpose could be thought of; nor would any new burthen, or variation in the present burthen, the subjects of this kingdom, implied in any proposals for this purpose, with me an objection to them, if the means were but found out of providing such a new burthen, either to the property or expences of our subjects, in such a manner as would be generally submitted and agreed to; and that such an expedient were found out, I wish for this general reason, that whatever in publick affairs is thought of great and general importance to be done at all, should be done, if possible, as soon as it appears to be so; that the most eligible methods for effecting it are such as may be tried on, and finished under the direction of the same persons who were engaged in them; and that the success of such measures should be as great as possible hazarded by the different sentiments of their successors. But a consideration will not go far in recommending the further increase of the Sinking Fund, by reducing the interest of the publick debts; which, upon suppositions on which my calculations have been made, would not, if the Sinking Fund were increased by reducing their interest to 2 per cent. be paid in less than 20 years and a half, or thereabouts; if to 1 per cent. in less than 18 years and 4 months; or if the creditors would be satisfied without interest at all till the payment of their principal, in less than sixteen years and eight months.

I am, SIR, &c.

[*End of Sir Nathaniel Gould's Essay on the Publick Debts of this Kingdom.*]

Article CCCLXV. In the foregoing excellent pamphlet of Sir Nathaniel Gould, pages 420 and 421, the judicious author, having occasion to make mention of the land-tax, takes notice of the hardship arising from inequality; but adds that, in his opinion, there was no great occasion to dwell upon that circumstance, because there was *then* a prospect that, by the operation of the Sinking Fund, the national debt would be wholly discharged in the course of a moderate number of years, after which he advises, and likewise supposes, that the taxes which had been before appropriated to the payment of the said debt, would be employed in defraying the ordinary expences of Government, so as to make the annuities of the land-tax and malt-tax become no longer necessary. But no (in April, 1782,) that pleasing prospect is at an end, and, instead of there is reason to apprehend, that it will be necessary, not only to continue the land-tax upon its present footing of four shillings in the pound, or raise by it the sum of two millions of pounds, sterling, *per annum*, for great number of years to come, (perhaps, for a hundred years,) but even to increase the said tax to double its present quantity, or to raise by it the sum of four millions of pounds, sterling, *per annum*, in order to make good the payment of the interest of the prodigious sum of money to which the national debt will, probably, have increased before the blessing of a general peace shall be completely restored to us;—more especially, if our ministers of state should resolve to apply a million, or twelve hundred thousand, pounds a year out of the Sinking Fund to the gradual diminution of the capital of the said debt in the manner that has been above recommended. The land-tax must, therefore, in our present unfortunate circumstances, be considered as a *permanent* part of the publick revenue; and consequently the very great inequality with which it is assessed on the inhabitants of different countries in the kingdom, and even on those different districts of the same country, is an object well worthy the attention of the Legislature; and, if it be really a grievance and a measure of general injustice (as, I must confess, it has always appeared to me to be, and Sir Nathaniel Gould in the passages above referred to, in pages 420, 421 seems to have considered it,) it is highly fit to be speedily corrected and redressed. I therefore hope I shall be excused, if, before I return from this political digression concerning the Sinking Fund and the national debt to the Doctrine of Life-annuities, (which is the proper subject of the book,) I lay before the reader some reflections on this subject of the inequality of the land-tax, and on the expediency of assessing it for the future in more equal manner, which were first drawn up and published in the *Publick Advertiser* in the month of January, 1780, together with a proposal to make the interest of all future sums of money that should be borrowed by the publick, liable to pay land-tax in the same proportion with the rents of land to the end that all the subjects of the Crown, those who are possessed

personal property as well as those who have landed estates, may both contribute, and be seen and known to contribute, in the same degree, in proportion to their respective annual incomes, to the publick expences which the exigencies of the state make necessary. These reflections are as follow :

On the expediency of an EQUAL ASSESSMENT of the LAND-TAX, and of making the annual interest of all new publick loans of money subject to the same tax.

CCCLXVI. It seems to be an indisputable maxim, founded both on equity and good policy, that, when taxes are necessary to the exigencies of the state, persons possessed of equal incomes should contribute equally towards them. And, whenever this maxim is departed from, even though the ground of such departure may be just by virtue of some original compact in favour of the persons who pay less than their share of the publick taxes, it is sure to create some jealousy and uneasiness in the other members of the state, and thereby to render the property so exempted somewhat less secure than it otherwise would be.

Of the very unequal assessment of the land-tax.

We have, indeed, in our government, two striking instances of a departure from this prudent and equitable maxim, which are frequently the subjects of very great complaints: I mean, the shamefully unequal assessment of the land-tax on the lands and houses of the kingdom, and the total exemption of all the interest of the publick funds from that and every other tax. By the unequal assessment of the land-tax, some people pay more than four shillings in the pound upon the rents of their lands, while others pay only eight-pence or nine-pence, or in some places, (as the two northern counties and the new buildings at Marybone) not more than four-pence upon theirs. And this is done by acts of parliament renewed every year, and not by any permanent and original act of parliament that could be considered as a plighting of the national faith to the purchasers of land, that their lands should always be taxed according to the rule of assessment then observed. On the contrary, the persons who have been lightly taxed have always feared, and those who have been heavily taxed have always hoped, that the Parliament would, one day or other, have a sufficient regard to justice to correct this gross inequality, and to impose the land-tax according to a new and equal assessment; or rather, indeed, according to a certain proportion of the rents received by every man; or, if the land is kept in the owner's hands, of the rents which were received for it when it was last lett, or which it might easily be lett for, in the judgment of the commissioners of the land-tax; as is done, if I mistake not, in the case of the

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the late house-tax. And they all have hoped, at times, (though now, I believe, that hope is at an end) that the land-tax would be reduced to two shillings in the pound, and sometimes even that it would be entirely taken off, or (to speak more correctly) permitted to expire without being reimposed; as I am fully persuaded it might have been, if prudent and economical measures had been pursued by our several ministers of state for these last forty years, and both this and the two last wars had been avoided. There is, therefore, as I apprehend, no weight in the reasoning of those who say that such a correction of the inequality of the land-tax would be unjust with respect to those purchasers of land who have bought their land at a greater price than they otherwise would have done, upon an expectation that the land-tax would continue to be raised according to the then present mode of assessment. The nation is not bound to continue in the practice of imposing this tax unequally, because these gentlemen have flattered themselves that they would do so.

I am the more confirmed in this opinion of the injustice of continuing the land-tax on its present unequal footing, because it was, that of Dr Benjamin Franklin, whom I consider as one of the most judicious and wisest men now living upon earth, and of whose talents we now feel the force since, by our attack upon one of the charters of the Americans, and our other alarming acts of authority against them, we have driven him to employ them against us. This truly great man used always to quote the continuance of the land-tax, upon its present very unequal footing, as a proof of the little regard that was had to justice and common sense in our national deliberations. And I remember once in particular, that when it was said that some people thought the correction of this inequality would be unjust with respect to those who had purchased land upon a supposition of its continuance, he replied with some quickness, "Unjust! yes, it would be unjust; for it would be doing but half of what strict justice would require; which would be to create a counter-inequality in the assessment of the land-tax, whereby the lands which had been heavily taxed should hereafter be taxed lightly, and those which had been lightly taxed should hereafter be taxed heavily, for the space of about fourscore years, or for a time that should be equal to the time during which the present unequal assessment had been permitted to continue." This counter-inequality he, perhaps, would not seriously have wished to see established; but, I think, *that* manner of expressing himself shewed strongly his opinion of the propriety of an exact equality for the future, and his contempt of the arguments derived from its supposed injustice with respect to purchasers.

This first deviation, therefore, from the rule of equal taxation in our government ought, as I conceive, to be corrected in the next land-tax act which shall be passed, there being no valid nor just objection to be made to such correction.

CCCLXVII. But the case is different with respect to the other instance of a deviation from the same rule, to wit, the exemption of the interest of the publick debts from the land-tax and all other taxes. For this interest cannot be made subject to the land-tax without a direct breach of the national faith to the proprietors of it, there being express clauses in the several acts of parliament by which the loans that constitute these debts have been established, which provide that the several annuities granted to the persons who have advanced their money to the Government, shall be *free from all taxes, charges, and impositions whatsoever.* * And the money so advanced to Government has been advanced at a lower rate of interest in consequence of these clauses. And much of it, I believe, has been lent to Government in the war of 1741, at the moderate interest of about four per cent. which, if it had been left subject to the land-tax (as all other personal estate is, according to the strict letter of the land-tax acts, though, from the difficulty of coming at it, the tax is seldom actually paid upon it;) could not have been obtained for less than five per cent. The owners of this debt may, therefore, be said to have paid the land-tax upon it in the very act of lending it on the terms proposed to them; since, in consideration of their exemption from that and other taxes, they consented to take four, instead of five, per cent. for their money. Nevertheless, in process of time, these original compacts grow to be in a manner forgot by the generality of mankind, who are apt to consider this exemption of the stock-holders from paying the land-tax, as an unjust distinction in their favour: and the land-holders in general are apt to hold this language, partly, perhaps, from ignorance of the aforesaid original clause of exemption, and partly from the bias of self-interest, which makes them wish to see the stock-holders bear a share of the burthen which they labour under, whether they have, or have not, been so exempted. I have known men of very good understandings and education talk in this manner, and, when they have been told of the said clause of exemption in the several acts of parliament for borrowing the said money, either refuse to believe that there was such a clause, or, if convinced of the existence of it, deny its efficacy and validity, in point of justice and good policy, to entitle the stock-holders to be exempted. This opinion and inclination in the land-owners of the kingdom, and perhaps in other classes of men that are not themselves stock-holders, certainly contributes to make the publick funds less secure than they would be, if they had not been so exempted in their first establishment.

Of the exemption of the publick funds from the payment of taxes.

The ground of it.

The said exemption is odious to the land-holders, and thereby lessens the security of the said funds.

* Thus, for example, in the statute of the 21st of George the Second, chap. ii, st. 15, it is enacted, "That all the several and respective annuities, payable in pursuance of this act, after the rate of four pounds *per centum per annum*, on all and every the principal sums for which the same are payable, shall be free from all taxes, charges, and impositions whatsoever."

ment, but had been left liable (like all other property, both real and personal,) to pay their proportion of the land-tax; for then, as both the class would constantly and visibly have contributed at the same time to the relief of the exigencies of the state, the land-holders would have had no pretence to grudge the stock-holders the enjoyment of the interest of their money, which would, in every view, have been as much their rightful property as the rents of freehold land are the property of their respective owners.

It would tend to the security of the publick funds, if the proprietors of them would consent to make the interest of them liable to the land-tax.

This diminution of the security of the publick funds, arising from their being thus exempted from taxes, appears to me to be a matter of so much importance, (as their security evidently depends, in a considerable degree, on the opinion the nation at large entertains of the justice of continuing them) that I should be ready, with respect to the little property in them, to give up my right to the said exemption, and make it liable to pay the land-tax at that which may be supposed to be the medium rate at which it is levied upon the lands of the kingdom, which have heard people estimate at about eighteen pence in the pound, or, most, two shillings, when the land-tax is called four shillings in the pound, or (to speak more correctly) when the sum raised by the land-tax in England and Wales, (exclusive of Scotland) is about two millions: for it is supposed that an equal assessment of the land-tax in England and Wales eighteen-pence or two shillings in the pound, would raise the said sum two millions. This, therefore, would be the rate at which property in the publick funds ought to be charged to the land-tax, if it could, consistent with justice and the preservation of the national faith, be charged to it at all. And at this rate, I say, I would freely consent to have my property the charged, notwithstanding my aforesaid right to an exemption, if all the other proprietors of stock, or a majority of them, were willing to do the same and this, not so much from a motive of publick spirit (though I hope the motive is not without its influence) as from a desire of rendering the remaining part of the interest due to me, the eighteen shillings and six pence in the pound, more secure: for then nothing but the most shameless and bare-faced injustice could ever prompt any land-holder to wish for, or encourage, any attempt to get rid of the publick debt by any other method than that of fairly paying it off. But such a general consent of the stockholders to let their stock be made liable to pay the land-tax, is what we can hardly expect to see; more especially as many of them are foreigners resident in Holland and Switzerland, who will probably conceive themselves to be no way obliged to pay taxes for the support of our government. And to subject their stock to the land-tax by act of parliament, without their consent, or even with the consent of some, but not all of them, would undoubtedly be a breach of the national faith, and an act of bankruptcy, *pro tanto*. With respect, therefore, to the immense publick debt already subsisting, I do not expect, or wish, to see any endeavours used to

But this is hardly to be expected.

Nor can such a measure be taken, consistently with justice, without their consent.

public authority to carry this measure, of making it liable to the land-tax, But it would be a useful measure in execution. But with respect to the loans of the present year, 1780, the government, of the following years, (if more such are to be expected,) I must own to make the interest of the following years, (if more such are to be expected,) I must own to make the interest of all future loans to be no more than the interest of the present year, 1780, the government, should like to see it tried. It would certainly have the good effect of making the security of the money so lent, for the reason already suggested, public loans of money liable to the land-tax: and it would be no immediate diminution of the interest received by the government for the said money; because, the interest which would be given to the land-tax, would be proportionally greater than if lent under a clause of exemption. And yet Government would be no loser by it, since it would receive back, in the shape of interest on the said interest, the additional interest it would be obliged to give on this account. This, it may be said, is mere trifling and doing nothing, but you give with one hand what you take back with the other. But the advantage resulting from it is this: the land-holder, and the stock-holder, the stock is originally made liable to this tax, will ever after run their fortune, and experience the same increase or diminution of their respective incomes, as the affairs of the state are prosperous or unfortunate. Contrary to all present appearance, the land-tax should ever again be less than four shillings in the pound, the stock-holder upon this new establishment would enjoy a proportional diminution of the tax upon his property: and, on the other hand, if it should become necessary, in the course of this most ruinous and unhappy war, to make the land-tax double or triple of what it now is, (which seems to be a much more likely event than the former) the stock-holder will pay a double or treble tax as well as the owner of land, and visibly bear his proportion of the common burthen, and thereby escape the dangerous envy of being considered as a kind of favoured, or neutral person unconcerned in the welfare or calamities of the country.

An ingenious and public-spirited writer of some letters that have appeared in the *Whitehall Evening Post*, dated from Windsor, and which have been lately collected and published in a pamphlet printed for Doddsley, recommends a measure of this kind, with respect to the public debt already existing, and has given very powerful reasons in support of it, which, together with the many other important particulars contained in those letters, are well worth the most serious attention of the public. But, as such a step would be attended with considerable difficulties, I cannot but doubt whether it be, upon the whole, advisable.

But the measure which I have here ventured to recommend, and which relates only to the stock hereafter to be created, would be attended with the same advantages, as far as it went, and would not be liable to any of the same objections.

And,

It would, probably, be expedient to admit the owners of such stock in the public funds as should be made liable to pay the land-tax, to a right of voting for members of parliament,

And, perhaps, if this measure were adopted, either with respect to interest of future loans of money, or to that of the money already due the public creditors, it might be advisable at the same time to admit stock-holders who would thus become contributors to the land-tax, a right of voting for members of the House of Commons, by whom the tax is granted. Every proprietor of such stock, who was of the male and a native of Great Britain, or Ireland, or any of the British dominions and had been in possession of an annuity of 10 pounds a year, standing in his own name, in any of the public funds for more than a whole year, had resided for more than a year together in any particular county in England, Wales, or Scotland, might, as I imagine, without any inconvenience be permitted to vote at the election of the knights of the shire, or commissioners of the shire, in which he had so resided. Perhaps the offering such a privilege might induce some of the present proprietors of the public funds to consent that the interest of their shares of the national debt should for the future be made liable to the land-tax: more especially, if they apprehend themselves to be under a kind of necessity (from the enormity of the present load upon the public revenue,) of making some sacrifice of this nature, or of giving up a part of their annual income, arising from the funds, in order to preserve the remainder; which the *Earl of Stair* has very able and most interesting pamphlet on the State of the Public Revenue, published about January, 1782, and intitled, *Facts, and Consequences*, &c. positively declares it will be absolutely necessary that it should do.

When the stock-holders should have thus become liable to the land-tax, and, in consideration of their thereby contributing, like the owners of land to the common burthens of the nation, should have been admitted to share in the election of the national representative, the security of the property in the public funds would be rendered as complete as any law or public regulations, can make it: though they would still have reason to wish, at least as heartily as any other class of men in the kingdom, the blessing of peace may speedily be restored to us, and that then Government may adopt such measures of vigour for increasing the public revenue, and such measures of oeconomy in the management of it, as enable them gradually to discharge some part of this enormous debt, thereby render the payment of the remainder of it less precarious.

[End of the reflections on the Land-tax and the Interest of the National Debt]

CCCLXVIII. I now return from these political digressions to the time of Life-Annuities, which is the proper subject of this treatise.

And, as I have gone through every thing that has appeared to be necessary to the illustration of the doctrine of annuities for single lives, I now proceed to subjoin the like short and convenient expressions for values of annuities dependent on more than one life as were given in Art. 86 *et seq.* for the values of annuities depending on a life.

best expression of the value of an annuity of one pound per annum for a given number of years, depending on the joint continuance of two lives of given ages, according to a given table of the probabilities of the duration of human life, and a given rate of the interest of money.

CCCLXIX. Let r be, as before, the sum of one pound together with its interest for one year according to the given rate of interest. Let N be the number of years in the age of the younger of the two lives on the joint continuance of whose lives the annuity is to depend; $N+a$ the number of years in the age of the older of the said persons; E the greatest number of years through which it is supposed to be payable for human life to be extended, according to the table of the probabilities of the duration of human life adopted for the calculation; which number is in Monsieur de Parcieux's table 94 years. Let n be any number of years not greater than $E - |N+a$, or $E - N - a$, or than the least number of years during which it is possible that the older of the lives may be prolonged. And let the annuity of one pound *per annum* granted for the term of n years, provided both the said persons of the ages of N years and $N+a$ years shall so long live, but otherwise to cease at the death of either of them. Let P be the number of persons denoted in Monsieur de Parcieux's table of the probabilities of the duration of human life, (or in such other table of those probabilities as is thought proper by the calculator to be adopted as the ground of his calculation) as being all living together at the said age of N years; and P' the number of persons represented in the said table to be living at the age of $N+1$ years; and P'' the number living at the age of $N+2$ years; P''' the number living at the age of $N+3$ years; and P^{iv} , P^v , P^{vi} , P^{vii} , P^{viii} , P^x , &c. the numbers living at the several following ages of $N+4$ years, $N+5$ years, $N+6$ years, $N+7$ years, $N+8$;

the term $\frac{P_n \times Q^n}{r^n}$, (which term will in this case be $\frac{PE-N-a \times QE-N-a}{r^n}$),

will be the value of an annuity of one pound *per annum* for the whole joint continuance of the two given lives of N years and $N+a$ years: but, if n is less than the said complement of $N+a$ years to E , or to the utmost duration of human life, the said expression will be less than the value of an annuity of one pound *per annum* for the whole joint continuance of the two given lives of the ages of N years and $N+a$ years, and will be the value of an immediate, but imperfect, life-annuity of one pound *per annum* during n years of the joint continuance of the lives of two persons of the said ages. This is evident from Art. XLVIII, XLIX, LI, pages 48, 49, 50, 51, 52, 53.

An example of the computation of the value of an immediate and complete life-annuity of one pound per annum for the whole joint continuance of the lives of two persons of given ages, by means of the foregoing expression.

CCCLXXI. Let it be required to find the value of an annuity of one pound *per annum* for the whole joint continuance of the lives of two persons of the ages of 75 years and 80 years, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and upon a supposition that the interest of money is 3 per cent.

Here n , or the number of years through which the annuity is to continue, in case both the lives (of which the older is of the age of fourscore years,) shall last so long, is the greatest possible number of years through which, according to Monsieur de Parcieux's table, a life of fourscore years of age can be extended, that is, (94—80 years, or) 14 years. Therefore the series

$$\frac{P^i \times Q^i}{r^i} + \frac{P^{i+1} \times Q^{i+1}}{r^{i+1}} + \frac{P^{i+2} \times Q^{i+2}}{r^{i+2}} + \frac{P^{i+3} \times Q^{i+3}}{r^{i+3}} + \frac{P^{i+4} \times Q^{i+4}}{r^{i+4}} + \frac{P^{i+5} \times Q^{i+5}}{r^{i+5}} + \frac{P^{i+6} \times Q^{i+6}}{r^{i+6}} + \frac{P^{i+7} \times Q^{i+7}}{r^{i+7}} + \text{\&c.}$$

in the foregoing expression must be continued to 14 terms; which terms may be computed as follows.

Here P is = 211, P^i = 192, P^{i+1} = 173, P^{i+2} = 154, P^{i+3} = 136, P^{i+4} = 118, P^{i+5} = 101, P^{i+6} = 85, P^{i+7} = 71, P^{i+8} = 58, P^{i+9} = 48, P^{i+10} = 38, P^{i+11} = 29, P^{i+12} = 22, and P^{i+13} = 16; and Q is = 118, Q^i = 101, Q^{i+1} = 85, Q^{i+2} = 71, Q^{i+3} = 59, Q^{i+4} = 48, Q^{i+5} = 38, Q^{i+6} = 29, Q^{i+7} = 22, Q^{i+8} = 16, Q^{i+9} = 11, Q^{i+10} = 8, Q^{i+11} = 5, Q^{i+12} = 3, Q^{i+13} = 1.

$\mathcal{Q}^v = 48$, $\mathcal{Q}^{vi} = 38$, $\mathcal{Q}^{vii} = 29$, $\mathcal{Q}^{viii} = 22$, $\mathcal{Q}^x = 16$, $\mathcal{Q}^z = 11$, $\mathcal{Q}^{xi} = 7$, $\mathcal{Q}^{xii} = 4$, $\mathcal{Q}^{xiii} = 2$, and $\mathcal{Q}^{xiv} = 1$. And r is $= 1.03$, and $\frac{1}{r} = \frac{1}{1.03} = .9708$, and $\frac{1}{r^2} = .9425$, $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, $\frac{1}{r^9} = .7664$, $\frac{1}{r^{10}} = .7440$, $\frac{1}{r^{11}} = .7224$, $\frac{1}{r^{12}} = .7013$, $\frac{1}{r^{13}} =$

.6809, and $\frac{1}{r^{14}} = .6611$. Therefore the expression $\frac{1}{P \times \mathcal{Q}} \times$ the serie

$$\frac{P^i \times \mathcal{Q}^i}{r} + \frac{P^{ii} \times \mathcal{Q}^{ii}}{r^2} + \frac{P^{iii} \times \mathcal{Q}^{iii}}{r^3} + \frac{P^{iv} \times \mathcal{Q}^{iv}}{r^4} + \frac{P^v \times \mathcal{Q}^v}{r^5} + \frac{P^{vi} \times \mathcal{Q}^{vi}}{r^6} + \frac{P^{vii} \times \mathcal{Q}^{vii}}{r^7} + \frac{P^{viii} \times \mathcal{Q}^{viii}}{r^8} + \frac{P^x \times \mathcal{Q}^x}{r^9} + \frac{P^{xi} \times \mathcal{Q}^{xi}}{r^{10}} + \frac{P^{xii} \times \mathcal{Q}^{xii}}{r^{11}} + \frac{P^{xiii} \times \mathcal{Q}^{xiii}}{r^{12}} + \frac{P^{xiv} \times \mathcal{Q}^{xiv}}{r^{13}}$$

+ $\frac{P^{xiv} \times \mathcal{Q}^{xiv}}{r^{14}}$ will be equal to $\frac{1}{211 \times 118} \times$ the serie

$$\begin{aligned} &+ 192 \times 101 \times .9708 + 173 \times 85 \times .9425 + 154 \times 71 \times .9151 \\ &+ 136 \times 59 \times .8884 + 118 \times 48 \times .8626 + 101 \times 38 \times .8374 \\ &+ 85 \times 29 \times .8130 + 71 \times 22 \times .7894 + 59 \times 16 \times .7664 \\ &+ 48 \times 11 \times .7440 + 38 \times 7 \times .7224 + 29 \times 4 \times .7013 \end{aligned}$$

+ $22 \times 2 \times .6809 + 16 \times 1 \times .6611 = \frac{1}{24,898} \times$ the serie

$$\begin{aligned} &+ 19,392 \times .9708 + 14,705 \times .9425 + 10,934 \times .9151 \\ &+ 8024 \times .8884 + 5664 \times .8626 + 3838 \times .8374 \\ &+ 2465 \times .8130 + 1562 \times .7894 + 944 \times .7664 \\ &+ 528 \times .7440 + 266 \times .7224 + 116 \times .7013 \end{aligned}$$

+ $44 \times .6809 + 16 \times .6611 = \frac{1}{24,898} \times$ the serie

$$\begin{aligned} &+ 18,825.7536 + 13,859.4625 + 10,005.7034 + 7,288.4334 \\ &+ 4,885.7664 + 3,213.9412 + 2,004.0450 + 1,233.0412 \\ &+ 723.4816 + 392.8320 + 192.1584 + 81.3598 + 29.9598 \end{aligned}$$

$$+ 29.9596 + 10.5776 = \frac{\text{£} 1}{24,898} \times 62,746.5085 = \text{£} 1 \times \frac{62746.5085}{24898} = \text{£} 1 \times 2.5201 = \text{£} 2.5201. \text{ Therefore the value of an annuity of one pound a year for the whole joint continuance of two lives of the ages of 75 years and 80 years, when the interest of money is 3 per cent. is } \text{£} 2.5201, \text{ or } 2\text{l. } 10\text{s. } 5\text{d. } \text{QE I.}$$

Note. This value 2.5201 exceeds the value found for the same annuity in Art. XLIX, to wit, £2.5197, by £.0004. But this very small difference (which begins only in the fourth and last place of decimal fractions,) is rather to be considered as a proof of the agreement of the two calculations with each other than of an error in either of them, and probably has arisen from the different order in which the arithmetical operations have been performed in them, the terms of the series having been separately divided by 24898 in Art. XLIX, and the several quotients thence arising added together into one sum, whereas in this latter calculation the same terms have been all added together into one sum without such division, and then the said sum has been divided by 24898.

An example of the computation of the value of an immediate, but imperfect, life-annuity, depending on the joint continuance of the lives of two persons of given ages, by means of the same expression.

CCCLXXII. Let it be required to find the value of an annuity of one pound *per annum* for the first five years of the joint continuance of the lives of two persons of the ages of 75 years and 80 years, according to the same table of the probabilities of the duration of human life, and the same rate of interest, as in the last example.

For this purpose we need only take the first five terms of the foregoing series $\frac{P_i \times Q_i}{r} + \frac{P_{i+1} \times Q_{i+1}}{r^2} + \frac{P_{i+2} \times Q_{i+2}}{r^3} + \frac{P_{i+3} \times Q_{i+3}}{r^4} + \frac{P_{i+4} \times Q_{i+4}}{r^5}$

&c. and multiply their sum into the fraction $\frac{\text{£}}{P \times Q}$; and the

product will be the value of the proposed annuity. These terms are $92 \times 101 \times .9708 + 173 \times 85 \times .9425 + 154 \times 71 \times .9151 + 136 \times 59 \times .8884 + 118 \times 48 \times .8626$; which are equal to

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$$19392 \times .9708 + 14,705 \times .9425 + 10,934 \times .9151 + 802 \\ \times .8834 + 5664 \times .8626 = 18,825.7536 + 13,859.462 \\ + 10,005.7034 + 7,288.4336 + 4,885.7664 = 54,865.1195$$

which being multiplied into the fraction $\frac{£}{p \times \mathcal{Q}}$, or $\frac{£}{211 \times 118}$,

$$\frac{£}{24,898}, \text{ is } = £1 \times \frac{54,865.1195}{24,898} = £1 \times 2.2035 = £2.2035$$

Therefore the value of an annuity of one pound, *per annum* for the first 75 years of the joint continuance of the lives of two persons of the ages of 75 years and 80 years, according to the foregoing suppositions concerning the duration of human life and the interest of money, is £2.2035 or 2l. 4s. 6d. $\frac{3}{4}$. QEI.

CCCLXXIII. And, if the annuity to be purchased is a remote one or is to be paid at the distance of more than one year, a short expression of its value, similar to the foregoing expressions, may be found as follows.

A short and general expression of the value of a remote annuity of one pound per annum for a given number of years, depending on the joint continuance of two lives of given ages, when the rate of the interest of money is also given.

Let r be, as before, the value of one pound together with its interest for a year at the given rate of the interest of money. And let m be the number of years at the end of which the annuity is to commence, so that the first payment of it shall be made at the end of $m+1$ years. And let N be the number of years in the age of the younger of the two persons upon the joint continuance of whose lives the annuity is to depend; and let $N+a$ be the number of years in the age of the older of the two persons; at the time of purchasing the said annuity. And let E be, as before, the whole number of years through which it is possible, according to the table of probabilities adopted in the calculation, for human life to be extended; which in Monsieur de Parcieux's table is 94 years. The will $N+a+m$ be the number of years in the age of the older of the said two persons at the time of the commencement of the said annuity and $N+a+m+1$ the number of years in his age at the time when the

Let payment of the said annuity will become due; and $E - \frac{N}{r-a+m}$, or $E - N - a - m$, will be the greatest possible number of years through which the life of the said older person can be extended after he shall have attained the age of $N + a + m$ years, and the annuity shall have commenced. Let n be any number of years not greater than $E - N - a - m$; and let P be the number of persons represented in M^r de Parcieux's table of the probabilities of the duration of human life, (or in such other table of those probabilities as shall be adopted as the ground of the calculation) as being alive at the age of N years, or of the younger of the said two persons, and $\mathcal{Q}$ be the number represented there as living at the age of $N + a$ years, or of the older of the said persons; and out of the P persons represented in the said table as living at the age of N years, let P_{m+1} , P_{m+2} , P_{m+3} , P_{m+4} , P_{m+5} , &c. be the numbers of persons represented therein as living at the subsequent ages of $N + m + 1$ years, $N + m + 2$ years, $N + m + 3$ years, $N + m + 4$ years, $N + m + 5$ years, &c. or at the ends of $m + 1$ years, $m + 2$ years, $m + 3$ years, $m + 4$ years, $m + 5$ years, &c. from the time of purchasing the annuity; and out of the $\mathcal{Q}$ persons represented therein as living at the age of $N + a$ years, let $\mathcal{Q}_{m+1}$, $\mathcal{Q}_{m+2}$, $\mathcal{Q}_{m+3}$, $\mathcal{Q}_{m+4}$, $\mathcal{Q}_{m+5}$, &c. be the numbers of persons represented therein as living at the subsequent ages of $N + a + m + 1$ years, $N + a + m + 2$ years, $N + a + m + 3$ years, $N + a + m + 4$ years, $N + a + m + 5$ years, &c. or at the ends of $m + 1$ years, $m + 2$ years, $m + 3$ years, $m + 4$ years, $m + 5$ years, &c. from the time of purchasing the annuity; and so on for all the following ages in the table.

Then will the value of an annuity of one pound *per annum*, to commence at the distance of m years, (so that the first payment of it shall be made at the end of $m + 1$ years,) and to continue during n years, provided two persons, who are of the ages of N years and $N + a$ years at the time of purchasing the annuity, shall so long live, but which shall cease soon as either of the said persons shall die;—I say, the value of such a joint annuity will be equal to the following expression, to wit,

$$\frac{1}{r} \times \text{the series } \frac{P_{m+1} \times \mathcal{Q}_{m+1}}{r^{m+1}} + \frac{P_{m+2} \times \mathcal{Q}_{m+2}}{r^{m+2}} + \frac{P_{m+3} \times \mathcal{Q}_{m+3}}{r^{m+3}} + \frac{P_{m+4} \times \mathcal{Q}_{m+4}}{r^{m+4}} + \frac{P_{m+5} \times \mathcal{Q}_{m+5}}{r^{m+5}} + \&c. \text{ continued to } n \text{ terms;}$$

to the term $\frac{P_{m+n} \times \mathcal{Q}_{m+n}}{r^{m+n}}$. This is evident from Art. LII, page 53,

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An example of the computation of the value of a remote life-annuity depending upon the joint continuance of two lives of given ages by means of the foregoing expression.

CCCLXXIV. Let it be required to find the value of an annuity of one pound *per annum* during the joint lives of two persons of the ages 75 and 80 years, but which shall not commence till five years after the purchase of it, so that the first payment of it shall not become due till the end of six years, or till the younger of the said two persons (if he shall be then living,) shall have attained the age of fourscore and one year, and the older of the said persons (if he shall be then living,) shall have attained the age of fourscore and six years; and which shall then continue during the whole remainder of the joint continuance of the lives of the said two persons; the interest of money being 3 per cent. (as in the example,) and the probabilities of the duration of human life being the same as they are represented to be in Monsieur de Parcieux's table of them.

Here N , or the number of years in the age of the younger of the said two persons, is 75; and $N+a$, or the number of years in the age of the older of the said two persons, is 80; and m , or the number of years before the proposed annuity is to commence, is 5; and consequently $N+a+m$, or the number of years in the age of the older of the said two persons at the time when the annuity is to commence, is 85. E , or the greatest number of years through which human life can be extended, is, according to Monsieur de Parcieux's table of probabilities, 94 years; and consequently $E - |N+a+m|$, or the greatest number of years through which it is possible that the life of the older of the said two persons can be extended after the annuity shall have commenced, is $94 - 85$, or 9 years. Therefore 9 years is likewise the greatest number of years through which it is possible that the lives of both the said persons should continue together in being. Therefore n , or the greatest number of years through which the annuity may continue, will be equal to 9 years; because the annuity, when once it has taken place, is supposed to continue during the whole remainder of the joint continuance of the lives of the said two persons. And consequently the series

$$\frac{P_{m+1} \times Q_{m+1}}{r^{m+1}} + \frac{P_{m+2} \times Q_{m+2}}{r^{m+2}} + \frac{P_{m+3} \times Q_{m+3}}{r^{m+3}} + \frac{P_{m+4} \times Q_{m+4}}{r^{m+4}} + \frac{P_{m+5} \times Q_{m+5}}{r^{m+5}} + \frac{P_{m+6} \times Q_{m+6}}{r^{m+6}} + \frac{P_{m+7} \times Q_{m+7}}{r^{m+7}} + \frac{P_{m+8} \times Q_{m+8}}{r^{m+8}} + \frac{P_{m+9} \times Q_{m+9}}{r^{m+9}}$$

will consist of nine terms. These terms may be computed as follows.

Since m is = v , we shall have $m+1 = vi$, and $m+11 = vii$,
 $m+111 = viii$, $m+1iv = ix$, $m+1v = x$, $m+1vi = xi$, $m+1vii$
 $m+1viii = xii$, and $m+1ix = xiv$; and in like manner
 $m+1 = 6$, $m+2 = 7$, $m+3 = 8$, $m+4 = 9$, $m+5 = 10$,
 $m+6 = 11$, $m+7 = 12$, $m+8 = 13$, and $m+9 = 14$. There-
the series $\frac{P_{m+1} \times Q_{m+1}}{r^{m+1}} + \frac{P_{m+11} \times Q_{m+11}}{r^{m+2}} + \frac{P_{m+111} \times Q_{m+111}}{r^{m+3}}$

$$\frac{P_{m+iv} \times Q_{m+iv}}{r^{m+4}} + \frac{P_{m+v} \times Q_{m+v}}{r^{m+5}} + \frac{P_{m+vi} \times Q_{m+vi}}{r^{m+6}}$$

$$\frac{P_{m+vii} \times Q_{m+vii}}{r^{m+7}} + \frac{P_{m+viii} \times Q_{m+viii}}{r^{m+8}} + \frac{P_{m+ix} \times Q_{m+ix}}{r^{m+9}}$$

$$\text{equal to } \frac{P_{vi} \times Q_{vi}}{r^6} + \frac{P_{vii} \times Q_{vii}}{r^7} + \frac{P_{viii} \times Q_{viii}}{r^8} + \frac{P_{ix} \times Q_{ix}}{r^9}$$

$$\frac{P_x \times Q_x}{r^{10}} + \frac{P_{xi} \times Q_{xi}}{r^{11}} + \frac{P_{xii} \times Q_{xii}}{r^{12}} + \frac{P_{xiii} \times Q_{xiii}}{r^{13}}$$

$$\frac{P_{xiv} \times Q_{xiv}}{r^{14}}.$$

But r is, as before, = 1.03; and consequently $\frac{1}{r}$ is = $\frac{1}{1.03}$ =

$$.98, \text{ and } \frac{1}{r^6} \text{ is } = .8374, \text{ and } \frac{1}{r^7} = .8130, \text{ and } \frac{1}{r^8} = .7894,$$

$$\frac{1}{r^9} = .7664, \text{ and } \frac{1}{r^{10}} = .7440, \text{ and } \frac{1}{r^{11}} = .7224, \text{ and } \frac{1}{r^{12}} =$$

$$.7013, \text{ and } \frac{1}{r^{13}} = .6809, \text{ and } \frac{1}{r^{14}} = .6611. \text{ And } P \text{ is } = 211, \text{ and}$$

$$s = 118, \text{ and } P_{vi} \text{ is } = 101, P_{vii} = 85, P_{viii} = 71, P_{ix} =$$

$$P_x = 48, P_{xi} = 38, P_{xii} = 29, P_{xiii} = 22, \text{ and } P_{xiv} =$$

$$\text{and } Q_{vi} \text{ is } = 38, \text{ and } Q_{vii} = 29, \text{ and } Q_{viii} = 22, \text{ and } Q_{ix}$$

$$= 11, \text{ and } Q_{xi} = 7, \text{ and } Q_{xii} = 4, \text{ and } Q_{xiii} =$$

and $Q_{xiv} = 1$. Therefore the expression $\frac{1}{P \times Q}$ $\times$ the series

$$\frac{P_{vi} \times Q_{vi}}{r^6}$$

$$\begin{aligned}
& \frac{P_{VI} \times Q_{VI}}{r^6} + \frac{P_{VII} \times Q_{VII}}{r^7} + \frac{P_{VIII} \times Q_{VIII}}{r^8} + \frac{P_{IX} \times Q_{IX}}{r^9} \\
& + \frac{P_{X} \times Q_{X}}{r^{10}} + \frac{P_{XI} \times Q_{XI}}{r^{11}} + \frac{P_{XII} \times Q_{XII}}{r^{12}} + \frac{P_{XIII} \times Q_{XIII}}{r^{13}} \\
& + \frac{P_{XIV} \times Q_{XIV}}{r^{14}} \text{ is equal to } \frac{1}{211 \times 118} \times \text{the series } 101 \times 38 \times .83 \\
& + 85 \times 29 \times .8130 + 71 \times 22 \times .7894 + 59 \times 16 \times .76 \\
& + 48 \times 11 \times .7440 + 38 \times 7 \times .7224 + 29 \times 4 \times .70 \\
& + 22 \times 2 \times .6809 + 16 \times 1 \times .6611 = \frac{1}{211 \times 118} \times \text{the series } 3838 \times .8374 \\
& + 2465 \times .8130 + 1562 \times .7894 + 944 \times .7664 + 528 \times .7440 + 266 \times .7224 + 116 \times .70 \\
& + 44 \times .6809 + 16 \times .6611 = \frac{1}{211 \times 118} \times \text{the series } 3213.9412 + 2004.0450 + 1233.0428 + 723.4816 + 392.83 \\
& + 192.1584 + 81.3508 + 29.9596 + 10.5776 = \frac{1}{211 \times 118} \times 7881.3890 = \frac{1}{24,898} \times 7881.3890 = \text{£}1.3165 = \text{£}0.3165 = 6s. 4d.
\end{aligned}$$

Therefore 6s. 4d. is the value of remote annuity of one pound *per annum*, that is to commence at the distance of five years, (or whereof the first payment is to be made at the end of six years,) and that is to continue during the joint lives of the persons who, at the time of purchasing it, are of the ages of 75 and years; according to Monsieur de Parcieux's table of the probabilities of life, and when the interest of money is 3 per cent. QEI.

CCCLXXV. The foregoing examples are, I presume, sufficient to illustrate the manner of computing the values of annuities, whether immediate or remote, that are to depend on the joint continuance of the lives of given ages. But, when a whole table of the values of such annuities is to be computed, it is not necessary to make a new calculation similar to that given above in Art. cccclxxi, for every different year

human life; but the values of these annuities for different pairs of lives whose ages differ from each other by the same number of years, may be reduced one from another by an easy arithmetical process similar to that of Mr. Morgan, above explained in Art. c, c1, c11, pages 109, 110, 111, 112, by which the value of an annuity for a single life of any given age is deduced from that of the like annuity for a life that is one year older. This method, (as well as that explained above in Art. c, c1, c11,) has been given by Mr. Morgan in his learned treatise on the Doctrine of Insurances and Annuities for Lives, page 73, and may be explained as follows.

A short and easy method of deducing from the value of an annuity of one pound a year during the joint continuance of any two given lives, the value of a like annuity for the joint continuance of two other lives that are one year younger than the former lives.

CCCLXXVI. Let the number of years in the age of the younger of the two given lives be called N , and the number of years in the age of the older of the said lives be called $N+a$. Then will the numbers of years in the ages of two lives that are younger than the given lives by one year, be $N-1$ years and $N+a-1$ years.

Let the value of an annuity of one pound a year for the joint continuance of the two given lives of the ages of N years and $N+a$ years be $\mathcal{P}$; and the value of the like annuity of one pound a year for the joint continuance of two other lives that are one year younger than the two given lives, and which consequently are of the ages of $N-1$ years and $N+a-1$ years, be $\mathcal{P}'$.

Let P be the number of persons who are supposed, in the table of probabilities of the duration of human life which is adopted as the ground of the calculation, to be living at the age of the younger of the two given lives, that is, at the age of N years, and $P+a$ the number of persons who are therein represented as living at the age which is younger than the former age by one year, that is, at the age of $N-1$ years; and let $\mathcal{Q}$ be the number of persons who are therein represented as living at the age of the older of the two given lives, or at the age of $N+a$ years, and $\mathcal{Q}+e$ the number of persons represented as living at the age which is younger than the said older age by one year, or at the age of $N+a-1$ years.

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And

And let r be the value of one pound together with its interest one year, according to the rate of interest supposed in the calculation.

Then will $\mathcal{V}$, or the value of an annuity of one pound a year for the joint continuance of two lives that are of the ages of $N-1$ years and $N+a-1$ years, or are respectively younger than the two given lives one year, be equal to $\frac{1}{r} \times \frac{P \times \mathcal{Q}}{P+d \times \mathcal{Q}+e} \times \frac{\mathcal{L}}{1+\mathcal{V}}$, or to the quantity

which arises by adding one pound to $\mathcal{V}$, the value of an annuity one pound a year for the joint continuance of the two given lives of the ages of N years and $N+a$ years, and then multiplying the sum $1+\mathcal{V}$ by $\mathcal{L}$, first, into the fraction $\frac{1}{r}$, and, secondly, into the fraction

$\frac{P \times \mathcal{Q}}{P+d \times \mathcal{Q}+e}$. This may be demonstrated in the manner following.

DEMONSTRATION.

CCCLXXVII. It is evident from Art. cccclxx, that $\mathcal{V}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the given ages of N years and $N+a$ years, is (if we make use of the notation used in Art. cccclxix, cccclxx) equal to the expression

$\frac{\mathcal{L}}{P \times \mathcal{Q}} \times$ the series $\frac{P^1 \times \mathcal{Q}^1}{r} + \frac{P^{11} \times \mathcal{Q}^{11}}{r^2} + \frac{P^{111} \times \mathcal{Q}^{111}}{r^3} + \frac{P^{1111} \times \mathcal{Q}^{1111}}{r^4} + \&c.$ continued to $E-N-a$ terms, and then

$\mathcal{V}$, or the value of a like annuity of one pound a year for the joint continuance of two lives of the ages of $N-1$ years and $N+a-1$ years, is equal to the expression

$\frac{\mathcal{L}}{P \times \mathcal{Q}} \times$ the series $\frac{P^1 \times \mathcal{Q}^1}{r} + \frac{P^{11} \times \mathcal{Q}^{11}}{r^2} + \frac{P^{111} \times \mathcal{Q}^{111}}{r^3} + \frac{P^{1111} \times \mathcal{Q}^{1111}}{r^4} + \&c.$ continued to $E-N-a+1$ terms.

Now this last expression (which is $= \overset{f}{\mathcal{V}}$) may be derived from the former (which is $= \overset{f}{\mathcal{V}}$) by adding 1 to it, and then multiplying the sum, first, into the fraction $\frac{1}{r}$, and, secondly, into the fraction

$$\frac{P \times \mathcal{Q}}{P + a \times \mathcal{Q} + e}.$$

For, if we add 1 to the expression $\frac{1}{P \times \mathcal{Q}}$ $\times$ the series $\frac{P^1 \times \mathcal{Q}^1}{r}$ $+$ $\frac{P^{11} \times \mathcal{Q}^{11}}{r^2}$ $+$ $\frac{P^{111} \times \mathcal{Q}^{111}}{r^3}$ $+$ $\frac{P^{1111} \times \mathcal{Q}^{1111}}{r^4}$ $+$ &c. continued

to $E - N - a$ terms, it will become equal to $1 + \frac{1}{P \times \mathcal{Q}}$ $\times$ the series $\frac{P^1 \times \mathcal{Q}^1}{r}$ $+$ $\frac{P^{11} \times \mathcal{Q}^{11}}{r^2}$ $+$ $\frac{P^{111} \times \mathcal{Q}^{111}}{r^3}$ $+$ $\frac{P^{1111} \times \mathcal{Q}^{1111}}{r^4}$

$+$ &c. continued to $E - N - a$ terms, $= \frac{1}{P \times \mathcal{Q}}$ $\times$ $P \times \mathcal{Q}$ $+$ $\frac{1}{P \times \mathcal{Q}}$ $\times$ the series $\frac{P^1 \times \mathcal{Q}^1}{r}$ $+$ $\frac{P^{11} \times \mathcal{Q}^{11}}{r^2}$ $+$ $\frac{P^{111} \times \mathcal{Q}^{111}}{r^3}$

$+$ $\frac{P^{1111} \times \mathcal{Q}^{1111}}{r^4}$ $+$ &c. continued to $E - N - a$ terms, $= \frac{1}{P \times \mathcal{Q}}$ $\times$ the series $P \times \mathcal{Q}$ $+$ $\frac{P^1 \times \mathcal{Q}^1}{r}$ $+$ $\frac{P^{11} \times \mathcal{Q}^{11}}{r^2}$ $+$ $\frac{P^{111} \times \mathcal{Q}^{111}}{r^3}$

$+$ $\frac{P^{1111} \times \mathcal{Q}^{1111}}{r^4}$ $+$ &c. continued to $E - N - a + 1$ terms. And this

quantity, being multiplied by $\frac{1}{r}$, will become equal to $\frac{1}{P \times \mathcal{Q}}$ $\times$ the

series $\frac{P \times \mathcal{Q}}{r}$ $+$ $\frac{P^1 \times \mathcal{Q}^1}{r^2}$ $+$ $\frac{P^{11} \times \mathcal{Q}^{11}}{r^3}$ $+$ $\frac{P^{111} \times \mathcal{Q}^{111}}{r^4}$ $+$ $\frac{P^{1111} \times \mathcal{Q}^{1111}}{r^5}$ $+$ &c. continued to $E - N - a + 1$ terms; and,

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being further multiplied by the fraction $\frac{P \times Q}{P + d \times |Q + e|}$, it will become

equal to $\frac{1}{P + d \times |Q + e|} \times$ the series $\frac{P \times Q}{r} + \frac{P^i \times Q^i}{r^2} + \frac{P^{ii} \times Q^{ii}}{r^3} + \frac{P^{iii} \times Q^{iii}}{r^4} + \frac{P^{iv} \times Q^{iv}}{r^5} + \&c.$ continued to $E - N - a - 1$ terms, that is, to $\mathcal{X}$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P + d \times |Q + e|} \times \frac{1}{1 + \mathcal{V}} = \mathcal{X}$. QED.

CCCLXXVIII. I will now proceed to give a few examples of this method of deducing the values of annuities for joint lives from those of annuities for joint lives that are one year older, by which it will become familiar to the reader.

Examples of the foregoing method.

CCCLXXIX. Let us therefore suppose that the interest of money is $3\frac{1}{2}$ per cent. and that the two lives for whose joint continuance an annuity of one pound a year is to be granted are, successively, of the ages of 83 years and 93 years, of 82 years and 92 years, of 81 years and 91 years, of 80 years and 90 years, of 79 years and 89 years, of 78 years and 88 years, of 77 years and 87 years, of 76 years and 86 years, of 75 years and 85 years, of 74 years and 84 years, of 73 years and 83 years, of 72 years and 82 years, of 71 years and 81 years, and of 70 years and 80 years; in all which pairs of lives the difference of the two ages is constantly 10 years. And let the probabilities of the duration of human life be supposed to be such as they are represented to be in Monsieur de Parcieux's table of them.

CCCLXXX. Then, in the first place, we must compute the value of an annuity of one pound a year for the joint continuance of the two oldest lives, to wit, those of the ages of 83 years and 93 years, by means of the expression $\frac{1}{P \times Q} \times$ the series $\frac{P^i \times Q^i}{r} + \frac{P^{ii} \times Q^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} + \&c.$ continued to $E - N - a$ terms, which is given for that purpose in Art. cccclxx.

Here

Here r is = 1.035; and E is 94 years; N , or the number of years in the age of the younger of the two lives, is 83; a , or the difference of the ages of the two lives, is 10 years; $N+a$, or the number of years in the age of the older of the two lives, is 93; $E-N-a$ is 4—93 years, or 1 year; P , or the number of persons living at the age of the younger life, or the age of 83 years, is 71; P' , or the number of persons living at the age of 84 years, is 59; Q , or the number of persons living at the age of the older of the two lives, or at the age of 93 years, is 52; and Q' , or the number of persons living at the age of 94 years, is 41; and Q'' , Q''' , Q^{IV} , &c. or the numbers of persons living at the ages of 95 years, 96 years, 97 years, &c. are equal to 0. Therefore all

the terms of the series $\frac{P^I \times Q^I}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3}$

+ $\frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. after the first term $\frac{P^I \times Q^I}{r}$, are equal to 0; and

consequently the expression $\frac{1}{P \times Q} \times$ the series $\frac{P^I \times Q^I}{r} + \frac{P^{II} \times Q^{II}}{r^2}$

+ $\frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. is = $\frac{1}{P \times Q} \times \frac{P^I \times Q^I}{r}$

= $\frac{1}{71 \times 2} \times \frac{59 \times 1}{1.035} = \frac{1}{142} \times \frac{59}{1.035} = \pounds 1 \times \frac{59}{142 \times 1.035} = \pounds 1$

$\times \frac{59}{146.970} = \pounds 1 \times .401,442 = \pounds 0.401,442$. Therefore, according to

Monsieur de Parcieux's table of the probabilities of the duration of human life, the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 83 years and 93 years, when the interest of money is $3\frac{1}{2}$ per cent. is $\pounds 0.401,442$, or $85\frac{1}{4}d$. QEI .

CCCLXXXI. Having thus found the value of an annuity of one pound a year for the joint continuance of the oldest pair of lives by means

the expression $\frac{1}{P \times Q} \times$ the series $\frac{P^I \times Q^I}{r} + \frac{P^{II} \times Q^{II}}{r^2}$

+ $\frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. given for that purpose in

Art.

Art. cccclxx, we may now proceed to find the values of a like annuity for the joint continuance of all the younger pairs of lives above-mentioned, by means of the expression $\frac{1}{r} \times \frac{P \times Q}{P+d \times 12+e} \times \overline{1+V} |$

This may be done in the manner following.

CCCLXXXII. To find the value of an annuity of one pound year for the joint continuance of two lives of the ages of 82 years and 92 years, we must proceed as follows.

The value of an annuity of one pound a year for the joint continuance of two lives of the ages of 83 years and 93 years has been found to be £0.401,442. Therefore V is = 0.401,442. The numbers of persons living at the ages of 83 years and 82 years are 71 and 85; and the numbers of persons living at the ages of 93 years and 92 years are 2 and 1. Therefore P is = 71, and $P+d$ is = 85, and Q is = 2, and $Q+e$ is = 4. And r is = 1.035. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d \times 12+e} = \frac{1}{1.035} \times \frac{71 \times 2}{85 \times 4} \times £1.401,442 = \frac{1}{1.035} \times \frac{71}{85} \times £1.401,442 = \frac{1}{1.035} \times \frac{71 \times 2}{85 \times 4} \times £1.401,442 = \frac{1}{1.035} \times \frac{71}{170} \times £1.401,442 = \frac{1}{1.035 \times 170} \times 71 \times £1.401,442 = \frac{1}{175.950} \times £99.502,382 = \frac{99.502,382}{175.950} = £0.565,515. Therefore V , or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 82 years and 92 years, is = £0.565,515, or 11s. 3d. $\frac{1}{2}$. Q.E.I.$

CCCLXXXIII. When the two lives are of the ages of 81 years and 91 years, we shall have V = 0.565,515, and consequently $1+V$ = £1.565,515. And P will be = 85, and $P+d$ = 101, and Q = 1, and $Q+e$ = 7. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d \times 12+e} \times \overline{1+V} |$

$$= \frac{1}{1.035} \times \frac{85 \times 4}{101 \times 7} \times \text{£}1.565,515 = \frac{1}{1.035} \times \frac{340}{707} \times$$

$$\text{£}1.565,515 = \frac{340 \times \text{£}1.565,515}{1.035 \times 707} = \frac{340 \times \text{£}1.565,515}{731.745} = \frac{\text{£}532.275,100}{731.745}$$

= £0.727,405. Therefore $\overset{\text{£}}{X}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 81 years and 91 years, is = £0.727,405, or 14s. 6d. $\frac{1}{2}$ Q.E.I.

CCCLXXXIV. When the two lives are of the ages of 80 years and 90 years, we shall have $\overset{\text{£}}{V} = 0.727,405$, and consequently $1 + \overset{\text{£}}{V} = \text{£}1.727,405$. And P will be = 101, $P + d = 118$, $\overset{\text{£}}{Q} = 7$, and $1 + e = 11$. Therefore $\frac{1}{r} \times \frac{P \times \overset{\text{£}}{Q}}{P + d \times | \overset{\text{£}}{Q} + e} \times \frac{1}{1 + \overset{\text{£}}{V}} \text{£}$ will

$$= \frac{1}{1.035} \times \frac{101 \times 7}{118 \times 11} \times \text{£}1.727,405 = \frac{707}{1.035 \times 118 \times 11}$$

$$\text{£}1.727,405 = \frac{1221.275,335}{1.035 \times 118 \times 11} = \frac{1221.275,335}{1.035 \times 1298} = \frac{1221.275,335}{1343.430}$$

= £0.909,072. Therefore $\overset{\text{£}}{X}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 80 years and 90 years, is = £0.909,072, or 18s. 2d. $\frac{1}{2}$. Q.E.I.

CCCLXXXV. When the two lives are of the ages of 79 years and 89 years, we shall have $\overset{\text{£}}{V} = 0.909,072$, and consequently $1 + \overset{\text{£}}{V} = \text{£}1.909,072$. And P will be = 118, and $P + d = 136$, and $\overset{\text{£}}{Q} = 11$,

$$1 + e = 16. \text{ Therefore } \frac{1}{r} \times \frac{P \times \overset{\text{£}}{Q}}{P + d \times | \overset{\text{£}}{Q} + e} \times \frac{1}{1 + \overset{\text{£}}{V}} \text{£}$$

will

$$\begin{aligned} \text{will be} &= \frac{1}{1.035} \times \frac{118 \times 11}{136 \times 16} \times £1.909,072 = \frac{1298 \times 1.909,072}{1.035 \times 136 \times 16} \\ &= \frac{1298 \times 1.909,072}{1.035 \times 2176} = \frac{2477.975,456}{2252.160} = £1,100,266. \quad \text{Therefore} \end{aligned}$$

$\bar{X}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 79 years and 89 years, is £1,100,266 or 1*l.* 2*s.* 0*d.* Q E I.

CCCLXXXVI. When the two lives are of the ages of 78 years and 88 years, we shall have $\bar{V} = 1,100,266$, and consequently $1 + \bar{V} = £2,100,266$. And P will be = 136, and $P + d = 154$, and $\bar{Q} = 1$ and $\bar{Q} + e = 22$. Therefore $\frac{1}{r} \times \frac{P \times \bar{Q}}{P + d \times | \bar{Q} + e } \times \frac{1}{1 + \bar{V}}$ will be = $\frac{1}{1.035} \times \frac{136 \times 16}{154 \times 22} \times £2,100,266 = \frac{1}{1.035} \times \frac{217}{338}$ $\times £2,100,266 = \frac{4570.178,816}{3506.580} = £1,303,315$. Therefore $\bar{X}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 78 years and 88 years, is £1,303,315, 1*l.* 6*s.* 0*d.* $\frac{3}{4}$. Q E I.

CCCLXXXVII. When the two lives are of the ages of 77 years and 87 years, we shall have $\bar{V} = 1,303,315$, and consequently $1 + \bar{V} = £2,303,315$. And P will be = 154, and $P + d = 173$, and $\bar{Q} = 1$ and $\bar{Q} + e = 29$. Therefore $\frac{1}{r} \times \frac{P \times \bar{Q}}{P + d \times | \bar{Q} + e } \times \frac{1}{1 + \bar{V}}$ will be = $\frac{1}{1.035} \times \frac{154 \times 22}{173 \times 29} \times £2,303,315 = \frac{1}{1.035} \times \frac{3388}{173 \times 29}$ $\times £2,303,315 = \frac{1}{1.035} \times \frac{3388}{5017} \times £2,303,315 = \frac{3388}{5192.595}$ $\times £2,303,315$

$\pounds 303,315 = \frac{\pounds 7803.631,220}{5192.595} = \pounds 1.502,838$. Therefore $\mathcal{V}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 77 years and 87 years, is $\pounds 1.502,838$, or 1*l.* 10*s.* $\frac{1}{4}$ *d.* QEI.

CCCLXXXVIII. When the two lives are of the ages of 76 and 86 years, we shall have $\mathcal{V} = \pounds 1.502,838$, and consequently $1 + \mathcal{V} = \pounds 2.502,838$. And P will be = 173, $P + d = 192$, and $\mathcal{Q} = 29$, and $\mathcal{Q} + e = 38$. Therefore $\frac{1}{r} \times \frac{P \times \mathcal{Q}}{P + d \times \mathcal{Q} + e} \times \frac{1}{1 + \mathcal{V}} \pounds$ will be = $\frac{1}{1.035} \times \frac{173 \times 29}{192 \times 38} \times \pounds 2.502,838 = \frac{1}{1.035} \times \frac{5017}{7296} \times \pounds 2.502,838 = \frac{12556.738,246}{7551.360} = \pounds 1.662,844$. Therefore $\mathcal{V}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 76 years and 86 years, is $\pounds 1.662,844$, or 1*l.* 13*s.* 3*d.* QEI.

CCCLXXXIX. When the two lives are of the ages of 75 years and 85 years, we shall have $\mathcal{V} = \pounds 1.662,844$, and consequently $1 + \mathcal{V} \pounds = \pounds 2.662,844$. And P will be = 192, and $P + d = 211$, and $\mathcal{Q} = 38$, and $\mathcal{Q} + e = 48$. Therefore $\frac{1}{r} \times \frac{P \times \mathcal{Q}}{P + d \times \mathcal{Q} + e} \times \frac{1}{1 + \mathcal{V}} \pounds$ will be = $\frac{1}{1.035} \times \frac{192 \times 38}{211 \times 48} \times \pounds 2.662,844 = \frac{1}{1.035} \times \frac{7296}{211 \times 48} \times \pounds 2.662,844 = \frac{1}{1.035} \times \frac{7296}{10128} \times \pounds 2.662,844 = \frac{19,428.109,824}{10482.480} = \pounds 1.853,388$. Therefore $\mathcal{V}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 75 years and 85 years, is $\pounds 1.853,388$, or 1*l.* 17*s.* 0*d.* $\frac{3}{4}$. QEI.

P P P

CCCXC. When

CCCCXC. When the two lives are of the ages of 74 years and 82 years, we shall have $V = £1.853,388$, and consequently $\frac{1}{1+V} = £2.853,388$. And P will be $= 211$, and $P+d = 231$, and $Q = 4$ and $Q+e = 59$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d \times |Q+e|} \times \frac{1}{1+V}$ will be $= \frac{1}{1.035} \times \frac{211 \times 48}{231 \times 59} \times £2.853,388 = \frac{1}{1.035} \times \frac{10128}{13029} \times £2.853,388 = \frac{28,899.113,664}{14,106.015} = £2.048,708$. Therefore V , or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 74 years and 84 years, is $£2.048,708$, 2l. 0s. 11d. $\frac{2}{3}$. QEI.

CCCCXCI. When the two lives are of the ages of 73 years and 82 years, we shall have $V = £2.048,708$, and consequently $\frac{1}{1+V} = £3.048,708$. And P will be $= 231$, and $P+d = 251$, and $Q = 5$ and $Q+e = 71$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d \times |Q+e|} \times \frac{1}{1+V}$ will be $= \frac{1}{1.035} \times \frac{231 \times 59}{251 \times 71} \times £3.048,708 = \frac{1}{1.035} \times \frac{136}{178} \times £3.048,708 = \frac{41,550.841,332}{18,444.735} = £2.252,720$. Therefore V , the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 73 years and 83 years, is $£2.252,720$ or 2l. 5s. $\frac{1}{2}$ d. QEI.

CCCCXCII. When the two lives are of the ages of 72 years and 82 years, we shall have $V = £2.252,720$, and consequently $\frac{1}{1+V} = £3.252,720$. And P will be $= 251$, and $P+d = 271$, and $Q = 7$ and $Q+e = 85$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d \times |Q+e|} \times \frac{1}{1+V} =$

$$= \frac{1}{1.035} \times \frac{251 \times 71}{271 \times 85} \times £3.252,720 = \frac{1}{1.035} \times \frac{17821}{23035} \times$$

$$£3,252,720 = \frac{57,966.723,120}{23,841.225} = £2.431,365. \text{ Therefore } \mathcal{V}, \text{ or the}$$

value of an annuity of one pound a year for the joint continuance of two lives of the ages of 72 years and 82 years, is = £2.431,365, or 2l. 8s. 7d. $\frac{1}{2}$. QE1.

CCCXCIII. When the two lives are of the ages of 71 years and

71 years, we shall have $\mathcal{V} = 2.431,365$, and consequently $\overline{i + V} | £ = £3,431,365$. And P will be = 271, and $P + d = 291$, and $\mathcal{Q} = 85$,

and $\mathcal{Q} + e = 101$. Therefore $\frac{1}{r} \times \frac{P \times \mathcal{Q}}{P + d | \times | \mathcal{Q} + e} \times \overline{i + V} | £$ will

$$= \frac{1}{1.035} \times \frac{271 \times 85}{291 \times 101} \times £3.431,365 = \frac{1}{1.035} \times \frac{23035}{29391}$$

$$£3.431,365 = \frac{79,041.492,775}{30,419.685} = £2.598,366. \text{ Therefore } \mathcal{V}, \text{ or}$$

the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 71 years and 81 years, is = £2.598,366, or 2l. 11s. 14d. $\frac{1}{2}$. QE1.

CCCXCIV. When the two lives are of the ages of 70 years and

70 years, we shall have $\mathcal{V} = 2.598,366$, and consequently $\overline{i + V} | £ = £3,598,366$. And P will be = 291, and $P + d = 310$, and $\mathcal{Q} = 101$,

and $\mathcal{Q} + e = 118$. Therefore $\frac{1}{r} \times \frac{P \times \mathcal{Q}}{P + d | \times | \mathcal{Q} + e} \times \overline{i + V} | £$

$$\text{will be} = \frac{1}{1.035} \times \frac{291 \times 101}{310 \times 118} \times £3.598,366 = \frac{1}{1.035} \times \frac{29,391}{36,580}$$

$$£3.598,366 = \frac{105,759.605,106}{37,860.300} = £2.793,416. \text{ Therefore } \mathcal{V}, \text{ or}$$

the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 80 years, is = £2.793,416, or 2l. 15s. 10d. $\frac{1}{2}$. QE1.

End of the examples of the foregoing method.

CCCXCV. The foregoing examples are, I presume, sufficient to illustrate, and render familiar to the reader, the method given in Art. cccxxvi of deriving the values of annuities for two joint lives from those of equal annuities for two joint lives one year older than the former by means of the expression $\frac{1}{r} \times \frac{P \times Q}{P+d \times 12+e} \times \frac{1}{1+V}$

A remark on the foregoing method of computing a table of the values of annuities for two joint lives.

And this is undoubtedly the best method that can be taken for the purpose of computing a table of these annuities. Yet it is liable to the inconvenience mentioned above explained in Art. cxiv, page 208, as belonging to the other method above explained in Art. c, ci, cii, by which the values of annuities for single lives are successively deduced from the values of annuities for the next older lives by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \frac{1}{1+V} L$, to wit, that, as the values of the annuities are deduced by it one from another in regular succession from the older ages to the younger, any error that should happen to be made in computing the value of any one of them would affect the values of all the following life-annuities, which would belong to younger ages than those in which the error arose. But this inconvenience Mr. Morgan has enabled us to remove by giving us a method of examining and proving the truth of our calculations, as fast as we make them, which is similar to the method given by him for the like purpose in the case of annuities for single lives, which has been stated and explained above in Art. cxv, cxvi, cxvii, and cxviii. This method is as follows.

Of Mr. Morgan's method of proving the truth of the computations of the values of annuities for two joint lives, that are made by means of the foregoing expression, $\frac{1}{r} \times \frac{P \times Q}{P+d \times 12+e} \times \frac{1}{1+V} L$ as fast as they are made.

CCCXCVI. Let N be (as before, in Art. cccxxvi,) the number of years in the age of the younger of any two given lives, upon the joint continuance of which an annuity of one pound *per annum* is to depend and $N+a$ be the number of years in the older of the said lives. And let $P, P', P'', P''', P^{iv}, P^v, P^vi$, &c. be the numbers of persons represented in the table of the probabilities of life that is adopted as the ground of the calculation, as living at the ages of N years, $N+1$ year

years, $N+2$ years, $N+3$ years, $N+4$ years, $N+5$ years, $N+6$ years, &c. respectively; and $\mathcal{Q}$, $\mathcal{Q}'$, $\mathcal{Q}''$, $\mathcal{Q}'''$, $\mathcal{Q}^{IV}$, $\mathcal{Q}^V$, $\mathcal{Q}^{VI}$, &c. the numbers of persons represented therein as living at the ages of $N+a$ years, $N+a+1$ years, $N+a+2$ years, $N+a+3$ years, $N+a+4$ years, $N+a+5$ years, $N+a+6$ years, &c. respectively. Also let A be the number of persons represented in the said table as living at the youngest age in the said table; (which in Monsieur de Parcieux's table of these probabilities is the age of 3 years;) and let K be the number of persons represented therein as living at the age which is greater than the said youngest age by a years. And let the number of years by which the age of N years, or of the younger of the two given lives, exceeds the youngest age in the table, be m years. Then will the age of $N-m$ years be the youngest age in the table, at which the number of persons represented in the table as living is A ; and $N-m+a$ will be a number of years in the age which exceeds the said youngest age by a years, and at which the number of persons represented in the table as living is K .

CCCXCVII. These things being premised, it is evident, in the first Preliminary proposition, That the present value of a single future payment of one pound, to be received at the end of m years in case two persons, who are now of ages of $N-m$ years (or the youngest age in the table,) and $N-m+a$ years, shall both be then alive, is $\frac{1}{r^m} \times \frac{P \times \mathcal{Q}}{A \times K}$. This is evident from Morgan's method.

Problem III, Art. XLV, pages 44, 45, 46.

In the second place, it follows from Art. CCCLXXIII, that the value of remote annuity of one pound a year, that is to commence at the end of m years, (or whereof the first payment is to be received at the end of $m+1$ years,) and that is to continue during the joint lives of two persons of the ages of $N-m$ years (or the youngest age in the

table,) and $N-m+a$ years, will be $\frac{1}{A \times K} \times$ the series $\frac{P' \times \mathcal{Q}'}{r^{m+1}}$

$$\frac{P' \times \mathcal{Q}'}{r^{m+2}} + \frac{P'' \times \mathcal{Q}''}{r^{m+3}} + \frac{P^{IV} \times \mathcal{Q}^{IV}}{r^{m+4}} + \frac{P^V \times \mathcal{Q}^V}{r^{m+5}} + \&c.$$

added to the end of the table, and consequently will be =

$$\times \text{the series } \frac{P' \times \mathcal{Q}'}{A \times K \times r^{m+1}} + \frac{P'' \times \mathcal{Q}''}{A \times K \times r^{m+2}} + \frac{P^{IV} \times \mathcal{Q}^{IV}}{A \times K \times r^{m+3}} + \frac{P^V \times \mathcal{Q}^V}{A \times K \times r^{m+4}} + \&c.$$

$$+ \frac{P^{iv} \times Q^{iv}}{A \times K \times r^{m+4}} + \frac{P^v \times Q^v}{A \times K \times r^{m+5}} + \&c. \text{ continued to the end of the table.}$$
 Let the value of this remote annuity be called R .

And, in the third place, it is evident that the value of an immediate life-annuity of one pound a year (or one of which the first payment is to be received at the end of a year,) that is to continue during the joint lives of two persons of the ages of N years and $N+a$ years, is :

$$\frac{1}{P \times Q} \times \text{the series } \frac{P^i \times Q^i}{r} + \frac{P^{ii} \times Q^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} + \frac{P^v \times Q^v}{r^5} + \&c. \text{ continued to the end of the table.}$$

Let this value be (as before,) called V .

The said principle, proposition itself,

CCCCXCVIII. Now, if the last of these three quantities (which is V) be multiplied into the first of them, to wit, $\frac{1}{r^m} \times \frac{P \times Q}{A \times K}$, or the present value of a future single payment of one pound, to be received at the end of m years in case of the joint continuance of two lives of the ages of $N-m$ years and $N-m+a$ years, the product thence arising will be equal to the second of them, (to wit, R) $\times$ the series

$$+ \frac{P^{ii} \times Q^{ii}}{A \times K \times r^{m+2}} + \frac{P^{iii} \times Q^{iii}}{A \times K \times r^{m+3}} + \frac{P^{iv} \times Q^{iv}}{A \times K \times r^{m+4}} + \frac{P^v \times Q^v}{A \times K \times r^{m+5}} + \&c. \text{ continued to the end of the table,}$$
 or to the value

a remote annuity of one pound a year, that is to commence at the distant of m years, or so that the first payment of it shall be received at the end of $m+1$ years, and that shall continue during the joint lives of two persons of the ages of $N-m$ years (or the youngest age in the table) and $N-m+a$ years.

DEMONSTRATION.

For $\frac{1}{r^m} \times \frac{P \times Q}{A \times K} \times \text{the series } \frac{P^i \times Q^i}{r} + \frac{P^{ii} \times Q^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} + \frac{P^v \times Q^v}{r^5} + \&c. \text{ continued to the end of the table.}$

the end of the table, is $= \frac{1}{r^m} \times \frac{1}{A \times K} \times$ the same series $\frac{P^1 \times Q^1}{r}$
 $+ \frac{P^{11} \times Q^{11}}{r^2} + \frac{P^{111} \times Q^{111}}{r^3} + \frac{P^{1111} \times Q^{1111}}{r^4} + \frac{P^v \times Q^v}{r^5}$
 + &c. continued to the end of the table, $= f_1 \times$ the series
 $\frac{P^1 \times Q^1}{A \times K \times r^{m+1}} + \frac{P^{11} \times Q^{11}}{A \times K \times r^{m+2}} + \frac{P^{111} \times Q^{111}}{A \times K \times r^{m+3}} + \frac{P^{1111} \times Q^{1111}}{A \times K \times r^{m+4}}$
 $+ \frac{P^v \times Q^v}{A \times K \times r^{m+5}} + \&c.$ continued to the end of the table, =

QED.

CCCCXCIX. If, therefore, in computing a table of the several values of an annuity of one pound a year for the joint continuance of several successive pairs of lives whose ages differ from each other by the same number of years, or s years, we at the same time compute the corre-

sponding values of the expression $\frac{1}{r^m} \times \frac{P \times Q}{A \times K}$ and of the ex-

pression $f_1 \times$ the series $\frac{P^1 \times Q^1}{A \times K \times r^{m+1}} + \frac{P^{11} \times Q^{11}}{A \times K \times r^{m+2}} + \frac{P^{111} \times Q^{111}}{A \times K \times r^{m+3}} + \frac{P^{1111} \times Q^{1111}}{A \times K \times r^{m+4}} + \frac{P^v \times Q^v}{A \times K \times r^{m+5}} + \&c.$ continued

to the end of the table, or of R , and then multiply the expression

$\frac{P \times Q}{A \times K}$ into V , or the value of the annuity for two lives which we have before computed, and it shall appear that the product thereby obtained is equal to R , or to $f_1 \times$ the series $\frac{P^1 \times Q^1}{A \times K \times r^{m+1}}$

$\frac{P^{11} \times Q^{11}}{A \times K \times r^{m+2}} + \frac{P^{111} \times Q^{111}}{A \times K \times r^{m+3}} + \frac{P^{1111} \times Q^{1111}}{A \times K \times r^{m+4}} + \frac{P^v \times Q^v}{A \times K \times r^{m+5}} + \&c.$ continued to the end of the table, we may safely conclude that the value of V has been accurately computed.

CCCC. To

An explanation of the manner of applying the foregoing rule, or method of Mr. Morgan to the proof of the truth of the computations of table of the values of annuities for two joint lives.

CCCC. To make the manner of applying this rule of Mr. Morgan more apparent, it will be proper to set down again in regular order, a new table, the values of the life-annuities for the joint continuance of two given lives, which we have computed above in Art. cccclxxxccclxxxi, ccclxxxii, &c.—ccccxciv, from Monsieur de Parcieux

table by means of the expression $\frac{1}{1.035} \times \frac{P \times Q}{P + d \times \frac{Q}{12} + e} \times \frac{1}{1 + V}$

And, as, in obtaining the said values one from another by means of the expression, we proceeded upwards, or from the older lives to the youngest it will be convenient to set down the said values in the same order in the said new table in a column adjoining to two other columns that contain the numbers of years in the two ages corresponding to the said values. After these three columns, (containing the years in the ages of the given lives, and the values of the corresponding annuities,) I shall set down, in a fourth column, the several successive values of the expression

$\frac{1}{1.035} \times \frac{P \times Q}{A \times K}$, or the present values of a single payment of one pound

to be received at the ends of 81 years, 80 years, 79 years, 78 years, and every following lesser number of years down to 70 years, in case there be persons of the ages of 3 years (which is the youngest age in the table) and 13 years shall be then living. And then, in a fifth column, I shall set down the sums of the terms in the foregoing series of values contained in the fourth column, as they arise; so that every term in this fifth column that is even with any two given ages in the first and second columns, shall be equal to the sum of all the terms in the fourth column that correspond to the ages that are older than the said two given ages. Thus, for example, the term in the fifth column that is even with the ages of 75 years and 85 years in the first and second columns, is equal to the sum of all the terms in the fourth column that correspond to the ages that are older than the ages of 75 and 85 years. These sums, contained in this fifth column, will be equal to the values of remote annuities one pound a year for the joint continuance of the lives of two persons of the ages of 3 years and 13 years, that are to commence at the distance of 80 years, 79 years, 78 years, &c. down to 70 years, or that the first payments of them shall become due at the ends of 81 years

to years, 79 years, 78 years, &c. down to 71 years; and they will comprize all the different values of the quantity which in Art. cccxcvii is called R , that relate to remote periods that are greater than 70 years. For they are the sums of the successive values of the expression

$$\begin{aligned} & \frac{P \times Q}{A \times K}, \text{ which, it is evident, are the same with the terms of the} \\ & \frac{P^1 \times Q^1}{A \times K \times r^m + 1} + \frac{P^{11} \times Q^{11}}{A \times K \times r^m + 2} + \frac{P^{111} \times Q^{111}}{A \times K \times r^m + 3} + \frac{P^{1111} \times Q^{1111}}{A \times K \times r^m + 4} \\ & + \frac{P^v \times Q^v}{A \times K \times r^m + 5} + \&c. \text{ continued to the end of the table, which,} \\ & \text{is} = R. \end{aligned}$$

And, lastly, in a sixth column, I shall set down the products that arise by multiplying the terms of the third column, or the values of the annuities for the joint continuance of two lives of the several ages set down in the first and second columns, by the corresponding terms, or terms that are placed even with them, in the fourth column, or by the values of single future payments of one pound, depending on the joint continuance of two lives of the ages of 3 years and 13 years.

And, when we have thus obtained the numbers that are to be placed in these several columns, we must compare those in the sixth column with those that stand even with them in the fifth column: and so far as we find them to be equal to the said numbers in the fifth column, we may conclude that the numbers in the third column, or the values of an annuity of one pound for two joint lives of the ages set down in the first and second columns, have been rightly computed.

CCCCI. The only difficulty that can occur in forming a table of this kind is in the computation of the successive values of the expression

$$\frac{P \times Q}{A \times K}.$$

But this will be found to be a work of no great labour, $\frac{1}{r^m} \times \frac{P \times Q}{A \times K}$.

Of the computation of the successive values of the expression

$$\frac{1}{r^m} \times \frac{P \times Q}{A \times K}.$$

may be performed in the manner following.

Qq q

CCCCII. A is

CCCCII. A is the number of persons represented in Monsieur de Parcieux's table of probabilities as living at the age of 3 years, which is 1000; and K is the number of persons therein represented as living at the age of 13 years, which is 860. Therefore $A \times K$ is $= 1000 \times 860 = 860,000$. And r is $= 1.035$; and therefore (by Mr. Smart's second table of compound interest, page 60 *et seq.*) $\frac{1}{r}$ is $= .966,183,57$

$$\text{and } \frac{1}{r^{81}} \text{ is } = .061,635,61; \quad \frac{1}{r^{80}} \text{ is } = .063,792,85;$$

$$\frac{1}{r^{79}} \text{ is } = .066,025,60; \quad \frac{1}{r^{78}} \text{ is } = .068,336,50;$$

$$\frac{1}{r^{77}} \text{ is } = .070,728,27; \quad \frac{1}{r^{76}} \text{ is } = .073,203,76;$$

$$\frac{1}{r^{75}} \text{ is } = .075,765,90; \quad \frac{1}{r^{74}} \text{ is } = .078,417,70;$$

$$\frac{1}{r^{73}} \text{ is } = .081,162,32; \quad \frac{1}{r^{72}} \text{ is } = .084,003,00;$$

$$\frac{1}{r^{71}} \text{ is } = .086,943,11; \quad \frac{1}{r^{70}} \text{ is } = .089,986,12;$$

$$\frac{1}{r^{69}} \text{ is } = .093,135,63; \quad \frac{1}{r^{68}} \text{ is } = .096,395,38;$$

$$\text{and } \frac{1}{r^{67}} \text{ is } = .099,769,22. \quad \text{And the values of } P \text{ and } Q$$

when m is $= 81$, or the numbers of persons represented in Monsieur de Parcieux's table of probabilities as living at the ages of $(81 + 3$ years or) 84 years and $(81 + 13$ years, or) 94 years, are 59 and 1; and the following values of P and Q at the following lesser ages of 83 years and 93 years, 82 years and 92 years, 81 years and 91 years, 80 years and 90 years, 79 years and 80 years, 78 years and 88 years, 77 years and 87 years, 76 years and 86 years, 75 years and 85 years, 74 years and 84 years, 73 years and 83 years, 72 years and 82 years, 71 years and 81 years, and 70 years and 80 years, are 71 and 2, 85 and 4, 101 and 7, 118 and 11, 136 and 16, 154 and 22, 173 and 29, 192 and 38, 211 and 48, 231 and 59, 251 and 71, 271 and 85, 291 and 101, and, lastly, 310 and 118.

CCCCIII. Therefore

CCCCIII. Therefore the first value of the expression $\frac{\pounds}{r^m} \times \frac{P \times Q}{A \times K}$,

or that which it has when m is = 81, is = $\pounds 0.061,635,61 \times \frac{59 \times 1}{860,000}$

$$= \frac{\pounds 59 \times 0.061,635,61}{860,000} = \frac{3.636,400}{860,000} = \pounds 0.000,004,22.$$

The second value of the expression $\frac{\pounds}{r^m} \times \frac{P \times Q}{A \times K}$, or that which

it has when m is = 80, is = $\pounds 0.063,792,85 \times \frac{71 \times 2}{860,000} =$

$$\frac{\pounds 42 \times 0.063,792,85}{860,000} = \frac{9.058,584}{860,000} = \pounds 0.000,010,53.$$

The third value of the expression $\frac{\pounds}{r^m} \times \frac{P \times Q}{A \times K}$, or that which

it has when m is = 79, is = $\pounds 0.066,025,60 \times \frac{85 \times 4}{860,000} =$

$$\frac{\pounds 40 \times 0.066,025,60}{860,000} = \frac{34 \times 0.066,025,60}{86,000} = \frac{2.244,870}{86,000} = \pounds 0.000,026,10.$$

The fourth value of the expression $\frac{\pounds}{r^m} \times \frac{P \times Q}{A \times K}$, or that which

it has when m is = 78, is = $\pounds 0.068,336,50 \times \frac{101 \times 7}{860,000} =$

$$\frac{\pounds 27 \times 0.068,336,50}{860,000} = \frac{48.313,905}{860,000} = \pounds 0.000,056,17.$$

The fifth value of $\frac{£}{r_m} \times \frac{P \times Q}{A \times K}$ is = £0.070,728,27

$$\frac{118 \times 11}{860,000} = \frac{1298 \times £0.070,728,27}{860,000} = \frac{£91,805,094}{860,000} = £0.000,106,75.$$

The sixth value of $\frac{£}{r_m} \times \frac{P \times Q}{A \times K}$ is = £0.073,203,76

$$\frac{136 \times 16}{860,000} = \frac{2176 \times £0.073,203,76}{860,000} = \frac{£159,291,381}{860,000} = £0.000,185,22.$$

The seventh value of $\frac{£}{r_m} \times \frac{P \times Q}{A \times K}$ is = £0.075,765,90

$$\frac{154 \times 22}{860,000} = \frac{3388 \times £0.075,765,90}{860,000} = \frac{£256,694,869}{860,000} = £0.000,298,48.$$

The eighth value of $\frac{£}{r_m} \times \frac{P \times Q}{A \times K}$ is = £0.078,417,70

$$\frac{173 \times 29}{860,000} = \frac{5017 \times £0.078,417,70}{860,000} = \frac{£393,421,600}{860,000} = £0.000,457,46.$$

The ninth value of $\frac{£}{r_m} \times \frac{P \times Q}{A \times K}$ is = £0.081,162,32

$$\frac{192 \times 38}{860,000} = \frac{7296 \times £0.081,162,32}{860,000} = \frac{£592,160,286}{860,000} = £0.000,688,55.$$

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$$\text{The tenth value of } \frac{\frac{£}{r_m}}{\frac{£}{r_m}} \times \frac{P \times Q}{A \times K} \text{ is } = £0.084,003,00 \times$$

$$\frac{211 \times 48}{860,000} = \frac{10,128 \times 0.084,003}{860,000} = \frac{850,782,384}{860,000} = £0.000,989,28.$$

$$\text{The eleventh value of } \frac{\frac{£}{r_m}}{\frac{£}{r_m}} \times \frac{P \times Q}{A \times K} \text{ is } = £0.086,943,11 \times$$

$$\frac{231 \times 59}{860,000} = \frac{13629 \times 0.086,943,11}{860,000} = \frac{1184,947,646}{860,000} = £0.001,377,84.$$

$$\text{The twelfth value of } \frac{\frac{£}{r_m}}{\frac{£}{r_m}} \times \frac{P \times Q}{A \times K} \text{ is } = £0.089,986,12 \times$$

$$\frac{251 \times 71}{860,000} = \frac{17821 \times 0.089,986,12}{860,000} = \frac{1603,642,644}{860,000} = £0.001,864,70.$$

$$\text{The thirteenth value of } \frac{\frac{£}{r_m}}{\frac{£}{r_m}} \times \frac{P \times Q}{A \times K} \text{ is } = £0.093,135,63 \times$$

$$\frac{271 \times 85}{860,000} = \frac{23035 \times 0.093,135,63}{860,000} = \frac{2145,379,237}{860,000} = £0.002,494,62.$$

$$\text{The fourteenth value of } \frac{\frac{£}{r_m}}{\frac{£}{r_m}} \times \frac{P \times Q}{A \times K} \text{ is } = £0.096,395,38 \times$$

$$\frac{291 \times 101}{860,000} = \frac{29391 \times 0.096,395,38}{860,000} = \frac{2833,156,513}{860,000} = £0.003,294,36.$$

And

$$\text{And the fifteenth value of } \frac{\pounds}{r^m} \times \frac{P \times Q}{A \times K} \text{ is } = \pounds 0.099,769,22 \times \frac{310 \times 118}{860,000} = \frac{\pounds 36580 \times 0.099,769,22}{860,000} = \frac{\pounds 3649.558,067}{860,000} = \pounds 0.004,243,67.$$

CCCCIV. These several successive values of the expression $\frac{\pounds}{r^m} \times \frac{P \times Q}{A \times K}$, if ranged in order, will be as follows.

$\pounds$	0.000,004,22,
	0.000,010,53,
	0.000,026,10,
	0.000,056,17,
	0.000,106,75,
	0.000,185,22,
	0.000,298,48,
	0.000,457,46,
	0.000,688,55,
	0.000,989,28,
	0.001,377,84,
	0.001,864,70,
	0.002,494,62,
	0.003,294,36,
	0.004,243,67.

These numbers therefore will constitute the fourth column of the ensuing table.

CCCCV. The

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CCCCV. The sums that arise by the continual addition of these numbers will be as follows.

£	0.000,004,22,
	0.000,014,75,
	0.000,040,85,
	0.000,097,02,
	0.000,203,77,
	0.000,388,99,
	0.000,687,47,
	0.001,144,93,
	0.001,833,48,
	0.002,822,76,
	0.004,200,60,
	0.006,065,30,
	0.008,559,92,
	0.011,854,28,
	0.016,097,95.

The sums of the said successive values of the expression

$$\sum \frac{1}{m} \times \frac{P \times Q}{A \times K}.$$

These numbers therefore will constitute the fifth column of the following table.

CCCCVI. This

CCCCVI. This table is as follows.

T A B L E XXVIII.

Consisting of six columns of numbers; in the first of which the number of years in the several ages of human life, that differ from each other by a year, from the age of 84 years to the age of 70 years, inclusively, are set down in regular order; and in the second column are set down the numbers of years in the several ages of human life, that differ from each other by a year, from the age of 94 years to the age of 80 years inclusively; and in the third column are set down the several values of an annuity of one pound a year for the joint continuance of the lives of two persons of the ages set down in the first and second columns even with the said values; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. and in the fourth column are set down the present values of a single payment of one pound, to be received at the ends of 81 years, 80 years, 79 years, 78 years, and every following lesser number of years down to 67 years, inclusively, if two persons of the ages of 3 years and 13 years shall both be living at the ends of the said years; and in the fifth column are set down the numbers that arise by the continual addition of the numbers set down in the fourth column; so that each number in the said fifth column is equal to the sum of all the numbers in the said fourth column that are placed above it, or that correspond to the preceding, or older, ages; and in the sixth and last column are set down the products that arise by multiplying the terms of the third column, (or the values of a life-annuity of one pound a year for the joint continuance of two lives of the ages set down in the first and second columns,) by the corresponding terms of the fourth column, respectively.

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Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Present values of a single payment of one pound, to be received at the ends of 81 years, 80 years, 79 years, 78, 77, 76, &c. years, in case two persons of the ages of 3 years and 13 years shall both be living at the ends of those years respectively.	Sums of the values in the fourth co- lumn.	Products of the mul- tiplication of the numbers in the third and fourth columns.
94	£ 0.000,000	£ 0.000,004,22	£ 0.000,004,22	£ 0,000,004,22
93	0.401,442	0.000,010,53	0.000,014,75	0.000,014,76
92	0.565,515	0.000,026,10	0.000,040,85	0.000,040,85
91	0.727,405	0.000,056,17	0.000,097,02	0.000,097,04
90	0.909,072	0.000,106,75	0.000,203,77	0.000,203,79
89	1.100,266	0.000,185,22	0.000,388,99	0.000,389,01
88	1.303,315	0.000,298,48	0.000,687,47	0.000,687,48
87	1.502,838	0.000,457,46	0.001,144,93	0.001,144,95
86	1.662,844	0.000,688,55	0.001,833,48	0.001,833,51
85	1.853,388	0.000,989,28	0.002,822,76	0.002,822,89
84	2.048,708	0.001,377,84	0.004,200,60	0.004,200,64
83	2.252,720	0.001,864,70	0.006,065,30	0.006,065,33
82	2.431,365	0.002,494,62	0.008,559,92	0.008,559,95
81	2.598,366	0.003,294,36	0.011,854,28	0.011,854,33
80	2.793,416	0.004,243,67		

R r r SCHOLIUM.

S C H O L I U M.

CCCCVII. In the foregoing table I have computed only the fourteen values (reckoning from the oldest ages to the younger,) of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 10 years. It would have been difficult (as the reader must perceive,) to complete the table by computing the values of an annuity of one pound a year for the joint continuance of all the younger lives whose ages differ from each other the same difference of 10 years, down to the value of a like annuity for the joint continuance of two lives of the ages of 3 years and 13 years. But this was not necessary to the design with which the foregoing computations were undertaken; which was only to shew how such a table

might be formed by means of the expression $\frac{1}{r} \times \frac{P \times Q}{P + d \times Q}$ $\times \frac{1}{1 + \frac{1}{N}}$ £, and how the several numbers thereby obtained might be verified, as fast as they were computed, by multiplying them into the corresponding values of the expression $\frac{1}{r} \times \frac{P \times Q}{N \times K}$ or the com-

ponding numbers that are set down in the fourth column of the table, and comparing the products thence arising with the numbers the fifth column. This, I apprehend, is made sufficiently manifest by the computation of the fourteen values set down in the foregoing table, and therefore I have declined the trouble of continuing these computations any further.

CCCCVIII. Nevertheless, as it will be of great convenience to several persons as have occasion to deal in the purchase of annuities for joint lives, to have a *complete* table of the values of an annuity of one pound a year for the joint continuance of two lives whose ages differ from each other by 10 years, (of which the foregoing table contains only the first four numbers,) and likewise to have other tables of the values of the like annuities for the joint continuance of two lives whose ages differ from each other by more, or less, than 10 years, I have caused the foregoing table to be completed by another hand under the inspection of the learned Mr. Morgan, above-mentioned, (the present actuary of the Society for Equitable Assurances on Lives and Survivorships,) and also nine other tables of the same kind to be computed from Monsieur de Parcieux's

table of the probabilities of the duration of human life upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. (as it is supposed to be in the foregoing table,) to wit, a table of the values of an annuity of one pound a year for the lives of two persons of equal ages, another of the values of the like annuity for the lives of two persons whose ages differ from each other by five years, a third for two lives whose ages differ by 20 years, a fourth for two lives whose ages differ by 30 years, and a fifth, sixth, seventh, eighth, and ninth, table, for two lives whose ages differ by 40 years, 50 years, 60 years, 70 years, and 80 years. These tables have all been computed by means of the expression

$$\frac{1}{i} \times \frac{P \times Q}{P + Q} \times \frac{1}{1 + \sqrt{1 + i}} \times f, \quad \text{in the same manner as the}$$

numbers above computed in Table XXVIII; and the computations have also been verified in the same manner, to wit, by computing

$$\text{the successive values of the expression } \frac{1}{r^m} \times \frac{P \times Q}{A \times K}, \quad \text{and by finding the}$$

sums of those successive values, and, lastly, by multiplying the values of the several annuities into the corresponding values of the expression

$$\frac{1}{i} \times \frac{P \times Q}{A \times K}, \quad \text{and observing that the products thereby obtained were}$$

$$\text{equal to the corresponding sums of the successive values of } \frac{1}{r^m} \times \frac{P \times Q}{A \times K}.$$

As I have not thought it necessary to cause all these latter numbers (which are only to prove the truth of the computations) to be printed, and therefore I shall present the reader with only the values of the annuities themselves, as was done in the tables of the values of an annuity of one pound a year for a single life given above in Art. cci, Tables XII, XIII, XIV, XV, — XXIII, pages 221, 222, 223, — 232. And, as it is most usual, in exhibiting tables of the values of life-annuities, to begin with those of the younger ages and proceed on to those of older ages, I shall observe the same order in setting down the values contained in the foregoing tables, notwithstanding they were computed one from another proceeding in a contrary order, or from the older ages to the younger. These tables are as follows.

T A B L E XXIX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons of the same age, when the interest of money is $3\frac{1}{2}$ per cent.-----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life

Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	3	15.785,24	34	34	14.172,18	65	65	5.699,541
4	4	16.363,93	35	35	14.008,32	66	66	5.373,930
5	5	16.731,89	36	36	13.838,74	67	67	5.061,732
6	6	16.994,35	37	37	13.663,10	68	68	4.764,788
7	7	17.170,57	38	38	13.437,90	69	69	4.485,940
8	8	17.287,49	39	39	13.203,01	70	70	4.229,522
9	9	17.378,30	40	40	12.957,85	71	71	3.967,857
10	10	17.397,65	41	41	12.701,78	72	72	3.735,261
11	11	17.338,47	42	42	12.434,14	73	73	3.506,636
12	12	17.194,84	43	43	12.154,19	74	74	3.285,035
13	13	17.045,84	44	44	11.861,14	75	75	3.075,111
14	14	16.891,22	45	45	11.554,15	76	76	2.843,825
15	15	16.730,68	46	46	11.232,32	77	77	2.625,379
16	16	16.563,92	47	47	10.933,91	78	78	2.429,114
17	17	16.432,30	48	48	10.620,90	79	79	2.223,694
18	18	16.296,21	49	49	10.330,55	80	80	2.057,238
19	19	16.155,41	50	50	10.025,94	81	81	1.906,337
20	20	16.009,67	51	51	9.743,490	82	82	1.785,767
21	21	15.900,58	52	52	9.484,577	83	83	1.649,066
22	22	15.788,72	53	53	9.213,854	84	84	1.471,612
23	23	15.673,96	54	54	8.930,288	85	85	1.301,203
24	24	15.556,18	55	55	8.669,387	86	86	1.148,822
25	25	15.435,19	56	56	8.396,670	87	87	1.041,569
26	26	15.310,85	57	57	8.111,004	88	88	0.873,177
27	27	15.183,00	58	58	7.847,174	89	89	0.708,630
28	28	15.051,43	59	59	7.571,510	90	90	0.551,722
29	29	14.915,97	60	60	7.282,755	91	91	0.410,102
30	30	14.776,38	61	61	6.979,449	92	92	0.299,800
31	31	14.632,46	62	62	6.659,908	93	93	0.241,546
32	32	14.483,96	63	63	6.356,826	94	94	0.000,000
33	33	14.330,63	64	64	6.037,443			

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TICK B L E XXX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 5 years; when the interest of money is $3\frac{1}{4}$ per cent. --- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Value of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Value of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Value of an annuity of one pound a year for the joint continuance of both lives.
3	8	£ 16.510,17	32	37	£ 14.035,08	61	66	£ 6.065,430
4	9	16.854,05	33	38	13.843,23	62	67	5.748,630
5	10	17.051,58	34	39	13.643,78	63	68	5.447,890
6	11	17.155,02	35	40	13.436,29	64	69	5.150,625
7	12	17.171,55	36	41	13.220,25	65	70	4.858,151
8	13	17.154,48	37	42	12.995,13	66	71	4.567,925
9	14	17.120,70	38	43	12.739,85	67	72	4.299,865
10	15	17.048,09	39	44	12.473,04	68	73	4.040,373
11	16	16.933,54	40	45	12.193,97	69	74	3.792,444
12	17	16.795,58	41	46	11.901,58	70	75	3.560,612
13	18	16.652,69	42	47	11.616,67	71	76	3.314,346
14	19	16.504,62	43	48	11.317,93	72	77	3.088,057
15	20	16.351,08	44	49	11.025,10	73	78	2.876,561
16	21	16.213,13	45	50	10.718,15	74	79	2.663,173
17	22	16.090,90	46	51	10.416,04	75	80	2.477,972
18	23	15.964,95	47	52	10.137,24	76	81	2.292,901
19	24	15.835,08	48	53	9.845,200	77	82	2.129,559
20	25	15.701,10	49	54	9.556,736	78	83	1.964,259
21	26	15.583,33	50	55	9.273,595	79	84	1.770,308
22	27	15.462,37	51	56	8.994,268	80	85	1.595,707
23	28	15.338,05	52	57	8.718,822	81	86	1.437,314
24	29	15.210,20	53	58	8.449,496	82	87	1.316,220
25	30	15.078,62	54	59	8.167,754	83	88	1.149,833
26	31	14.943,13	55	60	7.889,259	84	89	0.969,173
27	32	14.803,51	56	61	7.597,410	85	90	0.793,411
28	33	14.659,54	57	62	7.290,800	86	91	0.630,011
29	34	14.510,98	58	63	7.002,970	87	92	0.495,243
30	35	14.357,56	59	64	6.700,900	88	93	0.351,339
31	36	14.199,03	60	65	6.382,877	89	94	0.000,000

T A B L E XXXI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 10 years; when the interest of money is $3\frac{1}{2}$ per cent.---Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	13	£ 6.366,17	31	41	£ 13.500,30	59	69	£ 5.611,286
4	14	16.585,56	32	42	13.282,31	60	70	5.336,698
5	15	16.688,70	33	43	13.055,10	61	71	5.054,106
6	16	16.732,59	34	44	12.818,10	62	72	4.784,148
7	17	16.749,69	35	45	12.570,68	63	73	4.523,082
8	18	16.734,46	36	46	12.312,20	64	74	4.260,822
9	19	16.703,36	37	47	12.063,41	65	75	3.999,072
10	20	16.634,79	38	48	11.784,38	66	76	3.728,183
11	21	16.547,42	39	49	11.513,43	67	77	3.470,693
12	22	16.418,12	40	50	11.229,92	68	78	3.233,054
13	23	16.284,59	41	51	10.953,88	69	79	2.996,397
14	24	16.146,60	42	52	10.685,81	70	80	2.793,419
15	25	16.003,93	43	53	10.405,59	71	81	2.598,368
16	26	15.856,33	44	54	10.112,29	72	82	2.431,367
17	27	15.723,54	45	55	9.825,467	73	83	2.252,722
18	28	15.586,50	46	56	9.525,226	74	84	2.048,709
19	29	15.444,98	47	57	9.227,309	75	85	1.853,389
20	30	15.298,75	48	58	8.935,094	76	86	1.602,845
21	31	15.167,59	49	59	8.645,309	77	87	1.502,839
22	32	15.032,49	50	60	8.341,630	78	88	1.303,316
23	33	14.893,27	51	61	8.038,570	79	89	1.100,267
24	34	14.749,68	52	62	7.735,712	80	90	0.909,073
25	35	14.601,50	53	63	7.437,175	81	91	0.727,405
26	36	14.448,46	54	64	7.123,728	82	92	0.565,515
27	37	14.290,29	55	65	6.808,550	83	93	0.421,442
28	38	14.104,15	56	66	6.496,025	84	94	0.004,000
29	39	13.910,74	57	67	6.186,700			
30	40	13.709,61	58	68	5.894,503			

T A B L E XXXII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 20 years; when the interest of money is $3\frac{1}{2}$ per cent. --- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	23	£ 15.560,29	27	47	£ 12.402,10	51	71	£ 5.515,096
4	24	15.772,84	28	48	12.146,36	52	72	5.249,786
5	25	15.876,39	29	49	11.900,87	53	73	4.984,023
6	26	15.925,03	30	50	11.644,53	54	74	4.719,681
7	27	15.929,42	31	51	11.398,29	55	75	4.469,896
8	28	15.902,97	32	52	11.162,98	56	76	4.202,853
9	29	15.861,35	33	53	10.917,97	57	77	3.943,096
10	30	15.784,00	34	54	10.662,54	58	78	3.706,498
11	31	15.667,99	35	55	10.417,61	59	79	3.462,599
12	32	15.510,67	36	56	10.162,63	60	80	3.246,444
13	33	15.347,68	37	57	9.896,823	61	81	3.039,033
14	34	15.178,75	38	58	9.625,224	62	82	2.848,658
15	35	15.003,54	39	59	9.342,070	63	83	2.646,551
16	36	14.821,70	40	60	9.046,436	64	84	2.409,133
17	37	14.651,58	41	61	8.737,294	65	85	2.173,497
18	38	14.452,13	42	62	8.413,488	66	86	1.953,728
19	39	14.244,52	43	63	8.095,180	67	87	1.766,127
20	40	14.028,29	44	64	7.761,737	68	88	1.527,607
21	41	13.821,34	45	65	7.411,735	69	89	1.292,917
22	42	13.605,76	46	66	7.064,716	70	90	1.065,724
23	43	13.381,12	47	67	6.733,989	71	91	0.846,497
24	44	13.146,84	48	68	6.408,777	72	92	0.646,370
25	45	12.902,34	49	69	6.102,706	73	93	0.444,598
26	46	12.646,97	50	70	5.807,267	74	94	0.000,000

T A B L E XXXIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 30 years; when the interest of money is $3\frac{1}{4}$ per cent. --- Compute from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	33	£ 14.605,10	24	54	£ 10.805,23	45	75	£ 4.717,841
4	34	14.761,38	25	55	10.556,78	46	76	4.427,255
5	35	14.812,79	26	56	10.298,13	47	77	4.152,480
6	36	14.810,22	27	57	10.028,53	48	78	3.892,549
7	37	14.763,70	28	58	9.769,124	49	79	3.631,597
8	38	14.662,37	29	59	9.499,177	50	80	3.399,171
9	39	14.542,30	30	60	9.217,864	51	81	3.182,288
10	40	14.384,49	31	61	8.924,272	52	82	2.990,528
11	41	14.186,34	32	62	8.617,370	53	83	2.779,762
12	42	13.945,55	33	63	8.317,989	54	84	2.533,005
13	43	13.694,31	34	64	8.005,274	55	85	2.295,974
14	44	13.432,04	35	65	7.678,014	56	86	2.071,761
15	45	13.158,08	36	66	7.356,765	57	87	1.876,901
16	46	12.871,77	37	67	7.042,733	58	88	1.628,765
17	47	12.611,02	38	68	6.726,106	59	89	1.381,241
18	48	12.338,55	39	69	6.419,796	60	90	1.137,781
19	49	12.075,74	40	70	6.126,861	61	91	0.903,979
20	50	11.801,15	41	71	5.828,088	62	92	0.686,040
21	51	11.551,46	42	72	5.547,756	63	93	0.467,103
22	52	11.312,81	43	73	5.267,684	64	94	0.000,000
23	53	11.064,30	44	74	4.990,020			

T A B L E XXXIV.

showing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 40 years; when the interest of money is $3\frac{1}{4}$ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the life of the older person.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the life of the younger person.	Years in the life of the older person.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the life of the younger person.	Years in the life of the older person.	Values of an annuity of one pound a year for the joint continuance of both lives.
43.	12.995,63	21	61	8.987,406	39	79	3.742,057
44	13.020,79	22	62	8.674,708	40	80	3.511,390
45	12.944,45	23	63	8.369,404	41	81	3.291,726
46	12.812,26	24	64	8.050,492	42	82	3.092,312
47	12.655,71	25	65	7.716,753	43	83	2.873,803
48	12.464,90	26	66	7.388,815	44	84	2.619,179
49	12.274,57	27	67	7.067,833	45	85	2.369,586
50	12.047,58	28	68	6.755,442	46	86	2.133,182
51	11.804,02	29	69	6.453,925	47	87	1.931,165
52	11.543,45	30	70	6.166,483	48	88	1.669,911
53	11.271,87	31	71	5.873,947	49	89	1.412,744
54	10.988,55	32	72	5.600,948	50	90	1.159,767
55	10.714,92	33	73	5.329,415	51	91	0.919,313
56	10.429,73	34	74	5.061,816	52	92	0.697,812
57	10.145,47	35	75	4.801,680	53	93	0.473,412
58	9.870,862	36	76	4.525,229	54	94	0.000,000
59	9.584,837	37	77	4.259,330			
60	9.286,549	38	78	4.003,965			

T A B L E XXXV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 50 years; when the interest of money is $3\frac{1}{2}$ per cent. Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	53	10.680,23	24	74	5.075,965
4	54	10.628,92	25	75	4.811,044
5	55	10.513,03	26	76	4.529,340
6	56	10.350,53	27	77	4.257,630
7	57	10.148,69	28	78	4.003,127
8	58	9.938,555	29	79	3.742,189
9	59	9.709,814	30	80	3.512,640
10	60	9.449,236	31	81	3.294,317
11	61	9.154,806	32	82	3.096,568
12	62	8.824,696	33	83	2.880,142
13	63	8.501,682	34	84	2.628,121
14	64	8.164,376	35	85	2.382,003
15	65	7.811,534	36	86	2.150,472
16	66	7.463,965	37	87	1.950,898
17	67	7.132,382	38	88	1.689,411
18	68	6.809,136	39	89	1.429,589
19	69	6.496,404	40	90	1.175,112
20	70	6.197,247	41	91	0.931,818
21	71	5.900,765	42	92	0.706,129
22	72	5.623,758	43	93	0.477,775
23	73	5.348,020	44	94	0.000.000

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TABLE XXXVI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 60 years; when the interest of money is $3\frac{1}{4}$ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	63	£8.049,257	20	80	£3.526,172
4	64	7.882,630	21	81	3.306,196
5	65	7.643,727	22	82	3.106,801
6	66	7.382,707	23	83	2.888,572
7	67	7.107,740	24	84	2.634,545
8	68	6.828,138	25	85	2.386,279
9	69	6.554,273	26	86	2.152,327
10	70	6.281,256	27	87	1.949,808
11	71	5.989,106	28	88	1.688,533
12	72	5.702,314	29	89	1.428,905
13	73	5.416,621	30	90	1.174,597
14	74	5.134,386	31	91	0.931,450
15	75	4.858,956	32	92	0.705,886
16	76	4.566,065	33	93	0.477,648
17	77	4.288,874			
18	78	4.028,814			
19	79	3.761,967			

T A B L E XXXVII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 70 years; when the interest of money is 34 per cent. Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	73	£ 5.137,907	14	84	£ 2.656,044
4	74	4.956,842	15	85	2.402,893
5	75	4.746,962	16	86	2.163,852
6	76	4.503,802	17	87	£ 1.959,233
7	77	4.258,192	18	88	1.695,616
8	78	4.027,334	19	89	1.433,647
9	79	3.777,677	20	90	1.176,850
10	80	3.557,527	21	91	0.933,060
11	81	3.341,255	22	92	0.706,947
12	82	3.137,624	23	93	0.478,200
13	83	2.914,904	24	94	0.000,000

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T A B L E XXXVIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 80 years; when the interest of money is $3\frac{1}{4}$ per cent.---
Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	83	£ 2.783,857	9	89	£ 1.432,098
4	84	2.574,553	10	90	1.180,457
5	85	2.351,322	11	91	0.937,542
6	86	2.133,541	12	92	0.709,889
7	87	1.940,957	13	93	0.479,721
8	88	1.686,249	14	94	0.000,000

CCCCIX. According

CCCCIX. According to Monsieur de Parcieux's table of the probabilities of the duration of human life, (which begins with the age of years, and ends with the age of 94 years, and supposes the utmost possible extent of human life to be somewhat less than 95 years,) it is evident that only one pair of lives can be found whose ages will differ from each other by 90 years, to wit, two lives of the ages of 3 and 93 years. And the value of an annuity of one pound a year for the joint continuance of the lives of these ages, upon a supposition that the interest of money is 3 per cent. is £0.468,599.

Remarks on the great number of tables (of the same kind as those above computed,) that would be necessary in order to exhibit the values annuities for two joint lives in all their possible varieties.

CCCCX. It would evidently be a work of great use and convenience to those persons who have occasion to sell or purchase annuities for joint lives, to compute as many more tables, of the same kind as the foregoing ones, as there may be taken different numbers of years for the difference of the ages of the two lives upon which the annuity is to depend; so that, for example, if the difference of the ages of any two lives was 23 years, or 27 years, instead of 20 years, or 30 years, the value of an annuity of one pound a year during the joint continuance of both lives should be accurately exhibited in some one or other of the tables, the same manner as the values of the like annuities are exhibited in the foregoing tables when the difference of the ages is either 0, or 5 years, 10 years, or 20, 30, 40, 50, 60, 70, or 80, years. But this would require an immense quantity of calculation. For, in order to have the exact value of an annuity of one pound a year for two joint lives in all the possible varieties in which the ages of the two lives may be combined together, when the interest of money is $3\frac{1}{4}$ per cent. (as it is supposed to be in the foregoing tables,) it would be necessary to compute no fewer than different tables, or 83 more than the ten that are above computed. Besides the said ten tables, which exhibit those values when the ages of two lives are equal, and when they differ from each other by 5 years, 10 years, or 20 years, or 30 years, or 40 years, or 50 years, or 60 years, or 70 years, or 80 years, it would be necessary to compute an eleven table of the same kind that should exhibit the values of a like annuity the joint continuance of two lives whose ages differed from each other one year; and a twelfth table of the same kind when the difference of the ages is two years; and a thirteenth table of the same kind when the difference of the ages is three years; and, in general, a new table of

same kind for every new difference that can be taken between the said two ages; which differences (if we reckon 0 for one of them, or include the use of two lives of equal ages,) will, all together, (or including the several differences in the tables above-computed,) amount to the number of 93. If the youngest life set down in the table of probabilities is of the age of one year; or, if we make use of Monsieur de Parcieux's table of probabilities, (in which the youngest age is that of 3 years,) will amount to the number of 91. And, if we were desirous of having the values of the like annuities for two joint lives in all their possible varieties when the interest of money is either 2 per cent. or $2\frac{1}{2}$ per cent. or 3 per cent. or 4 per cent. or $4\frac{1}{2}$ per cent. or 5 per cent. or 6 per cent. or 7 per cent. or 8 per cent. or 9 per cent. or 10 per cent. as well as when it is $3\frac{1}{4}$ per cent. (as we have the values of the like annuities for single lives of all ages for all those different rates of interest in the tables exhibited above in pages 221, 222, — — — 232,) it would be necessary to compute the same number of tables of this kind, to wit, 93, or 91, tables, for each of these rates of interest; which would make, in all, 12 times 93, or 12 times 91, such tables, or more than eleven hundred tables. Now the computation of such a prodigious number of tables, would be a business of so much length and labour that it, probably, will never be undertaken; though, perhaps, it might be worth the while of the Government, or of some of the societies for making insurances upon lives, or of the dean and chapter of some rich cathedral church, whose lands are leased out upon lives, or of some other wealthy body of men, to whom such tables might be peculiarly useful, to cause two, or three, sets of such tables to be computed for two, or three, of the most common and useful rates of interest, as, for example, for $3\frac{1}{4}$ per cent. 4 per cent. and 5 per cent. If this were to be undertaken and carefully performed, under such encouragement, it would, I doubt not, be allowed on all hands to be a work of great merit and of general advantage to the publick.

A method of finding by Interpolation the values of such annuities for two joint lives, as are not contained in any of the foregoing tables.

CCCCXI. But in the mean while, and until such tables shall be published, it will be desirable to find out, if possible, some tolerably easy method of deriving the values of such annuities for two joint lives as are set down in the foregoing ten tables, from the values of those which therein exhibited. Now this may be done to a moderate degree of exactness, sufficient for common purposes, by a kind of *Interpolation*, which may be explained in the following manner.

CCCCXII. The

The principles of
the said method
of Interpolation.

CCCCXII. The principles upon which this method of interpolation is founded are as follows.

In the first place it is evident beyond a doubt, that the value of an annuity for the joint continuance of any two given lives is greater than the value of the like annuity for two other lives whereof the younger is of the same age with the younger of the two former lives, and the older is older than the older of the two former lives: or, in other words, the value of an annuity for two joint lives of N years and $N+n$ years is greater than the value of the same annuity for two joint lives of the ages of N years and $N+n+e$ years. Thus, for example, the value of an annuity of one pound a year for two joint lives of the ages of 23 years and 30 years is greater than the value of the like annuity for two joint lives of the ages of 23 years and 40 years.

In the second place it seems highly probable, that, if we take a moderate number of lives that are successively older the one than the other by one year, and combine them, one after another, with another life that is younger than any of them, the values of an annuity for these successive pairs of joint lives, (in all of which the youngest life is of the same age,) will be nearly in arithmetical proportion, as well as the ages of the older lives in these successive pairs of lives, which are supposed to increase by the equal difference of one year. Thus, if the younger life in each of these pairs of lives is of the age of N years, and the several older lives, with which this life is to be successively combined, are of the ages of $N+n$ years, $N+n+1$ years, $N+n+2$ years, $N+n+3$ years, $N+n+4$ years, $N+n+5$ years, &c. it is probable that the values of an annuity

for two joint lives of the ages of N years and $N+n$ years,
and for two joint lives of the ages of N years and $N+n+1$ years,
and for two joint lives of the ages of N years and $N+n+2$ years,
and for two joint lives of the ages of N years and $N+n+3$ years,
and for two joint lives of the ages of N years and $N+n+4$ years,
and for two joint lives of the ages of N years and $N+n+5$ years, &c.
will form, pretty nearly, an arithmetical progression, as well as the older ages, $N+n$ years, $N+n+1$ years, $N+n+2$ years, $N+n+3$ years, $N+n+4$ years, $N+n+5$ years, &c. themselves: only the series of those values will be a decreasing progression, whereas the series of the older ages is an increasing one. This, I say, seems probable, (though it is not absolutely evident,) and will be found to be sufficiently near the truth, when the number of the terms in these progressions is not greater than 11, to be the foundation of a very useful method of approximating to the values of these joint annuities.

CCCCXIII. The

CCCCXIII. These things being premised, let it be proposed to find ^{The said method} the value of an annuity of one pound a year for the joint continuance of ^{itself.} two lives of the ages of N years and $N + a$ years, in which a , the difference of the two ages, is not either 5 years, or 10 years, or 20 years, or 30 years, or 40 years, or 50 years, or 60 years, or 70 years, or 80 years, (which are the differences of the two ages in the foregoing tables,) but some intermediate number of years between some two of these differences, that are contiguous to each other.

Let the greatest difference (amongst the differences set down in the foregoing tables, which are either 5 years, or 10 years, or some multiple of 10 years,) than which the given difference a is greater, be $10 \times m$, or $10m$. Then, it is evidently the next greater difference, or the least difference (amongst the differences set down in the foregoing tables,) that is greater than the difference a , will be $10 \times m + 1$, or $10m + 10$. Therefore (by the first principle above-mentioned,) the value of the proposed annuity of one pound a year for the joint continuance of two lives of the ages of N years and $N + a$ years will be of an intermediate magnitude between the value of a like annuity for two joint lives of the ages of N years and $N + 10m$ years, and the value of a like annuity for two joint lives of the ages of N years and $N + 10m + 10$ years; both which values may be found in some of the foregoing ten tables. And thus we may obtain, by means of the first principle above-mentioned, two limits of the value of the proposed annuity, between which we may be sure it is of an intermediate magnitude.

And, by the second principle above-mentioned, we may make a nearer approximation to its true value by reasoning as follows.

The eleven following values of an annuity of one pound a year for two joint lives, (of which eleven values we can find the first and the last in the ten foregoing tables,) will form, pretty nearly, a decreasing arithmetical progression; to wit, the values of the said annuity for two joint lives of the ages of

N years and $N + 10m$ years,
 N years and $N + 10m + 1$ years,
 N years and $N + 10m + 2$ years,
 N years and $N + 10m + 3$ years,
 N years and $N + 10m + 4$ years,
 N years and $N + 10m + 5$ years,

First

N years

N years and $N + 10m + 6$ years,
 N years and $N + 10m + 7$ years,
 N years and $N + 10m + 8$ years,
 N years and $N + 10m + 9$ years,
 and N years and $N + 10m + 10$ years.

Subtract therefore the last of these values from the first, and divide the difference by 10; and the quotient thence arising will be the quantity which must be continually added to the last value, or subtracted from the first value, in order to form an arithmetical progression of terms between the first value and the last. Let these continual additions, or subtractions be made. And, amongst the terms thereby obtained, that which corresponds to the ages of N years and $N + a$ years (which latter age must be equal to one of the intermediate ages between $N + 10m$ years and $N + 10m + 10$ years,) will be a near value of the proposed annuity of one pound a year for two joint lives of the ages of N years and $N + a$ years. Q.E.I.

Examples of the foregoing method of discovering the values of the above mentioned intermediate, or omitted, annuities for two joint lives by Interpolation.

First example.

CCCCXIV. As an example of this method of interpolation, let the two lives for whose joint continuance an annuity of one pound a year to be granted, be of the ages of 70 years and 77 years.

Then it is evident, in the first place, that the value of this annuity is not contained in any of the foregoing ten tables; because 7 years is not the difference of the two ages in any of them.

But, in the second place, we may observe that the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 77 years must be less than the value of a like annuity for the joint continuance of two lives of the ages of 70 years and 75 years but greater than the value of the like annuity for the joint continuance of two lives of the ages of 70 years and 80 years.

In the third place we may observe that the values of the two last annuities, (between which the value of the proposed annuity lies,) to wit the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 75 years and of a like annuity for the joint continuance of two lives of the ages of 70 years and 80 years are both contained in the foregoing tables; the former of these values being contained in Table XXX, in which the difference of the ages is 5 years, and the latter of them being contained in Table XXXI, in which

the difference of the ages is 10 years. The former of these values appears in Table XXX to be = £3.560,612; and the latter of them appears in Table XXXI to be = £2.793,419. Therefore the value of the proposed annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 77 years is of an intermediate magnitude between £3.560,612 and £2.793,419.

In the fourth place we may reasonably suppose that the values of an annuity of one pound a year for the joint continuance of the six following pairs of lives, to wit,

- two lives of the ages of 70 years and 75 years,
- two lives of the ages of 70 years and 76 years,
- two lives of the ages of 70 years and 77 years,
- two lives of the ages of 70 years and 78 years,
- two lives of the ages of 70 years and 79 years,
- and two lives of the ages of 70 years and 80 years,

will form, pretty nearly, an arithmetical progression, or will decrease by nearly equal differences. Therefore, if we subtract the last of them, which is = £2.793,419, from the first, or £3.560,612, and divide the remainder, to wit, £0.767,193, by 5, the quotient, £0.153,438, will be the common difference by which these terms will decrease; and consequently, if this quotient be either continually added to the last term, £2.793,419, or continually subtracted from the first term, £3.560,612, we shall thereby obtain the values of the intermediate terms of the progression. If we proceed by addition, these intermediate terms will be as follows, to wit,

£	+	£	£
2.793,419	+	0.153,438,	or 2.946,857,
2.946,857	+	0.153,438,	or 3.100,295,
3.100,295	+	0.153,438,	or 3.253,732,
and 3.253,732	+	0.153,438,	or 3.407,171;

and, if we proceed by subtraction, they will be as follows, to wit,

£	—	£	£
3.560,612	—	0.153,438,	or 3.407,174,
3.407,174	—	0.153,438,	or 3.253,736,
3.253,736	—	0.153,438,	or 3.100,298,
and 3.100,298	—	0.154,438,	or 2.946,860.

T t t 2

Therefore

Therefore the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 76 years, 70 years and 77 years, 70 years and 78 years, and 70 years and 79 years,

Will be nearly equal to

£
3.407,174,
3.253,736,
3.100,298,
and 2.946,860;

of which values the second, to wit, £3.253,736, is that we were in search of. Q E I.

Note. This value, £3.253,736, (which is the third term of the arithmetical progression, consisting of six terms, whose first and last term are £3.560,612 and £2.793,419) might have been found separately, without finding the other intermediate terms of the said progression, by dividing the difference of the extreme terms, to wit, £0.767,193, by 5 so as to find the common difference of the terms, or £0.153,438, and then subtracting twice the said difference, or £0.306,876, from the first or greatest term, £3.560,612. For £3.560,612 — £0.306,876, = £3.253,736.

Second example.

CCCCXV. As another example of this method of Interpolation, let us suppose the ages of the two lives, for the joint continuance of which an annuity of one pound a year is to be granted, to be 59 years and 70 years.

Here we must observe in the first place, that the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years must be somewhat less than the value of a like annuity for two joint lives of the ages of 59 and 69 years; which appears by Table XXX to be = £5.611,286.

In the second place we must observe that the value of the said annuity of one pound a year for the joint continuance of two lives of the ages of 59 years and 70 years will be greater than the value of a like annuity for the joint continuance of two lives of the ages of 59 years and 79 years; which appears by Table XXXII to be = £3.462,599. Therefore the value of the said annuity of one pound a year for two joint lives of the ages of 59 years and 70 years is greater than £3.462,599, but less than £5.611,286. And it will evidently be much nearer to the greater of these values, or £5.611,286, than to the lesser value, £3.462,599.

In the third place, in order to make a nearer approach to the value of this annuity, we must suppose that the values of an annuity of one pound a year for the eleven following pairs of lives, to wit,

for two lives of the ages of 59 years and 69 years,
 two lives of the ages of 59 years and 70 years,
 two lives of the ages of 59 years and 71 years,
 two lives of the ages of 59 years and 72 years,
 two lives of the ages of 59 years and 73 years,
 two lives of the ages of 59 years and 74 years,
 two lives of the ages of 59 years and 75 years,
 two lives of the ages of 59 years and 76 years,
 two lives of the ages of 59 years and 77 years,
 two lives of the ages of 59 years and 78 years,
 and two lives of the ages of 59 years and 79 years,

will form, pretty nearly, an arithmetical progression, or decrease by nearly equal differences. And, if they do so decrease, the difference between the first and second of those values will be nearly the tenth part of the difference between the first and the last values. Now the first of these values has been shewn to be = £5.611,286, and the last of them has been shewn to be = £3.462,599: and the difference of these values is £2,148,687; of which the tenth part is £214,868. Therefore the second of the foregoing eleven values will be = £5.611,286 — £214,868, = £5.396,418. Therefore the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years, will be, nearly, equal to £5.396,418. Q.E.I.

CCCCXVI. There is another way of obtaining a near value of this annuity by means of the foregoing tables, besides this of Interpolation; in which it differs very little from it, and is founded on exactly the same principles. It is as follows.

It

It appears by Table XXX, (in which the difference of the ages of the two lives is 5 years,) that the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 59 years and 66 years is £6.700,900. And we have seen that the value of a like annuity for the joint continuance of two lives of the ages of 59 and 69 years is £5.611,286; which is less than £6.700,900 by the difference £1.089,614.

Now we may reasonably suppose that the values of an annuity of one pound a year for the joint continuance of the seven following pairs of lives, to wit,

two lives of the ages of 59 years and 64 years,
 two lives of the ages of 59 years and 65 years,
 two lives of the ages of 59 years and 66 years,
 two lives of the ages of 59 years and 67 years,
 two lives of the ages of 59 years and 68 years,
 two lives of the ages of 59 years and 69 years,
 and two lives of the ages of 59 years and 70 years,

will form, pretty nearly, an arithmetical progression, or will decrease in equal differences. And, if they do so decrease, the difference between the first of them (which is = £6.700,900,) and the last, (which is the value sought,) will be equal to six times their common difference; and consequently, if we can find the common difference of these values, the last of them may be derived from the first by subtracting from it six times the common difference. And it may also be derived from the sixth value (which we have seen to be = £5.611,286,) by subtracting from it the common difference itself. We must therefore inquire what is the common difference.

Now, since the first of these values (which are supposed to form an arithmetical progression,) is = £6.700,900, and the sixth of them is £5.611,286, it follows that the common difference of these values must be a sixth part of the difference of £6.700,900 and £5.611,286, that is a sixth part of £1.089,614, and therefore will be = £0.217,922. Therefore six times the said common difference will be = (six times £0.217,922 or) £1.307,532; which, being subtracted from £6.700,900, or the value of the foregoing seven values, leaves £5.393,368 for the last of the values, or the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years. Q.E.I.

Or, if we subtract the common difference, £0.217,922, itself, from £5.611,286, or the sixth of the foregoing values, the remaining quantity is £5.393,368.

£5393,364, will be the last of the said seven values, or the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years. **Q.E.I.**

Note. These values £5.393,368 and £5.393,364 are so nearly equal to the value found for this annuity in the last article, to wit, £5.396,418, that the difference is not worth attending to. But, if it were, we ought to consider the value last obtained, to wit, £5.393,368, as being nearer to the exact value of the proposed annuity than £5.396,418, because it is obtained by means of an arithmetical progression consisting of fewer terms than the other, in Art. ccccxv, by means of which the former value, £5.396,418, had been found.

CCCCXVII. As a third example of this method of Interpolation, Third example. at the ages of the two lives, upon the joint continuance of which an annuity of one pound a year is to depend, be 26 years and 53 years; the difference of which is 27 years.

Here we must observe, in the first place, that the value of this annuity is less than the value of a like annuity for two joint lives of the ages of 26 years and 46 years, and greater than the value of a like annuity for two joint lives of the ages of 26 years and 56 years; whence it follows, by Tables XXXII and XXXIII, that it is less than £12.646,97, and greater than £10.298,13.

In the second place we may reasonably suppose that the values of an annuity of one pound a year for the joint continuance of the following seven pairs of lives, to wit,

two lives of the ages of 26 years and 46 years,
 two lives of the ages of 26 years and 47 years,
 two lives of the ages of 26 years and 48 years,
 two lives of the ages of 26 years and 49 years,
 two lives of the ages of 26 years and 50 years,
 two lives of the ages of 26 years and 51 years,
 two lives of the ages of 26 years and 52 years,
 two lives of the ages of 26 years and 53 years,
 two lives of the ages of 26 years and 54 years,
 two lives of the ages of 26 years and 55 years,
 and two lives of the ages of 26 years and 56 years,

will

will form, pretty nearly, an arithmetical progression, or will decrease by nearly equal differences. And, if they do so decrease, their common difference will be nearly a tenth part of the difference of the extreme terms. Now the extreme terms are £12.646,97 and £10.298,13, the difference of which is £2.348,84; and the tenth part of this difference is £0.234,884. Therefore £0.234,884, is the common difference of the terms; and consequently £10.298,13 + $3 \times$ £0.234,884, (or £10.298,13 + £0.704,652, or £11.002,78 will be the value of the last term but three, or of the eighth term, or will be the value of the proposed annuity of one pound year for two joint lives of the ages of 26 years and 53 years. Q.E.I.

CCCCXVIII. These three examples will, I presume, be sufficient to illustrate this method of deriving the values of such annuities for two joint lives as *are not* contained in the foregoing ten tables, from the values of the annuities that *are* contained in those tables, by Interpolation. And therefore I shall not add any more examples of this method with the design.

Other examples of the foregoing method of Interpolation, which serve to shew to what degree of exactness the near values of these annuities for two joint lives, that are obtained by means of it, may be supposed co-incide with their true values.

CCCCXIX. But it will be necessary to give a few more instances of this method of Interpolation with *another* view, namely, in order to shew that the values of annuities for two joint lives which are obtained by means of it, are pretty nearly equal to their true values, and may consequently be used, on all common occasions, instead of the said true values, without any sensible inconvenience. Now this will best appear by computing, by means of this method of Interpolation, a few of the near values of an annuity of one pound a year for the joint continuance of two lives whose ages differ from each other by 5 years, and then comparing the said near values of these annuities, thereby obtained, with the exact values of the same annuities exhibited above in Table XXX, page 493.

First example.
CCCCXX. Let it therefore be required, in the first place, to find by this method of Interpolation, a near value of an annuity of one pound a year for the joint continuance of two lives of the ages of 10 years and 15 years; the exact value of which annuity appears in Table XXX to be £17.048,69.

Now it appears by Table XXX that the value of an annuity of one pound a year for two joint lives that are both of the same age of 10 years, is £17.397,65. And it appears from Table XXXI that the value of a like annuity of one pound a year for two joint lives of the ages of 10 years and 20 years is £16.634,79. Therefore, according to this method of interpolation, the value of an annuity of one pound a year for two joint lives of the ages of 10 years and 15 years will be nearly equal to an arithmetical mean proportional between £17.397,65 and £16.634,79, and consequently will be nearly equal to $\left(\frac{17.397,65}{2} + \frac{16.634,79}{2} \right)$, or $\frac{34.032,44}{2}$, or) £17.016,22. QEI.

The difference between this near value, £17.016,22, of the proposed annuity, and £17.048,09, its exact value, is £0.031,87, which is less than one 534th part of the said true value.

CCCCXXI. Let the two lives be of the ages of 20 years and 25 Second example.

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £15.701,10.

The near value of the same annuity will, according to the foregoing method of Interpolation, be an arithmetical mean proportional between £16.009,67, which appears by Table XXIX to be the value of a like annuity for two joint lives that are both of the age of 20 years, and £15.298,75, which appears by Table XXXI to be the value of a like annuity for two joint lives of the ages of 20 years and 30 years; and

consequently the said near value will be $\left(= \frac{16.009,67 + 15.298,75}{2} \right)$,

$$\left(\frac{31.308,42}{2} \right) = £15.654,21. \quad QEI.$$

The difference between this near value, £15.654,21, of the proposed annuity, and its more exact value, £15.701,10, is £0.046,89, which is less than the 334th part of the said exact value.

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CCCCXXII. Let

Third example.

CCCCXXII. 35 years.

Let the two lives be of the ages of 30 years and

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £14.357,56.

And the near value of the same annuity will, according to the foregoing method of Interpolation, be an arithmetical mean proportional between the value of a like annuity for two joint lives that are both of the same age of 30 years, which appears by Table XXIX to be £14.776,38, and the value of a like annuity for two joint lives of the ages of 30 years and 40 years, which appears by Table XXXI to be £13.709,61; and

consequently it will be = $\left(\frac{£14.776,38}{2} + \frac{£13.709,61}{2} \right) = \frac{£28.485,99}{2}$

£14.242,99. QEI.

The difference between this near value, £14.242,99, of the proposed annuity, and its more exact value, £14.357,56, is £0.114,57; which is less than the 125th part of the said exact value.

Fourth example.

CCCCXXIII. 45 years.

Let the two lives be of the ages of 40 years and

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £12.193,97.

The value of an annuity of one pound a year for two joint lives that are both of the age of 40 years, appears by Table XXIX to be £12.957,85, and the value of a like annuity for two joint lives of the ages of 40 years and 50 years appears by Table XXXI to be £11.229,92. Therefore the value of the like annuity for two joint lives of the ages of 40 years and 45 years is nearly equal to an arithmetical mean between £12.957,85 and

£11.229,92, and consequently is nearly equal to $\left(\frac{£12.957,85}{2} + \frac{£11.229,92}{2} \right)$

or $\frac{£24.187,77}{2}$, or) £12.093,88. QEI.

The difference between this near value, £12.093,88, of the proposed annuity, and its more exact value, £12.193,97, is £0.100,09; which is less than the 121st part of the said exact value.

CCCCXXIV. In like manner, if the two lives are of the ages of Fifth example.

50 years and 55 years, the exact value of an annuity of one pound during their joint continuance, given in Table XXX, is £9.273,595; and the near value of the same annuity, obtained by the foregoing method of Interpolation, will be an arithmetical mean between £10.025,94 and

£8.341,632, and consequently will be equal to $\frac{18.367,572}{2}$, or

£9.183,786. QEI.

The difference between this near value, £9.183,786, of the proposed annuity, and its more exact value, £9.273,595, is £0.089,809; which is less than the 103d part of the said exact value.

CCCCXXV. If the two lives are of the ages of 60 years and 65 years, the exact value of the annuity will be £6.382,877; and the near

value of it will be = $\left(\frac{7.282,755}{2} + \frac{5.336,698}{2} = \frac{12.619,453}{2} = \right)$

£6.309,726; which differs from the exact value, £6.382,877, by £0.073,151, which is less than the 87th part of the said exact value.

CCCCXXVI. If the two lives are of the ages of 70 years and 75 years, the exact value of the annuity will be £3.560,612; and the near

value of it will be = $\left(\frac{4.229,522}{2} + \frac{2.793,419}{2} = \frac{7.022,941}{2} = \right)$

£3.511,470; which differs from the exact value, £3.560,612, by £0.049,142; which is less than the 72d part of the said exact value.

CCCCXXVII. And, if the two lives are of the ages of 80 years Eighth example. and 85 years, the exact value of the annuity will be £1.595,707; and

the near value of it will be = $\left(\frac{2.057,238}{2} + \frac{0.909,073}{2} = \frac{2.966,311}{2} = \right)$

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£1.483,155;

less than 5 years, obtained by the same method of Interpolation, will differ from their true values by almost as small quantities, or by quantities not very different from the 72d part, or the 103d part, of the said true values. For in both cases the near values of the annuities sought are obtained by supposing that the principle laid down in Art. ccccxii is nearly true, or that the values of eleven annuities of one pound a year for the joint continuance of eleven pairs of lives of the following ages, to wit XXXXX

N years and N years,
 N years and $N + 1$ years,
 N years and $N + 2$ years,
 N years and $N + 3$ years,
 N years and $N + 4$ years,
 N years and $N + 5$ years,
 N years and $N + 6$ years,
 N years and $N + 7$ years,
 N years and $N + 8$ years,
 N years and $N + 9$ years,
 and N years and $N + 10$ years,

form, pretty nearly, an arithmetical progression, or decrease by nearly equal differences. If therefore we find upon trial (as we have done in the foregoing examples,) that the near value of an annuity for two joint lives of the ages of N and $N + 5$ years obtained in this manner, differs but by a small quantity from its true value, we may conclude with a high degree of probability, that the near values of the other intermediate annuities, (as, for example, of annuities for two joint lives of the ages of N years and $N + 3$ years or of the ages of N years and $N + 7$ years,) that are obtained in the same manner, will differ from their true values either by as small quantities, or by quantities that will be very little greater than the said difference between the near value and the true value of the said annuity for two joint lives of the ages of N years and $N + 5$ years, which is the middlemost annuity of the whole eleven.

CCCCXXX. And, if this conclusion be just, (which, I think, it is hardly possible to doubt of,) the near values of annuities for two joint lives, obtained by this method of Interpolation, may be considered as differing from their true values by only about the 72d part of the said true values, when the age of the older life is not greater than 75 years, and by only about the 103d part of the said true values when the age of the older life is not greater than 65 years, and by a much smaller part of the said true values when the age of the older life is only 30 or 20 years.

CCCCXXXI. Now

The values of annuities for two joint lives, obtained by the foregoing method of interpolation, are sufficiently exact for ordinary purposes.

CCCCXXXI. Now either the 103d part of the true value of an annuity, or the 72d part of it, is too small a difference to be of much importance in the bargains that are made for the purchase of life-annuities. And consequently this method of finding the values of annuities for two joint lives by Interpolation between the values of other contiguous annuities that have been already computed, may justly be considered a very useful and a sufficient supplement to the want of such complete tables of the values of these joint annuities, adapted to all the possible differences of ages in the two lives, as are mentioned in Art. ccccx.

End of the explanation and illustration of the foregoing method of finding the values of annuities for two joint lives by Interpolation.

CCCCXXXII. I shall now present the reader with another set of tables of the values of annuities for two joint lives, of the same kind as those above exhibited in Art. ccccviii, and which I have procured to be computed, (like the former,) under the inspection of the learned Mr. Morgan, from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $4\frac{1}{2}$ per cent. These tables are as follows.

T A B L E

LIFE-ANNUITIES.

T A B L E XXXIX.

containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons of the same age, when the interest of money is $4\frac{1}{2}$ per cent.-----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of life.	Years in the age of life.	Years in the age of life.	Years in the age of life.
33	34	34	65
32	35	35	66
31	36	36	67
30	37	37	68
29	38	38	69
28	39	39	70
27	40	40	71
26	41	41	72
25	42	42	73
24	43	43	74
23	44	44	75
22	45	45	76
21	46	46	77
20	47	47	78
19	48	48	79
18	49	49	80
17	50	50	81
16	51	51	82
15	52	52	83
14	53	53	84
13	54	54	85
12	55	55	86
11	56	56	87
10	57	57	88
9	58	58	89
8	59	59	90
7	60	60	91
6	61	61	92
5	62	62	93
4	63	63	94
3	64	64	

T A B L E XL.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 5 years; when the interest of money is $4\frac{1}{2}$ per cent.-----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	8	£ 14.219,29	32	37	£ 12.556,49	61	66	£ 5.756,635
4	9	14.525,27	33	38	12.407,81	62	67	5.466,931
5	10	14.707,65	34	39	12.252,17	63	68	5.191,171
6	11	14.810,70	35	40	12.089,10	64	69	4.917,419
7	12	14.839,89	36	41	11.918,07	65	70	4.646,949
8	13	14.840,94	37	42	11.738,52	66	71	4.377,326
9	14	14.828,32	38	43	11.531,14	67	72	4.127,800
10	15	14.782,57	39	44	11.312,59	68	73	3.885,426
11	16	14.700,60	40	45	11.082,06	69	74	3.653,183
12	17	14.598,23	41	46	10.838,70	70	75	3.435,590
13	18	14.491,45	42	47	10.600,65	71	76	3.203,078
14	19	14.379,98	43	48	10.349,18	72	77	2.988,987
15	20	14.263,55	44	49	10.102,06	73	78	2.788,448
16	21	14.160,62	45	50	9.840,830	74	79	2.585,274
17	22	14.071,49	46	51	9.582,860	75	80	2.408,860
18	23	13.979,20	47	52	9.345,367	76	81	2.231,987
19	24	13.883,53	48	53	9.091,623	77	82	2.075,853
20	25	13.784,30	49	54	8.846,134	78	83	1.917,410
21	26	13.699,50	50	55	8.601,572	79	84	1.730,370
22	27	13.612,10	51	56	8.359,590	80	85	1.561,668
23	28	13.521,96	52	57	8.120,294	81	86	1.408,369
24	29	13.428,91	53	58	7.885,844	82	87	1.291,505
25	30	13.332,79	54	59	7.638,860	83	88	1.129,847
26	31	13.233,41	55	60	7.393,970	84	89	0.953,637
27	32	13.130,57	56	61	7.135,517	85	90	0.781,711
28	33	13.024,06	57	62	6.861,983	86	91	0.621,495
29	34	12.913,66	58	63	6.605,443	87	92	0.489,284
30	35	12.799,11	59	64	6.333,493	88	93	0.347,978
31	35	12.680,15	60	65	6.045,498	89	94	0.000,000

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TJA B L J E B XLII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 10 years; when the interest of money is $4\frac{1}{2}$ per cent. --- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	13	£ 13.534,84	31	41	£ 12.132,94	59	69	£ 5.339,366
4	14	14.344,57	32	42	11.959,77	60	70	5.087,882
5	15	14.446,47	33	43	11.777,89	61	71	4.827,611
6	16	14.498,41	34	44	11.586,69	62	72	4.578,320
7	17	14.528,31	35	45	11.385,50	63	73	4.336,530
8	18	14.531,10	36	46	11.173,61	64	74	4.092,576
9	19	14.520,95	37	47	10.969,90	65	75	3.848,068
10	20	14.478,80	38	48	10.737,84	66	76	3.593,607
11	21	14.420,71	39	49	10.512,31	67	77	3.350,952
12	22	14.326,17	40	50	10.274,39	68	78	3.126,500
13	23	14.227,96	41	51	10.042,43	69	79	2.902,025
14	24	14.125,85	42	52	9.816,964	70	80	2.709,443
15	25	14.019,61	43	53	9.579,450	71	81	2.523,916
16	26	13.908,99	44	54	9.328,882	72	82	2.365,251
17	27	13.811,42	45	55	9.083,299	73	83	2.194,845
18	28	13.710,22	46	56	8.824,214	74	84	1.999,081
19	29	13.605,12	47	57	8.566,170	75	85	1.811,170
20	30	13.495,90	48	58	8.312,358	76	86	1.627,327
21	31	13.400,16	49	59	8.059,770	77	87	1.473,044
22	32	13.301,14	50	60	7.793,079	78	88	1.279,463
23	33	13.198,65	51	61	7.525,773	79	89	1.081,750
24	34	13.092,47	52	62	7.257,462	80	90	0.895,079
25	35	12.982,38	53	63	6.992,041	81	91	0.717,241
26	36	12.868,14	54	64	6.711,295	82	92	0.558,568
27	37	12.749,48	55	65	6.427,546	83	93	0.397,601
28	38	12.605,79	56	66	6.144,923	84	94	0.000,000
29	39	12.455,44	57	67	5.863,955			
30	40	12.297,99	58	68	5.597,977			

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T A B L E XLII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 20 years; when the interest of money is $4\frac{1}{2}$ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the younger life.	Years in the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the younger life.	Years in the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the younger life.	Years in the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	23	£ 13.566,18	27	47	£ 11.235,80	51	71	£ 5.249,784
4	24	13.764,63	28	48	11.025,14	52	72	5.006,611
5	25	13.869,97	29	49	10.823,13	53	73	4.761,978
6	26	13.928,96	30	50	10.610,56	54	74	4.517,666
7	27	13.950,52	31	51	10.406,55	55	75	4.286,359
8	28	13.946,14	32	52	10.212,00	56	76	4.037,427
9	29	13.929,47	33	53	10.008,01	57	77	3.794,414
10	30	13.882,15	34	54	9.793,822	58	78	3.572,788
11	31	13.801,26	35	55	9.588,700	59	79	3.343,173
12	32	13.684,05	36	56	9.373,704	60	80	3.139,598
13	33	13.561,85	37	57	9.148,020	61	81	2.943,841
14	34	13.434,35	38	58	8.916,202	62	82	2.764,131
15	35	13.301,20	39	59	8.672,806	63	83	2.572,536
16	36	13.162,05	40	60	8.416,828	64	84	2.345,809
17	37	13.033,30	41	61	8.147,138	65	85	2.119,938
18	38	12.878,23	42	62	7.862,472	66	86	1.908,778
19	39	12.715,57	43	63	7.581,639	67	87	1.728,597
20	40	12.544,83	44	64	7.285,200	68	88	1.497,798
21	41	12.382,05	45	65	6.971,573	69	89	1.269,895
22	42	12.211,23	46	66	6.659,066	70	90	1.048,546
23	43	12.031,83	47	67	6.360,345	71	91	0.834,287
24	44	11.843,25	48	68	6.065,302	72	92	0.638,300
25	45	11.644,84	49	69	5.786,990	73	93	0.440,344
26	46	11.435,90	50	70	5.517,470	74	94	0.000,000

T A B L E
XLIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 30 years; when the interest of money is $4\frac{1}{2}$ per cent.-----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	33	£ 12.892,77	24	54	£ 9.910,650	45	75	£ 4.518,077
4	34	13.047,91	25	55	9.702,397	46	76	4.247,671
5	35	13.112,32	26	56	9.484,074	47	77	3.991,243
6	36	13.130,47	27	57	9.254,869	48	78	3.748,011
7	37	13.110,84	28	58	9.034,348	49	79	3.502,708
8	38	13.043,28	29	59	8.803,302	50	80	3.284,035
9	39	12.959,63	30	60	8.560,832	51	81	3.079,666
10	40	12.842,64	31	61	8.305,940	52	82	2.899,153
11	41	12.689,53	32	62	8.037,502	53	83	2.699,676
12	42	12.497,79	33	63	7.774,945	54	84	2.464,370
13	43	12.296,06	34	64	7.498,687	55	85	2.237,648
14	44	12.083,66	35	65	7.207,393	56	86	2.022,653
15	45	11.859,89	36	66	6.920,335	57	87	1.835,846
16	46	11.623,98	37	67	6.638,708	58	88	1.596,108
17	47	11.410,32	38	68	6.353,241	59	89	1.356,043
18	48	11.185,18	39	69	6.076,191	60	90	1.119,059
19	49	10.967,99	40	70	5.810,588	61	91	0.890,743
20	50	10.739,19	41	71	5.538,183	62	92	0.677,405
21	51	10.532,33	42	72	5.282,170	63	93	0.462,633
22	52	10.334,98	43	73	5.025,291	64	94	0.000,000
23	53	10.128,01	44	74	4.769,599			

T A B L E XLIV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 40 years; when the interest of money is $4\frac{1}{2}$ per cent.—
Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	43	£ 11.664,55	21	61	£ 8.361,570	39	79	£ 3.606,689
4	44	11.706,30	22	62	8.087,980	40	80	3.390,203
5	45	11.657,84	23	63	7.820,090	41	81	3.183,644
6	46	11.559,58	24	64	7.538,184	42	82	2.996,187
7	47	11.439,70	25	65	7.240,908	43	83	2.789,655
8	48	11.288,63	26	66	6.947,583	44	84	2.547,150
9	49	11.138,05	27	67	6.659,348	45	85	2.308,580
10	50	10.953,87	28	68	6.377,813	46	86	2.082,011
11	51	10.754,12	29	69	6.105,244	47	87	1.888,493
12	52	10.538,20	30	70	5.844,810	48	88	1.636,142
13	53	10.311,44	31	71	5.578,317	49	89	1.386,792
14	54	10.073,02	32	72	5.329,295	50	90	1.140,577
15	55	9.842,622	33	73	5.080,618	51	91	0.905,794
16	56	9.600,677	34	74	4.834,663	52	92	0.689,009
17	57	9.358,650	35	75	4.594,865	53	93	0.468,882
18	58	9.124,664	36	76	4.338,334	54	94	0.000,000
19	59	8.879,210	37	77	4.090,833			
20	60	8.621,334	38	78	3.852,445			

T A B L E XLV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 50 years; when the interest of money is $4\frac{1}{2}$ per cent.---
Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint conti- nuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint conti- nuance of both lives.
3	53	£ 9.769,927	24	74	£ 4.848,025
4	54	9.740,534	25	75	4.603,719
5	55	9.652,684	26	76	4.342,180
6	56	9.522,340	27	77	4.089,075
7	57	9.355,740	28	78	3.851,485
8	58	9.181,316	29	79	3.606,634
9	59	8.989,406	30	80	3.391,199
10	60	8.767,436	31	81	3.185,907
11	61	8.513,132	32	82	3.000,036
12	62	8.224,342	33	83	2.795,513
13	63	7.940,830	34	84	2.555,537
14	64	7.642,510	35	85	2.320,372
15	65	7.327,995	36	86	2.098,610
16	66	7.016,758	37	87	1.907,556
17	67	6.718,992	38	88	1.655,071
18	68	6.427,600	39	89	1.403,200
19	69	6.144,730	40	90	1.155,588
20	70	5.873,408	41	91	0.918,076
21	71	5.603,355	42	92	0.697,208
22	72	5.350,681	43	93	0.473,203
23	73	5.098,132	44	94	0.000.000

T A B L E XLVI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 60 years; when the interest of money is $4\frac{1}{2}$ per cent.—
Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	63	£ 7.519,867	20	80	£ 3.404,081
4	64	7.378,607	21	81	3.197,263
5	65	7.169,216	22	82	3.009,860
6	66	6.938,290	23	83	2.803,637
7	67	6.693,300	24	84	2.561,759
8	68	6.442,915	25	85	2.324,537
9	69	6.196,954	26	86	2.100,434
10	70	5.950,820	27	87	1.906,501
11	71	5.685,410	28	88	1.654,218
12	72	5.423,923	29	89	1.402,532
13	73	5.162,329	30	90	1.155,084
14	74	4.902,882	31	91	0.917,714
15	75	4.648,840	32	92	0.696,969
16	76	4.376,827	33	93	0.473,078
17	77	4.118,650	34	94	0.000,000
18	78	3.875,876			
19	79	3.625,462			

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T A B L E XLVII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 70 years; when the interest of money is 4 per cent.---
Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	73	£ 4.898,531	14	84	£ 2.582,455
4	74	4.734,189	15	85	2.340,580
5	75	4.541,851	16	86	2.111,582
6	76	4.316,868	17	87	1.915,651
7	77	4.088,644	18	88	1.661,118
8	78	3.868,953	19	89	1.407,168
9	79	3.639,892	20	90	1.157,288
10	80	3.433,726	21	91	0.919,296
11	81	3.230,664	22	92	0.698,015
12	82	3.039,329	23	93	0.473,624
13	83	2.828,899	24	94	0.000,000

T A B L E XLVIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 80 years; when the interest of money is $4\frac{1}{2}$ per cent.---
Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
3	83	£ 2.702,580	9	89	£ 1.405,610
4	84	2.503,721	10	90	1.160,806
5	85	2.290,611	11	91	0.923,697
6	86	2.082,129	12	92	0.700,914
7	87	1.897,819	13	93	0.475,131
8	88	1.651,923	14	94	0.000,000

CCCCXXXIII. According

End

CCCCXXXIII. According to Monsieur de Parcieux's table of the probabilities of the duration of human life (which begins with the age of 3 years, and ends with the age of 94 years, and supposes the utmost possible extent of human life to be somewhat less than 95 years,) it is evident that only two pairs of lives can be found whose ages will differ from each other by 90 years, to wit, two lives of the ages of 3 years and 93 years, and two lives of the ages of 4 years and 94 years. And an annuity for the joint continuance of this latter pair of lives cannot, according to this table of probabilities, be of any value, because it is supposed to be certain, according to this table, that the older life, to wit, the life of 94 years, will be extinct before the end of the year, or before the payment of the annuity will become due. Therefore the only two lives, whose ages differ from each other by 90 years, for the joint continuance of which an annuity can be of any value, are two lives of the ages of 3 years and 93 years. And the value of an annuity of one pound a year for the joint continuance of two lives of these ages, when the interest of money is $4\frac{1}{2}$ per cent. is £50.464,115.

The value of an annuity of one pound a year for two joint lives of the ages of 3 years and 93 years.

CCCCXXXIV. When the interest of money is $4\frac{1}{2}$ per cent. and the difference of the ages of the two lives is not either 0, or 5 years, or 10 years, or 20 years, or 30 years, or 40, 50, 60, 70, 80, or 90, years, but some intermediate number of years lying between some two of these differences that are contiguous to each other, we must have recourse to the method of interpolation above explained in Art. cccxxxiii, in order to obtain a near value of an annuity of one pound a year for their joint continuance; as we did above when the interest of money was supposed to be $3\frac{1}{2}$ per cent. For this method of interpolation will be equally applicable to the discovery of these near values of annuities for two joint lives at one rate of the interest of money as at another.

When the value of an annuity for two joint lives, (upon a supposition that the interest of money is $4\frac{1}{2}$ per cent.) is not contained in any of the foregoing tables, recourse must be had to the method of interpolation above-described.

CCCCXXXV. We may therefore, by the help of the two foregoing sets of tables, together with the method of interpolation above-explained, discover either the true values, or tolerably near values, of all annuities for the joint continuance of two lives of any ages whatsoever, when the interest of money is either $3\frac{1}{2}$ per cent. or $4\frac{1}{2}$ per cent.

End of the directions for finding the values of annuities for two joint lives when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.

Of the values of annuities for two joint lives, when the interest of money is 3 per cent. 4 per cent. and 5 per cent.

These values may also be found to a tolerable degree of exactness by means of the two foregoing sets of tables.

The conjectural supposition by means of which the said values may be deduced from the values of the same annuities when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. which are given in the two foregoing sets of tables.

CCCCXXVI. And from the values of these annuities for two joint lives when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. we may find tolerably near values of the same annuities when the interest of money is 3 per cent. 4 per cent. and 5 per cent. by supposing that the values of an annuity for the joint continuance of any two given lives at the five following different rates of interest, to wit, 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent. (which differ from each other by only $\frac{1}{2}$ per cent.) form, pretty nearly, an arithmetical progression, or decrease by nearly equal differences. For, if this supposition be true, the third value of the annuity, or that which it has when the interest of money is 4 per cent. will be nearly equal to an arithmetical mean proportional between the second value of it, or that which it has when the interest of money is $3\frac{1}{2}$ per cent. and the fourth value of it, or that which it has when the interest of money is $4\frac{1}{2}$ per cent. which two extreme values may be found in the two foregoing sets of tables. We, therefore, need only subtract the fourth value of the annuity, or that which it has when the interest of money is $4\frac{1}{2}$ per cent. from the second value of it, or that which it has when the interest of money is $3\frac{1}{2}$ per cent. and divide the remainder by 2; and, if the quotient, thence arising, be subtracted from the second value of the annuity, or that which it has when the interest of money is $3\frac{1}{2}$ per cent. the remainder will be nearly equal to the third value of it, or that which it has when the interest of money is 4 per cent. And, in like manner, if we subtract the said quotient from the fourth value of the annuity, or that which it has when the interest of money is $4\frac{1}{2}$ per cent. the remainder will be the fifth value of the annuity, or that which it has when the interest of money is 5 per cent. and, if we add the said quotient to the second value of the annuity, or that which it has when the interest of money is $3\frac{1}{2}$ per cent. the sum will be equal to the first value of it, or that which it has when the interest of money is 3 per cent.

An example of the method of deducing the former values from the latter.

Thus, for example, if it were required to assign the values of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years. when the interest of money is 3 per cent. 4 per cent. and 5 per cent. by means of the two foregoing sets of tables, which exhibit the values of this annuity only when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. we must proceed in the following manner.

The value of an annuity of one pound a year for the joint continuance of two lives of the ages of 25 years and 35 years, when the interest of money is $3\frac{1}{2}$ per cent. appears by Table XXXI, page 494, to be = £14.601,50;

£14.601,50; and the value of the same annuity, when the interest of money is $4\frac{1}{2}$ per cent. appears by Table XLI to be = £12.982,38. The difference between £14.601,50 and £12.982,38 is £1.619,12; and half this difference is £0.809,56. Therefore the value of this annuity when the interest of money is 4 per cent. will be nearly equal to £14.601,50 — £0.809,56, or £13.791,94; and the value of it when the interest of money is 5 per cent. will be nearly equal to £12.982,38 — £0.809,56, or £12.172,82; and the value of it when the interest of money is 3 per cent. will be nearly equal to £14.601,50 + £0.809,56, or £15.411,06. Therefore the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 25 years and 35 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. are nearly equal to £15.411,06, £13.791,94, and £12.172,82. Q.E.I.

CCCCXXVII. In the foregoing article we have shewn how to derive a near value of any given annuity upon a supposition that the interest of money is 3 per cent. 4 per cent. and 5 per cent. from the two values of the same annuity when the interest of money is $3\frac{1}{4}$ per cent. and $4\frac{1}{4}$ per cent. which two values are either given exactly in some of the tables of the two foregoing sets of tables, or may be derived, by the aforesaid method of Interpolation, from the values which are there set down. And the method of doing this (and which we have described in the last article,) is very similar to the aforesaid method of Interpolation (described above in Art. ccccxiii,) and may itself likewise be called with propriety *a method of Interpolation*, as it proceeds (like the former method,) on a supposition that the unknown value of the proposed annuity, when the interest of money is 4 per cent. is nearly equal to an arithmetical mean between the two known values of the same annuity when the interest of money is $3\frac{1}{4}$ per cent. and $4\frac{1}{4}$ per cent. or consists in the *interpolation*, or *interposition*, of an arithmetical mean between those two known values. Indeed, as to the manner of determining the other two unknown values of the proposed annuity, or those which it has when the interest of money is 3 per cent. and 5 per cent. the word *Interpolation* is not quite so proper for it; because they are not found by *interpolating*, or *interposing*, any new terms between the two known values of the said annuity (which it has when the interest of money is $3\frac{1}{4}$ per cent. and $4\frac{1}{4}$ per cent.) but by adding new terms, at both ends, to the arithmetical progression, (consisting of three terms) whereof the two known values of the annuity (which it has when the interest of money is $3\frac{1}{4}$ per cent. and $4\frac{1}{4}$ per cent.) are the first and last terms, and the arithmetical mean between those two known values is the middle term. With respect, therefore, to these values of the proposed annuity, which it has when the interest of money is 3 per cent. and 5 per cent. and which are thus deter-

Of the similarity of this method of deducing the said former values from the latter, to the method of *interpolation* described above in Art. 413.

This method may be called, with propriety, the method of interpolation and continuation.

mined to a moderate degree of exactness by continuing the aforesaid arithmetical progression (consisting of the aforesaid three terms,) at both its ends, we ought rather to call this method of proceeding *the method of Continuation* than *the method of Interpolation*. And therefore, perhaps, it may be proper to call this whole method of finding all the said three unknown values of the proposed annuity, (which it has when the interest of money is 3 per cent. 4 per cent. and 5 per cent.) taken together, *the method of Interpolation and Continuation*. But, by whatever name we call them, the two parts of this method are both founded on the same principle, or supposition, to wit, "that, when three rates of the interest of money are taken that are successively greater the one than the other by only one half per cent. (as either the rates of 3 per cent. $3\frac{1}{2}$ per cent. and 4 per cent. or of $3\frac{1}{2}$ per cent. 4 per cent. and $4\frac{1}{2}$ per cent. or of 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent.) the values of an annuity for two joint lives of any given ages at these three rates of interest, will form, pretty nearly, an arithmetical progression, or will decrease by nearly equal differences;" which supposition (though I do not know any method of demonstrating it,) has so great an appearance of probability that, I imagine, it is hardly possible to doubt the truth of it.

Of the degree of exactness to which the near values of annuities for two joint lives, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. that are deduced, by the foregoing method of Interpolation and Continuation, from the values of the same annuities when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. may, upon reasonable grounds, be supposed to be true.

CCCCXXXVIII. But, though it can hardly be doubted that the near values of an annuity for two joint lives of any given ages when the interest of money is 3 per cent. and 4 per cent. and 5 per cent. that are obtained by the foregoing method of *Interpolation and Continuation* from the values of the same annuity when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. are not very different from their true values, yet it would be desirable (if it can be done,) to discover to what degree of exactness we may consider these near values of the proposed annuity as agreeing, or coinciding, with its correspondent true values.

CCCCXXXIX. Now, to do this in the most satisfactory manner, it would be necessary to compute the exact values of an annuity of one pound a year for seven or eight pairs of joint lives of different ages when the

the interest of money is 3 per cent. and 4 per cent. and 5 per cent. and then to compare the said exact values with the near values of the same annuity, at the same rates of the interest of money, that are derived, by means of the foregoing method of Interpolation and Continuation, from the values of the same annuity when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. which are given in the two foregoing sets of tables: upon which comparison I have no doubt that the differences between the said near values and exact values would be found to be very inconsiderable. But this comparison I am not at present able to make, not being possessed of the exact values of any annuities for two joint lives at the interest of 3 per cent. 4 per cent. or 5 per cent. computed from Monsieur de Parceux's table of the probabilities of the duration of human life. And I do not think it an object of sufficient importance to make it worth while, on this account only, to procure any of these exact values to be computed.

A probable method of discovering the said degree of exactness,

CCCCXL. But we may form a very probable conjecture concerning the degree of exactness to which the aforesaid near values of annuities for two joint lives at 3 per cent. 4 per cent. and 5 per cent. (which are obtained, by the foregoing method of *interpolation and continuation*, from the values of the same annuities at the interests of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) agree, or co-incide, with their exact values, by supposing what I imagine, it is hardly possible to doubt of, to wit, that there is such an analogy between the values of annuities for two joint lives and the values of annuities for single lives of the same ages with the older of the two joint lives, that, if the near values of annuities for single lives at the interests of 3 per cent. 4 per cent. and 5 per cent. (that are obtained from the exact values of the same annuities at the interests of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of *interpolation and continuation*;) are found to agree, or co-incide, with their true values to a certain degree of exactness, so as (for example,) to differ from the said true values by only a 100th, or an 1000th, part of the said true values, it may be concluded that the near values of annuities for two joint lives (of which the older is of the same age with the single life with which it is compared,) at the same interests of 3 per cent. 4 per cent. and 5 per cent. (that are obtained, by the said method of interpolation and continuation, from the exact values of the same annuities at the interests of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) will likewise agree, or co-incide, with their true values to the same, or very nearly the same, degree of exactness as the near values and the true values of the corresponding annuities for single lives were found to do, or so as to differ from the said true values by only about a 200th, or a 1000th, part of the said true values. Thus, for example, if we should derive the value of an annuity of one pound a year for a single life of the age of 35 years, when the interest of money is 4 per cent. from the two values of the

the same annuity for the same life when the interest of money is $3\frac{1}{4}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of interpolation, or by taking an arithmetical mean between those two values; and, upon comparing this near value of the said annuity with its exact value, (which is contained above in Table XVI, page 225,) we should find the difference between them to be only 2 200th, or 2 100th, part of the said exact value; we may conclude that the near value of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years, when the interest of money is 4 per cent. that is derived from the exact values of the same annuity for the same two joint lives, when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of interpolation or by taking an arithmetical mean between the said two exact values, will also differ from the true value of the same annuity by only about a 200th or a 100th, part of the said true value. If this conclusion be allowed to be just, (and it certainly appears so highly probable, that it seems difficult to doubt of the truth of it,) we may then try the degree of exactness to which this method of *interpolation and continuation* exhibits the values of annuities for two joint lives when the interest of money is 3 per cent. 4 per cent. and 5 per cent. by applying it to the determination of the values of the like annuities for single lives (of the same ages as the old lives in the corresponding joint lives,) when the interest of money is 3 per cent. 4 per cent. and 5 per cent. and comparing the near values of the said annuities for single lives, thereby obtained, with the exact values of the same annuities for single lives at the same rates of interest, which are exhibited above in Tables XIV, XVI, and XVIII. Of this I shall not proceed to give a few instances.

An example of the said method.

CCCCXLI. In the first place therefore, let us suppose that the two lives for whose joint continuance an annuity of one pound a year is to be granted, are of the ages of 25 years and 35 years.

We have seen in Art. ccccxvii, that the two exact values of this annuity, when the interest of money is $3\frac{1}{4}$ per cent. and $4\frac{1}{2}$ per cent. are £14.601,50 and £12.982,38, and that the near values of the same annuity, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. that are derived from the said two exact values by the foregoing method of interpolation and continuation, are £15.411,06, £13.791,94 and £12.172,82. We are therefore to endeavour to determine, (by a reasonable conjecture founded on the analogy just now supposed to take place between the values of annuities for joint lives and the values of annuities for single lives,) *to what degree of exactness* these three near values £15.411,06, £13.791,94, and £12.172,82, of the aforesaid annuity when the

the interest of money is 3 per cent. 4 per cent. and 5 per cent. agree, or coincide, with the true values of the same annuity at the same rates of interest. Now, in order to lay a ground for a conjecture of this kind, we may proceed as follows.

The value of an annuity of one pound a year for a single life of the age of 35 years, when the interest of money is 3 per cent. appears by Table XIV, page 223, to be £18.464,13; and the values of the same annuity, when the interest of money is $3\frac{1}{4}$ per cent. 4 per cent. $4\frac{1}{4}$ per cent. and 5 per cent. appear by Tables XV, XVI, XVII, and XVIII, to be £17.206,612, £16.084,014, £15.078,39, and £14.174,54. These are the true, or exact, values of this annuity at these five different rates of interest.

Now let the first, third, and fifth values of this annuity be derived from the second and fourth values of it, to wit, £17.206,612 and £15.078,39, by the aforesaid method of *interpolation and continuation*. This may be done in the manner following. The excels of the second term, £17.206,612, of this progression above the fourth term, £15.078,39, is £2.128,222; and half this difference is £1.064,111. Therefore the sum of the said five terms (which are supposed, for the present purpose, to constitute a decreasing arithmetical progression,) will be = £17.206,612 + £1.064,111, or £18.270,723; and the third term will be = £17.206,612 — £1.064,111, or £16.142,501; and the fifth term will be £15.078,39 — £1.064,111, or £14.014,279; and consequently the values of the proposed annuity of one pound a year for the life of a person of the age of 35 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. will be nearly equal to £18.270,723, £16.142,501, and £14.014,279. Now, if we compare these three near values of this annuity, at these three different rates of interest, with the three exact values of the same annuity at the same rates of interest, (which we have seen to be £18.464,13, £16.084,014, and £14.174,54,) we shall find, that the first near value, to wit, £18.270,723, is less than the corresponding true value, £18.464,13, by the difference £0.193,407, which is less than the 95th part of the said true value; and that the second near value, to wit, £16.142,501, is greater than the corresponding true value, £16.084,014, by the difference £0.058,487, which is less than the 275th part of the said true value; and that the third near value, £14.014,279, is less than the corresponding true value, £14.174,54, by the difference, £0.160,261, which is less than the 88th part of the said true value. Thus it appears that the near values of an annuity of one pound a year for a single life of the age of 35 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by this method of *interpolation and continuation*, differ from the corresponding true values of it

it by less than the 95th, the 276th, and the 88th, parts of the said true values respectively. And from hence we may conclude (by means of the analogy above-mentioned between the values of annuities for single lives and the values of annuities for joint lives,) that the near values of an annuity of one pound a year for two joint lives of the ages of 25 years and 3 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent which were obtained in Art. cccxxxv by the same method of *interpolation and continuation*, to wit, £15.411,06, £13.791,94, and £12.172,88 will likewise differ from the corresponding true values of the same annuity at those three rates of interest, by only about the 95th part, the 276th part, and the 88th part, of the said true values respectively.

Another example
of the said method.

CCCCXLII. In the second place, Let us suppose the two lives to be of the ages of 25 years and 45 years.

The value of an annuity of one pound a year for two joint lives of these ages, when the interest of money is $3\frac{1}{2}$ per cent. appears by Table XXXII, page 495, to be £12.902,34; and the value of the same annuity for the same joint lives, when the interest of money is $4\frac{1}{2}$ per cent. appears by Table XLII to be £11.644,84. The difference of these values is £1.257,50; and half this difference is £0.628,75. Therefore according to the foregoing method of *interpolation and continuation*, the value of the same annuity for the same joint lives, when the interest of money is 3 per cent. will be nearly equal to (£12.902,34 + £0.628,75, or £13.531,09; and the value of it, when the interest of money is 4 per cent. will be nearly equal to (£12.902,34 — £0.628,75, or) £12.273,59 and the value of it, when the interest of money is 5 per cent. will be nearly equal to (£11.644,84 — £0.628,75, or) £11.016,09.

Now, in order to form a probable conjecture concerning the degree of exactness to which these near values of the aforesaid annuity for two joint lives agree with its corresponding true values, let us derive the values of like annuity of one pound a year for a single life of the age of 45 years when the interest of money is 3 per cent. 4 per cent. and 5 per cent. from the values of it when the interest of money is $3\frac{1}{2}$ per cent. and 4 per cent. by the same method of *interpolation and continuation*; and then compare the near values, thereby obtained, with the correspondent true values of the same annuity at the same rates of interest, (to wit, 3 per cent. 4 per cent. and 5 per cent.) as exhibited above in Tables XIV, XVI, and XVIII. This may be done as follows.

The value of an annuity of one pound a year for a life of the age of 45 years, when the interest of money is $3\frac{1}{4}$ per cent. appears by Table XV to be £14.716,120; and the value of the same annuity for the same life, when the interest of money is $4\frac{1}{4}$ per cent. appears by Table XVII to be £13.163,92. The difference of these values is £1.552,200; and half the said difference is £0.776,100. Therefore the value of an annuity of one pound a year for a life of the age of 45 years, when the interest of money is $3\frac{1}{4}$ per cent. will be nearly equal to (£14.716,120 + £0.776,100, or) £15.492,220; and the value of the same annuity for the same life, when the interest of money is 4 per cent. will be nearly equal to (£14.716,120 - £0.776,100, or) £13.940,020; and the value of the same annuity for the same life, when the interest of money is 5 per cent. will be nearly equal to (£13.163,92 - £0.776,100, or) £12.387,820.

Now the true values of this annuity, when the interest of money is 1 per cent. 4 per cent. and 5 per cent. appear by Tables XIV, XVI, and XVIII, to be £15.608,74, £13.904,190, and £12.487,42. Therefore the list of the foregoing near values of the said annuity, to wit, £15.492,220, falls short of the corresponding true value of it, £15.608,74, by the difference, £0.116,520, which is less than the 133d part of the said true value; and the second near value of the said annuity, to wit, £13.940,020, exceeds the corresponding true value of it, to wit, £13.904,190, by the difference, £0.035,830, which is less than the 388th part of the said true value; and the third near value of the said annuity, to wit, £12.387,820, is less than the corresponding true value of it, to wit, £12.487,42, by the difference, £0.099,600, which is less than the 125th part of the said true value. We may therefore conclude (by means of the analogy above-mentioned between the values of annuities for single lives and the values of annuities for joint lives,) that the three near values of an annuity of one pound a year for two joint lives of the ages of 25 years and 45 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained above by the said method of *interpolation and continuation*, to wit, £13.531,09, £12.273,59, and £11.016,09, will differ from the corresponding true values of the same annuity by only about the 133d part, the 388th part, and the 125th part, of the said true values respectively.

CCCCXLIII. As a third example of this method of determining by analogy the degree of exactness of the values of annuities for two joint lives, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by the foregoing method of *interpolation and continuation*, let us suppose the two lives to be of the ages of 20 years and 70 years.

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The value of an annuity of one pound a year for two joint lives of these ages, when the interest of money is $3\frac{1}{2}$ per cent. appears by Table XXXV, page 495, to be £6.197,247; and the value of the same annuity for the same joint lives, when the interest of money is $4\frac{1}{2}$ per cent. appears by Table XLV to be £5.873,408. The difference of these values is £0.323,839; and half this difference is £0.161,919. Therefore, according to the foregoing method of *interpolation* and *continuation*, the values of the same annuity for the same joint lives, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. will be nearly equal to (£6.197,247 - £0.161,919, or) £6.359,166, (£6.197,247 - £0.161,919, or) £6.035,321 and (£5.873,408 - £0.161,919, or) £5.711,489, respectively.

Now, in order to form a conjecture concerning the degree of exactness of these near values of an annuity of one pound a year for these two joint lives of the ages of 20 years and 70 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. we must derive the values of like annuity for a single life of the age of 70 years, at the same rates of the interest of money, from the values of it, when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the same method of *interpolation* and *continuation*, and then compare the near values, thereby obtained, with the corresponding true values of the same annuity for a single life of the same age of 70 years, at the same rates of interest, as they are exhibited above in Tables XIV, XVI, and XVII. This may be done as follows.

The value of an annuity of one pound a year for a single life of the age of 70 years, when the interest of money is $3\frac{1}{2}$ per cent. appears by Table XV to be £6.575,357; and the value of the same annuity for the same life, when the interest of money is $4\frac{1}{2}$ per cent. appears by Table XVII to be £6.220,54. The difference of these values is £0.354,817 and half the said difference is £0.177,408. The sum of £6.575,357 and £0.177,408 is £6.752,765; and the difference of £6.575,357 and £0.177,408 is £6.397,949; and the difference of £6.220,54 and £0.177,408 is £6.043,132. Therefore £6.752,765 will be a near value of an annuity of one pound a year for a single life of the age of 70 years when the interest of money is 3 per cent. and £6.397,949 will be a near value of the same annuity for the same life, when the interest of money is 4 per cent. and £6.043,132 will be a near value of the same annuity for the same life, when the interest of money is 5 per cent.

Now the true values of an annuity of one pound a year for a single life of the age of 70 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. appear by Tables XIV, XVI, and XVII, to be £6.765,920, £6.393,749, and £6.055,280; which differ from the foregoing near values of the same annuity, obtained by the method of *interpolation*

interpolation and continuation, to wit, £6.752,765, £6.397,949, and £6.043,132, by only £0.013,155, £0.004,200, and £0.012,148, which are less than the 514th part, the 1522d part, and the 498th part, of the said true values £6.765,920, £6.393,749, and £6.055,280, respectively. We may therefore conclude (by means of the analogy above-mentioned between the values of annuities for single lives and the values of annuities for joint lives,) that the three near values of an annuity of one pound a year for two joint lives of the ages of 20 years and 70 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained above by the said method of *interpolation and continuation*, to wit, £6.339,166, £6.035,328, and £5.711,489, will differ from the corresponding true values of the same annuity by only about the 514th, the 1522d, and 498th, part of the said true values.

CCCCXLIV. It appears from the three foregoing examples, that the near values of an annuity of one pound a year for a single life of either 15 years, or 45 years, or 70 years, of age, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by the foregoing method of *interpolation and continuation*, differ but little from the corresponding true values of the same annuity; and further, that the differences of the near values of the said annuity, when the interest of money is 4 per cent. (which are obtained by *interpolation*, or by *interpolating* arithmetical means between the values of the same annuity, when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) from its corresponding true values, are much smaller than the differences of the near values of the said annuity from its true values, when the interest of money is 3 per cent. and 5 per cent. in which cases the said near values are not obtained by *interpolation*, but by *continuation*. Therefore, if the foregoing analogy between the values of annuities for single lives and the values of annuities for joint lives be allowed to subsist, it will follow, that the near values of annuities for two joint lives, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained, by the foregoing method of *interpolation and continuation*, from the values of the like annuities when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. will likewise differ but little from the true values of the same annuities; and that the differences of the near values of such annuities from their true values, will be still less, when the interest of money is 4 per cent. than when it is 3 per cent, or 5 per cent. But in all the three rates of interest the said near values of these annuities will be *near enough* to their true values for most of the common purposes of business.

Conclusion drawn from the three foregoing examples, concerning the degree of exactness of the near values of annuities for two joint lives obtained by means of the foregoing method of *interpolation and continuation*.

End of the explanation and illustration of the foregoing method of *Interpolation and Continuation*.

Of the values of annuities for two joint lives, when the interest of money is either lower than 3 per cent. or higher than 5 per cent.

CCCCXLV. We have seen in the foregoing articles how, by the help of the two sets of tables of the values of annuities for two joint lives, given above in Art. ccccxvi and ccccxvii, (which are founded on a supposition that the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) we may find tolerably near values of the same annuities when the interest of money is 3 per cent. or 4 per cent. or 5 per cent. by means of the foregoing method of *Interpolation and Continuation*. But what shall we do, may the reader ask, when the interest of money is not 3, 4, or 5, per cent. but 2 per cent. or 6 per cent. or 7 per cent. or 8, or 9, or 10, per cent? Will the same method of *Interpolation and Continuation* enable us to find the values of annuities for two joint lives at these several rates of interest to a tolerable degree of exactness? or, in other words, may it be supposed that the value of an annuity of one pound a year for two joint lives of given ages at the several following rates of interest of money, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. $5\frac{1}{2}$ per cent. 6 per cent. $6\frac{1}{2}$ per cent. 7 per cent. $7\frac{1}{2}$ per cent. 8 per cent. $8\frac{1}{2}$ per cent. 9 per cent. $9\frac{1}{2}$ per cent. and 10 per cent. (as well as at the interests of 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent.) will form, pretty nearly, an arithmetical progression, or decrease by nearly equal differences? For, if this supposition is true, it is evident that, when any two of the terms of this progression are known, all the other terms may be derived from them by additions, or subtractions, of the common difference by which the terms decrease; and consequently the values of the proposed annuity at all the other rates of interest may be deduced in this manner from its values when the interest of money is $3\frac{1}{2}$ per cent. and 4 per cent. which are exhibited in the two foregoing sets of tables. In answer to this question I must observe, that it does not seem probable that this supposition will be nearly true in such a variety of different rates of interest. For we have, in the last article, seen reason to conclude, that the near values of an annuity for two joint lives of given ages when the interest of money is 3 per cent. and 5 per cent. which are obtained by means of this supposition, differ much more from its corresponding true values than the near value of it (obtained by means of the same supposition,) when the interest of money is 4 per cent, differs from its true value. And therefore we have reason to conclude, that the near values of the same annuity when the interest of money is $2\frac{1}{2}$ per cent. or 2 per cent, or $5\frac{1}{2}$ per cent. or 6 per cent, or 7, 8, 9, or 10 per cent. that would be obtained by means of this supposition, would differ still more from its corresponding true values, and, probably, would differ from them too much to make it advisable to neglect the differences and consider the said near values, in practice, as equal to the corresponding true values: at least we may well suppose this to be the case at the

very high interests of 8, 9, and 10 per cent. It seems desirable, therefore, to discover, if possible, some other method of finding a tolerably near value of an annuity for two joint lives of given ages, when the interest of money is either lower than 3 per cent. or higher than 5 per cent, that may exempt us from the necessity of computing its exact value by means of the expression

$$\frac{1}{r} \times \text{the series } \frac{P_i \times Q_i}{r} + \frac{P_{ii} \times Q_{ii}}{r^2} + \frac{P_{iii} \times Q_{iii}}{r^3} + \frac{P_{iv} \times Q_{iv}}{r^4} + \frac{P_v \times Q_v}{r^5} + \frac{P_{vi} \times Q_{vi}}{r^6} + \frac{P_{vii} \times Q_{vii}}{r^7} + \&c. \text{ continued to the end of}$$

the table of the probabilities of the duration of human life; the computation of which, when the two lives are young, is very tedious. Now one method of doing this to a moderate degree of exactness (though not to so great a degree of exactness as might be desired) is as follows.

A method of approximating to the value of an annuity of one pound a year for the joint continuance of two lives of given ages, without computing the expression $\frac{1}{r} \times \text{the series } \frac{P_i \times Q_i}{r} + \frac{P_{ii} \times Q_{ii}}{r^2} + \frac{P_{iii} \times Q_{iii}}{r^3} + \frac{P_{iv} \times Q_{iv}}{r^4} + \frac{P_v \times Q_v}{r^5} + \&c. \text{ (continued to the end of the table of the probabilities,)} \text{ which is accurately equal to it.}$

CCCCXVI. It is evident that the series $\frac{P_i \times Q_i}{r} + \frac{P_{ii} \times Q_{ii}}{r^2} + \frac{P_{iii} \times Q_{iii}}{r^3} + \frac{P_{iv} \times Q_{iv}}{r^4} + \frac{P_v \times Q_v}{r^5} + \&c. \text{ continued to the end of the table of probabilities, will consist of the same number of terms as the series } \frac{Q_i}{r} + \frac{Q_{ii}}{r^2} + \frac{Q_{iii}}{r^3} + \frac{Q_{iv}}{r^4} + \frac{Q_v}{r^5} + \&c. \text{ continued likewise to the end}$

of the table of probabilities: because, when all the persons who are represented in the table of probabilities as living at the age of the older of the two lives, and whose number is originally Q , shall be dead, the annuity (which is to depend on the joint continuance of both the lives,) must necessarily cease, notwithstanding several of the persons who are there represented as living at the age of the younger of the two lives, and whose number is originally P , may be still alive.

CCCCXVII. Since

CCCCXLVII. Since therefore the two serieses $\frac{Q^I}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^V}{r^5} + \&c.$ and $\frac{P^I \times Q^I}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^V \times Q^V}{r^5} + \&c.$ consist of the same number of terms;—

and, since they also have the same quantities for the denominators of their several terms, to wit, the quantities $r, r^2, r^3, r^4, r^5, \&c.$ —and the numerators of the terms of the former series, to wit, $Q^I, Q^{II}, Q^{III}, Q^{IV}, Q^V, \&c.$ are involved in, or are factors of, the numerators of the terms of the latter series, to wit, $P^I \times Q^I, P^{II} \times Q^{II}, P^{III} \times Q^{III}, P^{IV} \times Q^{IV}, P^V \times Q^V, \&c.$ —it seems reasonable to suppose that the value of the whole latter series may, in some way or other, be deduced from the value of the whole former series, to a tolerable degree of exactness, so as to make it unnecessary to take the trouble of computing all the terms of the latter series and then adding them up into one sum. And, if this shall appear to be practicable, it will follow, that the value of an annuity of one pound a year for the joint continuance of two lives of any given ages, may be deduced from the value of an annuity of the like amount for the older of the same two lives singly; which value we shall always be able to find in one of the twelve tables above exhibited in Art. cci, Pages 221, 222, 223, &c. — 232. And thus the said tables of the values of annuities for single lives may be made subservient to the discovery of the values of the like annuities for two joint lives. For let A be the value of an annuity of one pound a year for the older of the two lives singly; and let B be the value of an annuity of one pound a year for the joint continuance of both lives. Then will A be equal to $\frac{1}{r} \times$ the series $\frac{Q^I}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^V}{r^5} +$

&c. and B will be equal to $\frac{1}{P \times Q} \times$ the series $\frac{P^I \times Q^I}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^V \times Q^V}{r^5} + \&c.$ and consequently the series $\frac{Q^I}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^V}{r^5} + \&c.$ will be $= Q \times A$, and the series

$$\text{series } \frac{P_1 \times Q_1}{r} + \frac{P_{11} \times Q_{11}}{r^2} + \frac{P_{111} \times Q_{111}}{r^3} + \frac{P_{1111} \times Q_{1111}}{r^4} + \frac{P_{11111} \times Q_{11111}}{r^5}$$

+ &c. will be = $P \times Q \times B$. If therefore we can derive the latter series from the former, we shall thereby obtain the value of $P \times Q \times B$, which, being divided by $P \times Q$, will give us B , or the value of an annuity of one pound a year for the two proposed joint lives.

We must therefore endeavour to find a method of deriving the value of the series $\frac{P_1 \times Q_1}{r} + \frac{P_{11} \times Q_{11}}{r^2} + \frac{P_{111} \times Q_{111}}{r^3} + \frac{P_{1111} \times Q_{1111}}{r^4} + \frac{P_{11111} \times Q_{11111}}{r^5}$ + &c. from the value of the series $\frac{Q_1}{r} + \frac{Q_{11}}{r^2} + \frac{Q_{111}}{r^3} + \frac{Q_{1111}}{r^4} + \frac{Q_{11111}}{r^5}$ + &c. which is equal to $Q \times A$. Now this may be done to a certain moderate degree of exactness, by means of a conjectural supposition that has an appearance of great probability, in the manner following.

CCCCXLVIII. Put S for the value of the simple series $\frac{Q_1}{r} + \frac{Q_{11}}{r^2} + \frac{Q_{111}}{r^3} + \frac{Q_{1111}}{r^4} + \frac{Q_{11111}}{r^5}$ + &c. and the Greek capital Σ for the value of the compound series $\frac{P_1 \times Q_1}{r} + \frac{P_{11} \times Q_{11}}{r^2} + \frac{P_{111} \times Q_{111}}{r^3} + \frac{P_{1111} \times Q_{1111}}{r^4} + \frac{P_{11111} \times Q_{11111}}{r^5}$ + &c. and put $g = \frac{Q_1}{r}$, and $b = \frac{Q_{11}}{r^2}$, and $i = \frac{P_1 \times Q_1}{r}$, and $k = \frac{P_{11} \times Q_{11}}{r^2}$.

And let G denote the value of an infinite series of terms in geometrical proportion, of which $\frac{Q_1}{r}$ and $\frac{Q_{11}}{r^2}$, or g and b , are the two first terms, and the Greek capital Γ denote the value of an infinite series of terms in geometrical proportion, of which $\frac{P_1 \times Q_1}{r}$ and $\frac{P_{11} \times Q_{11}}{r^2}$, or i and k , are the two first terms.

Then

Then will the infinite geometrical series G , or $g + b + \frac{b^2}{g} + \frac{b^3}{g^2} + \frac{b^4}{g^3} + \frac{b^5}{g^4} + \frac{b^6}{g^5} + \&c.$ be $= \frac{gg}{g-b}$, and the infinite geometrical series F , or $i + \frac{k}{i} + \frac{k^2}{i^2} + \frac{k^3}{i^3} + \frac{k^4}{i^4} + \frac{k^5}{i^5} + \&c.$ be $= \frac{i^2}{i-k}$; as will be evident from Art. LXXX, pages 91, 92.

CCCCXLIX. Now it seems reasonable to suppose that the geometrical series G , or $g + b + \frac{b^2}{g} + \frac{b^3}{g^2} + \frac{b^4}{g^3} + \frac{b^5}{g^4} + \frac{b^6}{g^5} + \&c.$ *ad infinitum*, will bear, pretty nearly, the same proportion to the simple series S , or $\frac{g}{r} + \frac{g^2}{r^2} + \frac{g^3}{r^3} + \frac{g^4}{r^4} + \frac{g^5}{r^5} + \frac{g^6}{r^6} + \frac{g^7}{r^7} + \frac{g^8}{r^8} + \&c.$ continued to the end of the table of probabilities, (of which series the two first terms, $\frac{g}{r}$ and $\frac{g^2}{r^2}$, are the same with the two first terms, g and b , of the said geometrical series,) as the geometrical series F , or $i + k + \frac{k^2}{i} + \frac{k^3}{i^2} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5} + \&c.$ *ad infinitum*, bears to the compound series Z , or $\frac{P_1 \times Q_1}{r} + \frac{P_{11} \times Q_{11}}{r^2} + \frac{P_{111} \times Q_{111}}{r^3} + \frac{P_{1111} \times Q_{1111}}{r^4} + \frac{P_{11111} \times Q_{11111}}{r^5} + \frac{P_{111111} \times Q_{111111}}{r^6} + \&c.$ continued to the end of the table of probabilities, of which series the two first terms $\frac{P_1 \times Q_1}{r}$ and $\frac{P_{11} \times Q_{11}}{r^2}$, are the same with the two first terms, i and k , of the said geometrical series. And, if this supposition is true, the series Z , or $\frac{P_1 \times Q_1}{r} + \frac{P_{11} \times Q_{11}}{r^2} + \frac{P_{111} \times Q_{111}}{r^3} + \frac{P_{1111} \times Q_{1111}}{r^4} + \frac{P_{11111} \times Q_{11111}}{r^5} + \frac{P_{111111} \times Q_{111111}}{r^6} + \&c.$

$$+ \frac{P_{VI} \times Q_{VI}}{r^6} + \frac{P_{VII} \times Q_{VII}}{r^7} + \&c. \text{ will be nearly equal to } \frac{1 \times S}{G}.$$

But we have seen in Art. XLVII, that the series Σ , or $\frac{P^1 \times Q^1}{r} + \frac{P_{II} \times Q_{II}}{r^2} + \frac{P_{III} \times Q_{III}}{r^3} + \frac{P_{IV} \times Q_{IV}}{r^4} + \frac{P_V \times Q_V}{r^5} + \frac{P_{VI} \times Q_{VI}}{r^6} + \frac{P_{VII} \times Q_{VII}}{r^7} + \&c.$ is $= P \times Q \times B$, and the series S , or $\frac{Q^1}{r} + \frac{Q_{II}}{r^2} + \frac{Q_{III}}{r^3} + \frac{Q_{IV}}{r^4} + \frac{Q_V}{r^5} + \frac{Q_{VI}}{r^6} + \frac{Q_{VII}}{r^7} + \&c.$ is $= Q \times A$. Therefore $P \times Q \times B$ will be nearly equal to $\frac{\Gamma \times Q \times A}{G}$.

But the series G is $= \frac{gg}{g-b}$, and the series Γ is $= \frac{ii}{i-k}$; as we have

seen in Art. cccclviii. Therefore $P \times Q \times B$ will be nearly equal to $\frac{\frac{P^1 \times Q^1}{g-b} \times \frac{ii}{i-k} \times \frac{g-b}{gg}}{\frac{gg}{g-b}}$, or $Q \times A \times \frac{ii}{i-k} \times \frac{g-b}{gg} \times \frac{ii}{i-k}$.

and B , or the value of an annuity of one pound a year for the two joint lives whose ages correspond to the numbers P and Q , will be nearly equal

$$\text{to } \frac{Q \times A}{P \times Q} \times \frac{ii}{gg} \times \frac{g-b}{i-k}, \text{ or } \frac{P^1}{P} \times \frac{ii}{gg} \times \frac{g-b}{i-k}.$$

But g is $= \frac{Q^1}{r}$, and b is $= \frac{Q_{II}}{r^2}$, and i is $= \frac{P^1 \times Q^1}{r}$, and k is $= \frac{P_{II} \times Q_{II}}{r^2}$. Therefore $i = g \times P^1$, and k is $= b \times P_{II}$. Consequently

$$ii \text{ is } = gg \times P^1 \times P^1, \text{ and } i-k \text{ is } = g \times P^1 - b \times P_{II}, \text{ and } \frac{ii}{gg} \times \frac{g-b}{i-k} \text{ is } =$$

A a a a

$$\text{is} = \frac{g \times P_i \times P_i}{g \times g} \times \frac{g-b}{g \times P_i - b \times P_{ii}} = P_i \times P_i \times \frac{g-b}{g \times P_i - b \times P_{ii}}$$

$$\text{Therefore } \frac{A}{P} \times \frac{i}{g} \times \frac{g-b}{i-k} \text{ is} = \frac{A}{P} \times P_i \times P_i \times \frac{g-b}{g \times P_i - b \times P_{ii}} =$$

$$A \times \frac{P_i}{P} \times \frac{P_i \times g - b}{g \times P_i - b \times P_{ii}} = A \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_{ii} - b \times P_{ii}}. \text{ There}$$

fore B , or the value of an annuity of one pound a year for the two joint lives whose ages correspond to the numbers P and Q , will be nearly equal to $A \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_{ii} - b \times P_{ii}}$, or may be derived from A , or the value of the like annuity for a single life of the age corresponding to Q , by multiplying it into the fraction $\frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_{ii} - b \times P_{ii}}$. Q.E.L.

Examples of the foregoing method of approximating to the values of annuities for two joint lives.

CCCCCL. I shall now proceed to try the truth of this expression $A \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_{ii} - b \times P_{ii}}$, by applying it to the computation of the values of an annuity of one pound a year for the joint continuance of two lives of some of the ages set down in the tables above exhibited in Art. ccccviii and ccccxiii. And, if we shall find that the values of those annuities which we shall obtain by means of this expression, differ but little from the values of them contained in those tables, (which have been accurately computed,) we shall have reason to conclude that the same expression will give us the like approximations to the true values of annuities for two joint lives in other instances.

Examples of the said method, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

CCCCCL. Let us therefore suppose that the two lives for the joint continuance of which an annuity of one pound a year is to be purchased are of the ages of 10 years and 20 years; and that the interest of money is $3\frac{1}{2}$ per cent. and that the table of probabilities of the duration of human life, by which the calculation is to be governed, is that of Monsieur de Parcieux.

First example; in which the difference of the ages of the two lives is 10 years.

Then we shall have (by Table XV.) $\overset{£}{A}$, or the value of an annuity of

one pound a year for a single life of the age of 20 years, $\overset{£}{P}_1 = 19,440,616$, and $P = 880$, $P_1 = 872$, $P_{11} = 866$, $\overset{£}{Q}_1 = 814$, $\overset{£}{Q}_1 = 806$, $\overset{£}{Q}_{11} = 798$,

$\overset{£}{P}_{11} = 1,035$, $\frac{1}{r} \left(= \frac{1}{1.035} \right) = 0.966, 183$, $\frac{1}{r^2} = 0.933, 510$; and con-

sequently $\overset{£}{g} \left(= \frac{\overset{£}{Q}_1}{r} = 806 \times .966, 183 \right) = 778.743, 498$, and $b \left(= \frac{\overset{£}{Q}_{11}}{r^2} \right)$

$= 798 \times .933, 510 = 744.940, 980$. Therefore $\overset{£}{g} \times P_1$ will be $(=$

$778.743, 498 \times 872) = 679,064.330, 256$, and $b \times P_1$ will be $= 744.940, 980$

$\times 872 = 649,588.534, 560$, and $b \times P_{11}$ will be $(= 744.940, 980 \times 866)$

$= 643,629.006, 720$; and consequently $\overset{£}{g} \times P_1 - b \times P_1$ will be $(=$

$79,064.330, 256 - 649,588.534, 560) = 29,475.795, 696$, and $\overset{£}{g} \times P_1 -$

$b \times P_{11}$ will be $(= 679,064.330, 256 - 643,629.006, 720) = 35,435.323, 536$.

Therefore $\frac{\overset{£}{g} \times P_1 - b \times P_1}{\overset{£}{g} \times P_1 - b \times P_{11}}$ will be $= \frac{29,475.795, 696}{35,435.323, 536} =$ (nearly)

$\frac{35,435}{35,435} = .8318$; and $\frac{P_1}{P} \times \frac{\overset{£}{g} \times P_1 - b \times P_1}{\overset{£}{g} \times P_1 - b \times P_{11}}$ will be $=$

(nearly) $\frac{P_1}{P} \times .8318 (= \frac{872}{880} \times .8318 = \frac{109}{110} \times .8318 = \frac{90.6662}{110})$

$\times .8242$. Therefore $\overset{£}{A} \times \frac{P_1}{P} \times \frac{\overset{£}{g} \times P_1 - b \times P_1}{\overset{£}{g} \times P_1 - b \times P_{11}}$ will be $(= \overset{£}{A} \times .8242$

$\times 19,440,616 \times .8242) = \overset{£}{16,022,955}$. Q.E.I.

The true value of this annuity for two joint lives appears by Table

XXI, page 494, to be $\overset{£}{16,634,79}$; which is greater than the near value

it just now found, to wit. $\overset{£}{16,022,955}$, by the difference $\overset{£}{0,611,835}$,

which is somewhat less than a 27th part of the said true value.

CCCCLII. Now let the two lives be of the ages of 20 years and

years.

In this case $\overset{£}{A}$ will be the value of an annuity of one pound a year for

single life of the age of 30 years, and therefore (by Table XV.) will

be $\overset{£}{A} = 22,222, 10$ years.

be $\overset{£}{A} = 22,222, 10$ years.

be $\overset{£}{A} = 22,222, 10$ years.

be $\overset{£}{A} = 22,222, 10$ years.

be $\overset{£}{A} = 22,222, 10$ years.

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be $\overset{£}{A} = 22,222, 10$ years.

be $\overset{£}{A} = 22,222, 10$ years.

be $\overset{£}{A} = 22,222, 10$ years.

be $\overset{£}{A} = 22,222, 10$ years.

be = 18.068,798. And P will be = 814, P^I = 806, P^{II} = 798, Q^I = 734, Q^{II} = 726, Q^{III} = 718, r = 1.035, $\frac{1}{r}$ ($= \frac{1}{1.035}$) = 0.966, 183 and $\frac{1}{r^2}$ = 0.933,510. Therefore $\frac{Q^I}{r}$ will be ($= 726 \times .966, 183$) =

701.418,858, and $\frac{Q^{II}}{r^2}$ will be ($= 718 \times .933,510$) = 670.260,180

that is, g will be = 701.448,858, and b will be = 670.260,180. Therefore $g \times P^I$ will be ($= 701.448,858 \times 806$) = 565,367.779,548, and $b \times P^I$ will be ($= 670.260,180 \times 806$) = 540,229.705,080, and $b \times P^{II}$ will be ($= 670.260,180 \times 798$) = 534,867.623,640; and consequently $g \times P^I - b \times P^I$ will be ($= 565,367.779,548 - 540,229.705,080$) = 25,138 074,468, and $g \times P^I - b \times P^{II}$ will be ($= 565,367.779,548$

534,867.623,640) = 30,500.155,808. Therefore

$\frac{g \times P^I - b \times P^I}{g \times P^I - b \times P^{II}}$ will

be = $\frac{25,138.074,468}{30,500.155,908}$ = (nearly) $\frac{25,138.074,468}{30,500}$ = .8241; and $\frac{P^I}{P}$

$\times \frac{g \times P^I - b \times P^I}{g \times P^I - b \times P^{II}}$ will be ($= \frac{P^I}{P} \times .8241 = \frac{806}{814} \times .8241 = \frac{40}{407}$

$\times .8241 = \frac{332.1123}{407}$) = .8160. Therefore $A \times \frac{P^I}{P} \times \frac{g \times P^I - b \times P^I}{g \times P^I - b \times P^{II}}$

will be = $A \times .8160 = 18.068,798 \times .8160 = 14.7441$. Q.E.I.

Difference between the foregoing near value and the true value of this annuity.

The true value of this annuity for the said joint lives of the ages of 2 years and 30 years appears by Table XXXI, page 494, to be £15.298,75 which is greater than the value just now found for it, to wit, £14.7441; the difference £0.554,65, which is less than the 27th part of the said true value, £15.298,75.

Five more examples of the same method, in all which the difference of the ages of the two lives is the same as in the two former examples, to wit, 10 years.

CCCCCLIII. If we derive in the same manner the values of an annuity of one pound a year for the joint continuance of the five following pairs of lives, to wit, two lives of the ages of 30 years and 40 years, two lives the ages of 40 years and 50 years, two lives of the ages of 50 years and 60 years, two lives of the ages of 60 years and 70 years, and two lives of the ages of 70 years and 80 years, from the values of the same annuity for single lives of the same ages as the older lives in each of these pairs of lives, respectively, that is, for single lives of the ages of 40 years, 50 years,

years, 70 years, and 80 years, (which values appear by Table XV to be £16.104,542, £13.183,083, £10.104,074, £6.575,357, and £3.661,781;) — I say, if we derive near values of the said annuity for the said pairs of joint lives from these values of the same annuity for the said single lives by means of the aforesaid expression $A \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$, those near values will be found to be as follows; to wit,

for two joint lives of the ages of 30 years and 40 years, £12.865,18;
for two joint lives of the ages of 40 years and 50 years, £10.917,67;
for two joint lives of the ages of 50 years and 60 years, £7.680,967;
for two joint lives of the ages of 60 years and 70 years, £5.074,649;
and for two joint lives of the ages of 70 years and 80 years, £2.645,883.

CCCCLIV. Now it appears from Table XXXI, that the true values of these last five annuities for joint lives are as follows, to wit,

The true values of the last five annuities.

for two joint lives of the ages of 30 years and 40 years, £13.709,61;
for two joint lives of the ages of 40 years and 50 years, £11.229,92;
for two joint lives of the ages of 50 years and 60 years, £8.341,630;
for two joint lives of the ages of 60 years and 70 years, £5.336,698;
and for two joint lives of the ages of 70 years and 80 years, £2.793,419.

CCCCLV. The difference between the near value, £12.865,18, of the first of these annuities, and its true value, £13.709,61, is £0.844,43; which is less than a 16th part of the said true value.

The differences between their foregoing near values and their true values.

The difference between the near value, £10.917,67, of the second of these annuities, and its true value, £11.229,92, is £0.312,25; which is less than a 35th part of the said true value.

The difference between the near value, £7.680,967, of the third of these annuities, and its true value, £8.341,630, is £0.660,663; which is less than a 12th part of the said true value.

The

The difference between the near value, £5.074,649, of the fourth of the foregoing annuities, and its true value, £5.336,698, is £0.262,049; which is less than the 20th part of the said true value.

And the difference between the near value, £2.645,883, of the fifth and last of the foregoing annuities, and its true value, £2.793,419, is £0.147,536; which is less than the 19th part of the said true value.

CCCCCLVI. In all these examples it appears that the near values of annuities for two joint lives, obtained by means of the expression $A \times \frac{P_1}{p}$ $\times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$, are less than their respective true values, and that the differences of the said near values and true values are in some cases about a 35th part of the said true values respectively, and in other cases a 27th, a 20th, a 19th, and a 16th part of the said true values, and in some cases almost a 12th part of them.

Five other examples of the foregoing method of finding near values of annuities for two joint lives; in which the difference of the ages of the two lives is 30 years.

CCCCCLVII. Let us now examine the near values of some annuities for two joint lives, that are obtained by means of the said expression, $A \times \frac{P_1}{p} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$, when the difference of the ages of the two lives is considerably greater than 10 years. Let this difference, therefore, be 30 years; and let us derive, by means of the said expression, $A \times \frac{P_1}{p}$ $\times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$, near values of an annuity of one pound a year for the five following pairs of joint lives, upon a supposition that the interest of money is (as it was supposed to be in the foregoing examples,) 3 $\frac{1}{4}$ per cent. to wit,

- two lives of the ages of 10 years and 40 years,
- two lives of the ages of 20 years and 50 years,
- two lives of the ages of 30 years and 60 years,
- two lives of the ages of 40 years and 70 years,
- and two lives of the ages of 50 years and 80 years,

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from the values of a like annuity of one pound a year for single lives of the ages of 40 years, 50 years, 60 years, 70 years, and 80 years, respectively, which are given above in Table XV, to wit, £16.104,542, £13.183,083, £10.104,074, £6.575,357, and £3,661,781. These near values will be found to be as follows; to wit,

for two joint lives of the ages of 10 years and 40 years, £13.892,33;	The near values of the said annuities.
for two joint lives of the ages of 20 years and 50 years, £11.068,05;	
for two joint lives of the ages of 30 years and 60 years, £8.560,176;	
for two joint lives of the ages of 40 years and 70 years, £5.931,793;	
and for two joint lives of the ages of 50 years and 80 years, £3.320,420.	

CCCCLVIII. Now it appears from Table XXXIII, that the true values of these last five annuities for two joint lives are as follows; to wit,

for two joint lives of the ages of 10 years and 40 years, £14.384,49;
 for two joint lives of the ages of 20 years and 50 years, £11.801,15;
 for two joint lives of the ages of 30 years and 60 years, £9.217,864;
 for two joint lives of the ages of 40 years and 70 years, £6.126,861;
 and for two joint lives of the ages of 50 years and 80 years, £3.399,171.

CCCCCLIX. The difference between the near value, £13.892,33, of the first of these annuities and its true value, £14.384,49, is £0.492,16; which is less than the 29th part of the said true value.

The difference between the near value, £11.068,05, of the second annuity, and its true value, £11.801,15, is £0.733,10; which is less than the 16th part of the said true value.

The difference between the near value, £8.560,176, of the third annuity, and its true value, £9.217,864, is £0.657,688; which is less than the 14th part of the said true value.

The difference between the near value, £5.931,793, of the fourth annuity, and its true value, £6.126,861, is £0.195,068; which is less than the 31st part of the said true value.

And

And the difference between the near value, £3.320,420, and the true value, £3.399,171, of the fifth and last annuity, is £0.078,751; which is less than the 43d part of the said true value.

CCCCIX. In these five examples it appears that the near values of annuities for two joint lives, whose ages differ from each other by 30 years obtained by means of the expression $A \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_2}{g \times P_1 - b \times P_2}$, are less than their respective true values, (as was the case with the near value of the annuities mentioned in Art. ccccl, &c. — cccclvi) and that the differences of the said near values and true values are in some cases less than the 43d part of the said true values respectively, and in other cases a 31st, a 29th, and a 16th part of the said true values, and in some cases about a 14th part of them.

These differences are rather smaller than the differences of the near values and true values of the annuities for two joint lives whose ages differ from each other by only 10 years, which are mentioned in Art. cccclvi.

Examples of the said method, upon a supposition that the interest of money is $4\frac{1}{2}$ per cent.

CCCCXLI. We will next examine the near values of annuities for two joint lives, which may be obtained by means of the expression $A \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_2}{g \times P_1 - b \times P_2}$, when the interest of money is $4\frac{1}{2}$ per cent.

Examples in which the differences of the ages of the two lives is 10 years.

Let there be seven pairs of lives whose ages differ from each other by 10 years, to wit,

- two lives of the ages of 10 years and 20 years,
- two lives of the ages of 20 years and 30 years,
- two lives of the ages of 30 years and 40 years,
- two lives of the ages of 40 years and 50 years,
- two lives of the ages of 50 years and 60 years,
- two lives of the ages of 60 years and 70 years,
- and two lives of the ages of 70 years and 80 years.

When the interest of money is $4\frac{1}{2}$ per cent. the near values of an annuity of one pound a year for the joint continuance of these several pairs of lives, that may be derived by means of the expression $A \times \frac{P_1}{P}$ x

$\frac{P_1 - b \times P_1}{P_1 - b \times P_{11}}$, from the values of the like annuity of one pound a year for single lives of the ages of 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years, (which values appear by Table XVII to be £16.623,93, £15.690,92, £14.254,42, £11.920,73, £9.346,46, £6.220,54, and £3.532,59,) will be found to be as follows; to wit,

The near value of an annuity of one pound a year for two joint lives of the ages of 10 years and 20 years, will be £14.655,20;

The near values of an annuity of one pound a year for the said seven pairs of joint lives.

for two joint lives of the ages of 20 years and 30 years, £13.359,42;
for two joint lives of the ages of 30 years and 40 years, £11.594,96;
for two joint lives of the ages of 40 years and 50 years, £10.126,50;
for two joint lives of the ages of 50 years and 60 years, £7.329,817;
for two joint lives of the ages of 60 years and 70 years, £4.889,327;
and for two joint lives of the ages of 70 years and 80 years, £2.580,00.

CCCCXII. Now it appears from Table XLI that the true values of these seven annuities of one pound a year for two joint lives are as follows;

The true values of the same annuity for the same pairs of joint lives.

for two joint lives of the ages of 10 years and 20 years, £14.478,80;
for two joint lives of the ages of 20 years and 30 years, £13.495,90;
for two joint lives of the ages of 30 years and 40 years, £12.297,99;
for two joint lives of the ages of 40 years and 50 years, £10.274,39;
for two joint lives of the ages of 50 years and 60 years, £7.793,079;
for two joint lives of the ages of 60 years and 70 years, £5.087,882;
and for two joint lives of the ages of 70 years and 80 years, £2.709,443.

The differences
between the said
near values and
true values.

CCCCCLXIII. The difference between the near value, £14.655,20, of the first of these annuities, and its true value, £14.478,80, (which, it is worth observing, is, less than the near value, though in all the former examples the true values of the annuities have been greater than their near values,) is £0.176,40; which is less than the 82d part of the said true value.

The difference between the near value, £13.359,42, of the second annuity, and its true value, £13.495,90, is £0.136,48; which is less than the 98th part of the said true value.

The difference between the near value, £11.594,96, of the third annuity, and its true value, £12.297,99, is £0.703,03; which is less than the 17th part of the said true value.

The difference between the near value, £10.126,50, of the fourth annuity and its true value, £10.274,39, is £0.147,89; which is less than the 69th part of the said true value.

The difference between the near value, £7.329,817, of the fifth annuity, and its true value, £7.793,079, is £0.463,262; which is less than the 17th part of the said true value.

The difference between the near value, £4.889,327, of the sixth annuity, and its true value, £5.087,882, is £0.198,555; which is less than the 25th part of the said true value.

And the difference between the near value, £2.580,00, of the seventh and last annuity, and its true value, £2.709,443, is £0.129,443; which is less than the 20th part of the said true value.

A remark on the
said differences.

These differences are smaller than those of the near values and the true values of the annuities mentioned in Art. cccclvii and cccclviii, for two joint lives whose ages differ from each other by 30 years, when the interest of money is $3\frac{1}{2}$ per cent. and are smaller in a still greater degree than the differences of the near values and the true values of the annuities for two joint lives, mentioned in Art. cccclvi, whose ages differ by only 10 years, when the interest of money is $3\frac{1}{4}$ per cent.

CCCCXLIV. We will now examine the near values of some annuities. Five other examples of the foregoing method of finding the values of annuities for two joint lives; in which the difference of the ages of the two lives is 30 years, and the interest of money (as in the two preceeding articles,) is $4\frac{1}{2}$ per cent.

Now, when the interest of money is $4\frac{1}{2}$ per cent. the near values of an annuity of one pound a year for the five following pairs of joint lives, to wit,

two joint lives of the ages of 10 years and 40 years,
 two joint lives of the ages of 20 years and 50 years,
 two joint lives of the ages of 30 years and 60 years,
 two joint lives of the ages of 40 years and 70 years,
 and two joint lives of the ages of 50 years and 80 years,

which may be obtained by means of the expreffion $A \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$, will be found to be as follows; to wit,

for two joint lives of the ages of 10 years and 40 years, £12.446,40;
 for two joint lives of the ages of 20 years and 50 years, £10.248,45;
 for two joint lives of the ages of 30 years and 60 years, £8.075,146;
 for two joint lives of the ages of 40 years and 70 years, £5.655,547;
 and for two joint lives of the ages of 50 years and 80 years, £3.214,679.

The near values of the said five annuities.

CCCCXLV. Now it appears from Table XLIII, that the true values of these last five annuities for two joint lives are as follows; to wit,

for two joint lives of the ages of 10 years and 40 years, £12 842,64;
 for two joint lives of the ages of 20 years and 50 years, £10.739,19;
 for two joint lives of the ages of 30 years and 60 years, £8.560,832;
 for two joint lives of the ages of 40 years and 70 years, £5.810,588;
 and for two joint lives of the ages of 50 years and 80 years, £3.284,035.

CCCCCLXVI. All these true values are greater than the near value of the same annuities obtained in Art. cccclxiv by means of the expression $\frac{A \times P_1}{P} \times \frac{g \times P_1 - b \times P_{11}}{g \times P_1 - b \times P_{11}}$, as was the case with respect to the true values of the annuities mentioned in all the preceding examples, except the first example in Art. cccclxii, in which it appeared that the true value of an annuity of one pound a year for two joint lives of the ages of 10 year and 20 years, when the interest of money is $4\frac{1}{2}$ per cent, to wit, £14,478,80 was less than its near value, £14,655,20, obtained by means of the expression $\frac{A \times P_1}{P} \times \frac{g \times P_1 - b \times P_{11}}{g \times P_1 - b \times P_{11}}$. This remarkable exception I do not know how to account for.

The differences between the said near values and true values.

CCCCCLXVII. The difference between the near value, £12,446,40 of the first of the five last-mentioned annuities for two joint lives, and its true value, £12,842,64, is £0.396,24; which is less than the 32d part of the said true value.

The difference between the near value, £10,248,45, of the second of the said five annuities, and its true value, £10,739,19, is £0.490,74; which is less than the 21st part of the said true value.

The difference between the near value, £8,075,146, of the third annuity, and its true value, £8,560,832, is £0.485,686; which is less than the 17th part of the said true value.

The difference between the near value, £5,655,547, of the fourth annuity, and its true value, £5,810,588, is £0.155,041; which is less than the 37th part of the said true value.

And the difference between the near value, £3,214,679, of the fifth and last annuity, and its true value, £3,284,035, is £0.069,356; which is less than the 47th part of the said true value.

A table of the foregoing near values of an annuity of one pound a year for different pairs of joint lives, and of the corresponding true values of the same annuity, and of the differences of the said near values and true values, and of the fractions that express the proportions of the said differences to the said true values.

CCCCLVIII. If the near values and true values of all the annuities mentioned in the foregoing articles, from Art. ccccl. to the last article, inclusively, be ranged in regular order in two contiguous columns, and their differences be set down in a third column adjoining to the second column, and the fractions that express the proportions of the said differences to the said true values, be set down in a fourth column adjoining to the said third column, the said several numbers will be as follows.

<i>The near values of an annuity of one pound a year for two joint lives of several different ages; obtained by means of the expression $A \times \frac{P_i}{p} \times \frac{g \times P_i - b \times P_{ii}}{g \times P_i - b \times P_{ii}}$ when the interest of money is $3\frac{1}{2}$ per cent.</i>	<i>The true values of the same annuity for the same joint lives, when the interest of money is $3\frac{1}{2}$ per cent.</i>	<i>The differences of the said near and true values.</i>	<i>The proportions of the said differences to the said true values.</i>
$\pounds$ 16.022,955 14.744,1 12.865,18 10.917,67 7.680,967 5.074,649 2.645,883	$\pounds$ 16.634,79 15.298,75 13.709,61 11.229,92 8.341,630 5.336,698 2.793,419	$\pounds$ 0.611,835 0.554,65 0.844,43 0.314,25 0.660,663 0.262,049 0.147,536	$\frac{1}{27}$ $\frac{1}{27}$ $\frac{1}{26}$ $\frac{1}{27}$ $\frac{1}{25}$ $\frac{1}{26}$ $\frac{1}{29}$
$\pounds$ 13.892,33 11.068,05 8.560,176 5.931,793 3.320,420	$\pounds$ 14.384,49 11.801,15 9.217,864 6.126,861 3.399,171	$\pounds$ 0.492,16 0.733,10 0.657,688 0.195,068 0.078,751	$\frac{1}{25}$ $\frac{1}{26}$ $\frac{1}{24}$ $\frac{1}{27}$ $\frac{1}{47}$

The near values of the same annuity for the same joint lives, obtained by means of the same expression $A \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$ when the interest of money is $4\frac{1}{2}$ per cent.	The true values of the same annuity for the same joint lives, when the interest of money is $4\frac{1}{2}$ per cent.	The difference of the said near and true values.	The proportions of the said differences to the said true values.
£ 14.655,20 13.359,42 11.594,96 10.126,50 7.329,817 4.889,327 2.580,00	£ 14.478,80 13.495,90 12.297,99 10.274,39 7.793,079 5.087,882 2.709,443	£ 0.176,40 0.136,48 0.703,03 0.147,89 0.463,262 0.198,555 0.129,443	$\frac{1}{15}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{16}$ $\frac{1}{17}$ $\frac{1}{16}$
£ 12.446,40 10.248,45 8.075,146 5.655,547 3.214,679	£ 12.842,64 10.739,19 8.560,832 5.810,588 3.284,035	£ 0.396,24 0.490,74 0.485,686 0.155,041 0.069,356	$\frac{1}{15}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{17}$

A second table of the same kind as the last, in which the near values of the annuities for two joint lives are derived from the former near values of them, exhibited in the last table, by multiplying the said former near values into the fraction $\frac{104}{100}$.

CCCCXIX. If the foregoing near values of an annuity of one pound a year for two joint lives (which were obtained by means of the expression $A \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$) be increased in the proportion of 104 to 100, or be multiplied into the fraction $\frac{104}{100}$, we shall thereby obtain a se-

cond set of near values of the said annuity for the same joint lives, that will, for the most part, differ less than the foregoing near values of it from its corresponding true values. This will appear upon tryal, by computing these second near values, (which will be equal to $A \times \frac{104}{100} \times \frac{P_i}{P} \times \frac{P_i - b \times P_i}{P_i - b \times P_i}$) and placing them one under another in a column (as the former near values were placed in the last article,) and then setting down the corresponding true values of the same annuity in a second column adjoining to the former, and the differences between the said near values and true values in a third column adjoining to the second, and the fractions that express the proportions of the said differences to the said true values in a fourth column adjoining to the third. This may be done in the manner following.

The near values of an annuity of one pound a year for two joint lives of several different ages, obtained by means of the expression $A \times \frac{104}{100} \times \frac{P_i - b \times P_i}{P_i - b \times P_i}$	The true values of the same annuity for the same joint lives.	The differences of the said near and true values.	The proportions of the said differences to the said true values.
$\frac{P_i}{P} \times \frac{P_i - b \times P_i}{P_i - b \times P_i}$ when the interest of money is $3\frac{1}{2}$ per cent.			
$\pounds$ 16.663,86 15.333,86 13.379,78 11.354,37 7.988,205 5.277,634 2.751,718	$\pounds$ 16.634,79 15.298,75 13.709,61 11.229,92 8.341,630 5.336,698 2.793,419	$\pounds$ 0.029,07 0.035,11 0.329,83 0.124,45 0.353,425 0.059,064 0.041,701	$\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$
$\pounds$ 14.448,02 11.510,77 8.902,583 6.169,064 3.453,236	$\pounds$ 14.384,49 11.801,15 9.217,864 6.126,861 3.399,171	$\pounds$ 0.063,53 0.290,38 0.315,281 0.042,203 0.054,065	$\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$ $\frac{1}{57\frac{1}{2}}$

The near values of the same annuity for the same joint lives, obtained by means of the same expression $d \times \frac{104}{100} \times \frac{P_1}{P}$	The true values of the same annuity for the same joint lives, when the interest of money is $4\frac{1}{2}$ per cent.	The difference of the said near values and true values.	The portions of the said differences to be added to the said true values.
$\times \frac{g \times P_1 - b \times P_{11}}{g \times P_1 - b \times P_{11}}$ when the interest of money is $4\frac{1}{2}$ per cent.			
$\pounds$ 15.241,40 13.893,89 12.057,85 10.531,56 7.623,009 5.084,900 2.683,200	$\pounds$ 14.478,80 13.495,90 12.297,99 10.274,39 7.793,079 5.087,882 2.709,443	$\pounds$ 0.762,60 0.397,99 0.240,14 0.257,17 0.170,070 0.002,982 0.026,243	$\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$
$\pounds$ 12.944,256 10.658,38 8.398,151 5.881,768 3.343,266	$\pounds$ 12.842,64 10.739,19 8.560,832 5.810,588 3.284,035	$\pounds$ 0.101,61 0.080,81 0.162,681 0.071,180 0.059,231	$\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{11}$

A remark on the near values contained in the last table.

CCCCCLXX. By these instances it appears that the expression $d \times \frac{104}{100} \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_1}$ almost always gives us the value of an annuity of one pound a year for two joint lives to a considerably greater degree of exactness than the expression $d \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_1}$ and, in most cases, to a sufficient degree of exactness for practical purposes. And from its answering so well to the instances here given, it seems reasonable to suppose that it will give the values of annuities for two joint lives to nearly the same

same degree of exactness in other instances, or where the ages of the two joint lives are different from those above-supposed: and this it seems likely to do as well at one rate of the interest of money as at another; though for want of tables of the true values of annuities for two joint lives at any other rates of interest than $3\frac{1}{4}$ per cent. and $4\frac{1}{2}$ per cent. to try it by, the foregoing examples are confined to annuities at only those two rates of interest. And therefore I think upon the whole, I may venture to recommend the said

expression $A \times \frac{104}{100} \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$ as a tolerably convenient

approximation to the value of an annuity of one pound a year for two joint lives, when the interest of money is either less than 3 per cent. or greater than 5 per cent. and consequently the value of such annuity cannot be found in either of the two foregoing sets of tables for two joint lives in Art. cccviii and cccxxxii, nor be derived from the values exhibited in those tables by either the method of *Interpolation* explained in Art. ccccxii, ccccxiii, &c. — ccccxix, or the method of *Interpolation and Continuation* explained in Art. cccxxxvi, cccxxxvii, &c.

I now proceed to consider the values of annuities that depend on the joint continuance of *three* lives.

Of the values of annuities depending on the joint continuance of three lives.

CCCLXXI. Let r be, as before, the sum of one pound, together with its interest for a year, according to any given rate of interest. And let N be the number of years in the age of the younger of the three persons on the joint continuance of whose lives the annuity is to depend; and $N + a$ the number of years in the age of the next older of the said three persons; and $N + a + b$ the number of years in the age of the oldest of the said three persons; and E the greatest number of years through which it is supposed to be possible for human life to be extended, according to the table of probabilities of the duration of human life adopted for the calculation; which number is in Monsieur de Parcioux's table 94 years.

Let n be any number of years not greater than $E - N + a + b$, or $E - N - a - b$, or than the greatest number of years during which it is possible that the oldest of the three lives may be prolonged. And let an annuity of one pound *per annum* be granted for the term of n years, provided all the three persons aforesaid, of the ages of N years, $N + a$ years, and $N + a + b$ years, shall so long live, but otherwise to cease upon the

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death

A short expression of the value of an annuity of one pound a year depending on the joint continuance of three lives of given ages.

death of either of them. Let P be the number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life, (or in such other table of those probabilities as is thought proper by the calculator to be adopted as the ground of his calculation,) as being all living together at the said first, or youngest, age of N years; and P_i the number of persons represented in the said table to be living at the age of $N + i$ years; and P_{ii} the number living at the age of $N + 2$ years; and P_{iii} the number living at the age of $N + 3$ years; and P_{iv} , P_{v} , P_{vi} , P_{vii} , P_{viii} , P_{ix} , P_{x} , &c. the numbers living at the several following ages of $N + 4$ years, $N + 5$ years, $N + 6$ years, $N + 7$ years, $N + 8$ years, $N + 9$ years, $N + 10$ years, &c. respectively. And let Q be the number of persons represented in the said table as living at the second, or next older, age of $N + a$ years; and Q_i the number of persons represented there as living at the age of $N + a + i$ years; and Q_{ii} the number living at the age of $N + a + 2$ years; and Q_{iii} the number living at the age of $N + a + 3$ years; and Q_{iv} , Q_v , Q_{vi} , Q_{vii} , Q_{viii} , Q_{ix} , Q_x , &c. the numbers living at the several following ages of $N + a + 4$ years, $N + a + 5$ years, $N + a + 6$ years, $N + a + 7$ years, $N + a + 8$ years, $N + a + 9$ years, $N + a + 10$ years, &c. respectively. And let R be the number of persons represented in the said table as living at the third, or oldest, age of $N + a + b$ years; and R_i the number of persons represented there as living at the age of $N + a + b + i$ years; and R_{ii} the number living at the age of $N + a + b + 2$ years; and R_{iii} the number of persons living at the age of $N + a + b + 3$ years; and R_{iv} , R_v , R_{vi} , R_{vii} , R_{viii} , R_{ix} , R_x , &c. the numbers living at the several following ages of $N + a + b + 4$ years, $N + a + b + 5$ years, $N + a + b + 6$ years, $N + a + b + 7$ years, $N + a + b + 8$ years, $N + a + b + 9$ years, $N + a + b + 10$ years, &c. respectively.

These things being supposed, the present value of an annuity of one pound a year, to be enjoyed during the space of n years, in case all the said three lives, of the ages of N years, $N + a$ years, and $N + a + b$ years

shall so long continue, will be equal to the expression, $i \times$ the series

$$\frac{P_i \times Q_i \times R_i}{P \times Q \times R} + \frac{P_{ii} \times Q_{ii} \times R_{ii}}{P \times Q \times R} + \frac{P_{iii} \times Q_{iii} \times R_{iii}}{P \times Q \times R} + \frac{P_{iv} \times Q_{iv} \times R_{iv}}{P \times Q \times R} + \frac{P_v \times Q_v \times R_v}{P \times Q \times R} + \frac{P_{vi} \times Q_{vi} \times R_{vi}}{P \times Q \times R} + \frac{P_{vii} \times Q_{vii} \times R_{vii}}{P \times Q \times R} + \frac{P_{viii} \times Q_{viii} \times R_{viii}}{P \times Q \times R} + \frac{P_{ix} \times Q_{ix} \times R_{ix}}{P \times Q \times R} + \frac{P_x \times Q_x \times R_x}{P \times Q \times R} + \text{&c. continued to } n \text{ terms, or to the term}$$

$$\frac{P_n \times Q_n \times R_n}{P \times Q \times R} \text{ or equal to the expression, } \frac{f_1}{P \times Q \times R} + \text{the series}$$

$$\frac{P^i \times Q^i \times R^i}{r} + \frac{P^{ii} \times Q^{ii} \times R^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii} \times R^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv} \times R^{iv}}{r^4} \\ + \frac{P^v \times Q^v \times R^v}{r^5} + \frac{P^{vi} \times Q^{vi} \times R^{vi}}{r^6} + \frac{P^{vii} \times Q^{vii} \times R^{vii}}{r^7} + \text{\&c. con-}$$

tinued to n terms, or to the term $\frac{P^n \times Q^n \times R^n}{r^n}$, or (dropping, for the sake

of brevity, the marks $\times$ of the several multiplications,) equal to the ex-

pression $\frac{P^i}{P} \times$ the series $\frac{Q^i R^i}{Q} + \frac{Q^{ii} R^{ii}}{Q^2} + \frac{Q^{iii} R^{iii}}{Q^3} + \frac{Q^{iv} R^{iv}}{Q^4} + \frac{Q^v R^v}{Q^5} + \frac{Q^{vi} R^{vi}}{Q^6} + \frac{Q^{vii} R^{vii}}{Q^7} +$

continued to n terms, or to the term $\frac{P^n Q^n R^n}{r^n}$. This is evident from

Prob. III, and its 6th and 7th Corollaries, Art. LIII, LIV, LV, LVI, pages 53, 54, 55, 56, 57.

CCCCXXII. If n years is the greatest number of years through which it is possible (according to the table of the probabilities of the duration of human life adopted in the calculation,) for the oldest of the three given lives, or the life of the age of $N + a + b$ years, to be extended; or, in other words, if n is equal to $E - N - a - b$, the said expression $\frac{P^i}{P} \times$ the series $\frac{Q^i R^i}{Q} + \frac{Q^{ii} R^{ii}}{Q^2} + \frac{Q^{iii} R^{iii}}{Q^3} + \frac{Q^{iv} R^{iv}}{Q^4} + \frac{Q^v R^v}{Q^5} + \frac{Q^{vi} R^{vi}}{Q^6} + \frac{Q^{vii} R^{vii}}{Q^7} + \text{\&c.}$

continued to n terms, or to the term $\frac{P^n Q^n R^n}{r^n}$, (which term in this case

will be $\frac{P E - N - a - b \times Q E - N - a - b \times R E - N - a - b}{r^n}$) will be the

value of an annuity of one pound *per annum* for the whole joint continuance of the three given lives of the ages of N years, $N + a$ years, and $N + a + b$ years. But, if n is less than $E - N - a - b$, or the complement of $N + a + b$ (the number of years in the age of the oldest life) to E , or to the utmost possible duration of human life, the said expression will be less than the value of an annuity of one pound a year for the whole joint continuance of the said three lives, of the ages of N years, $N + a$ years, and $N + a + b$ years, and will be the value of an immediate, but imperfect,

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imperfect, life-annuity of one pound *per annum* during n years of the joint continuance of the said three lives

An example of the computation of the value of an immediate and complete life-annuity of one pound per annum for the whole joint continuance of the lives of three persons of given ages, by means of the foregoing expression.

CCCCCLXXIII. Let it be required to find the value of an annuity of one pound *per annum* for the whole joint continuance of the lives of three persons of the ages of 75 years, 80 years, and 85 years, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and upon a supposition that the interest of money is 3 per cent.

Here n , or the number of years through which the annuity is to continue, in case all the three lives (of which the oldest is of the age of 85 years,) shall last so long, is the greatest possible number of years through which, according to Monsieur de Parcieux's table, a life of the age of 85 years can be extended, that is (94 — 85 years, or) 9 years. Therefore the series $\frac{P^I Q^I R^I}{r} + \frac{P^{II} Q^{II} R^{II}}{r^2} + \frac{P^{III} Q^{III} R^{III}}{r^3} + \frac{P^{IV} Q^{IV} R^{IV}}{r^4} + \dots$ in the foregoing expression, must be continued to 9 terms; which terms may be computed as follows.

Here P is = 211, P^I is = 192, P^{II} = 173, P^{III} = 154, P^{IV} = 136, P^V = 118, P^{VI} = 101, P^{VII} = 85, P^{VIII} = 71, and P^{IX} = 59; and Q is = 118, Q^I = 101, Q^{II} = 85, Q^{III} = 71, Q^{IV} = 59, Q^V = 48, Q^{VI} = 38, Q^{VII} = 29, Q^{VIII} = 22, and Q^{IX} = 16; and R is = 48, R^I = 38, R^{II} = 29, R^{III} = 22, R^{IV} = 16, R^V = 11, R^{VI} = 7, R^{VII} = 4, R^{VIII} = 2, and R^{IX} = 1. And r is = 1.03, and $\frac{1}{r} = \frac{1}{1.03} = .9708$, and $\frac{1}{r^2} = .9425$, and $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, and $\frac{1}{r^9} = .7664$. Therefore the expression

LIFE-ANNUITIES.

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$$\begin{aligned}
 & \frac{A^1}{p^2 R} \times \text{the series } \frac{P^1 Q^1 R^1}{r} + \frac{P^{II} Q^{II} R^{II}}{r^2} + \frac{P^{III} Q^{III} R^{III}}{r^3} + \frac{P^{IV} Q^{IV} R^{IV}}{r^4} + \frac{P^V Q^V R^V}{r^5} + \frac{P^{VI} Q^{VI} R^{VI}}{r^6} + \frac{P^{VII} Q^{VII} R^{VII}}{r^7} + \frac{P^{VIII} Q^{VIII} R^{VIII}}{r^8} \\
 & + \frac{P^{IX} Q^{IX} R^{IX}}{r^9} \text{ will be equal to } \frac{A^1}{211 \times 118 \times 48} \times \text{the series} \\
 & 192 \times 101 \times 38 \times .9708 \\
 & + 173 \times 85 \times 29 \times .9425 \\
 & + 154 \times 71 \times 22 \times .9151 \\
 & + 136 \times 59 \times 16 \times .8884 \\
 & + 118 \times 48 \times 11 \times .8626 \\
 & + 101 \times 38 \times 7 \times .8374 \\
 & + 85 \times 29 \times 4 \times .8130 \\
 & + 71 \times 22 \times 2 \times .7894 \\
 & + 59 \times 16 \times 1 \times .7664 \\
 & = \frac{A^1}{24,898 \times 48} \times \text{the series} \\
 & 19392 \times 38 \times 19708 \\
 & + 14705 \times 29 \times 9425 \\
 & + 10934 \times 22 \times 9151 \\
 & + 8024 \times 16 \times 8884 \\
 & + 5664 \times 11 \times 8626 \\
 & + 3838 \times 7 \times 8374 \\
 & + 2465 \times 4 \times 8130 \\
 & + 1562 \times 2 \times 7894 \\
 & + 944 \times 1 \times 7664 \\
 & = \frac{A^1}{24,898 \times 48} \times \text{the series} \\
 & 38 \times 18,825.7536 \\
 & + 29 \times 13,859.4625 \\
 & + 22 \times 10,005.7034 \\
 & + 16 \times 7,288.4336 \\
 & + 11 \times 4,885.7664 \\
 & + 7 \times 3,213.9412
 \end{aligned}$$

$$\begin{aligned}
 &+ 4 \times 2004.0450 \\
 &+ 2 \times 1233.0428 \\
 &+ 1 \times 723.4816 \\
 &= \frac{\pounds 1}{1,195,104} \times \text{the series} \\
 &\quad 715,378.6368 \\
 &+ 401,924.4125 \\
 &+ 220,125.4748 \\
 &+ 116,614.9376 \\
 &+ 53,743.4304 \\
 &+ 22,497.5884 \\
 &+ 8,016.1800 \\
 &+ 2,466.0856 \\
 &+ 723.4816
 \end{aligned}$$

$$= \frac{\pounds 1}{1,195,104} \times 1,541,490.2277 = \frac{1,541,490.2277}{1,195,104} = \pounds 1.2898. \text{ There-}$$

fore the value of an annuity of one pound a year for the whole joint continuance of three lives of the ages of 75 years, 80 years, and 85 years, when the interest of money is 3 Per cent. is $\pounds 1.2898$, or $\pounds 1 \text{ sh. } 9\frac{1}{4} \text{ d.}$

Q E L

S C H O L I U M.

CCCCCLXXIV. When three, or more, lives are combined together, the differences that may be taken between their several ages are so immensely numerous that it is totally impracticable to form tables that shall exhibit the values of all, or even any considerable part of, the several annuities that may be supposed to depend on them. And consequently, whenever the values of such annuities are wanted, it will be necessary to take the trouble of computing them.

And, if the accurate value of such an annuity is wanted, I believe there is no other way of finding it but that which has been above set forth by computing the expression, $\frac{\pounds 1}{P \text{ Q R}} \times \text{the series } \frac{P^1 \text{ Q}^1 \text{ R}^1}{r} + \frac{P^{11} \text{ Q}^{11} \text{ R}^{11}}{r^2}$

$$\frac{P_{III} Q_{III} R_{III}}{r^3} + \frac{P_{IV} Q_{IV} R_{IV}}{r^4} + \frac{P_V Q_V R_V}{r^5} + \frac{P_{VI} Q_{VI} R_{VI}}{r^6} + \frac{P_{VII} Q_{VII} R_{VII}}{r^7} + \&c. \text{ continued to the end of the table of probabilities}$$

of the duration of human life; as was done in the foregoing example. But this method of obtaining it is in all cases rather tedious, even when the oldest life is a very old one: but when the three lives are all young, the computation is so very long and troublesome that few persons will, probably, care to undertake it. It seems therefore to be highly expedient to endeavour to find out some less difficult method of obtaining the value of an annuity of this kind by a tolerable approximation. Now this may be done to a moderate degree of exactness by a method analogous to that explained in Art. cccclxvi, cccclxvii, cccclxviii, cccclxix, whereby the value of an annuity of one pound a year for two joint lives of given ages was derived from the value of the same annuity for the older of the two single lives by means of the expression $A \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{II}}$. For we shall

find, upon examination, that the value of an annuity of one pound a year for three joint lives of given ages may be derived in the like manner from the value of the same annuity for the joint continuance of the two oldest lives, by means of an expression exactly similar to the aforesaid expression $A \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{II}}$. For if B is the value of an annuity of one pound a year for the joint continuance of the two oldest of the three lives, and g is $= \frac{Q_1 R_1}{r}$, and $b = \frac{Q_{II} R_{II}}{r}$, the value of an annuity of one pound a year for the joint continuance of all the three lives will be, nearly, equal to $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{II}}$; as may be shewn in the manner following.

A method

A method of deriving the value of an annuity of one pound a year for the joint continuance of three lives of any given ages, from the value of the same annuity for the joint continuance of the two older of the said lives, by approximation.

CCCCCLXXV. Let S be put for the series $\frac{Q^i R^i}{r} + \frac{Q^{ii} R^{ii}}{r^2} + \frac{Q^{iii} R^{iii}}{r^3} + \frac{Q^{iv} R^{iv}}{r^4} + \frac{Q^{v} R^v}{r^5} + \frac{Q^{vi} R^{vi}}{r^6} + \frac{Q^{vii} R^{vii}}{r^7} + \&c.$ continued to the end of the table of the probabilities of the duration of human life. And let the Greek capital letter Σ be put for the series $\frac{P^i Q^i R^i}{r} + \frac{P^{ii} Q^{ii} R^{ii}}{r^2} + \frac{P^{iii} Q^{iii} R^{iii}}{r^3} + \frac{P^{iv} Q^{iv} R^{iv}}{r^4} + \frac{P^{v} Q^v R^v}{r^5} + \frac{P^{vi} Q^{vi} R^{vi}}{r^6} + \frac{P^{vii} Q^{vii} R^{vii}}{r^7} + \&c.$ continued likewise to the end of the same table of probabilities.

Then it is evident that B , or the value of an annuity of one pound a year for the joint continuance of the two older lives (which answer to the letters Q and R), will be $= \frac{f^i}{Q R} \times S$; and that C , or the value of the same annuity for the joint continuance of all the three lives, will be $= \frac{f^i}{P Q R} \times \Sigma$.

Let i be put $= \frac{P^i Q^i R^i}{r}$, and k be put $= \frac{P^{ii} Q^{ii} R^{ii}}{r^2}$.

Then, since g was taken $= \frac{Q^i R^i}{r}$, and b was taken $= \frac{Q^{ii} R^{ii}}{r^2}$, we shall have $i = g \times P^i$, and $k = b \times P^{ii}$.

Let G be put for the infinite geometrical progression $g + b + \frac{b b}{g} +$

$\frac{b^3}{g^3} + \frac{b^4}{g^3} + \frac{b^5}{g^4} + \frac{b^6}{g^5} + \&c.$ of which the two first terms, g and b , are respectively equal to, $\frac{Q^1 R^1}{r}$ and $\frac{Q^{11} R^{11}}{r^2}$, the two first terms of the series S . And let the Greek capital letter Γ be put for the infinite geometrical progression $i + k + \frac{k^2}{i} + \frac{k^3}{i^2} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5} + \&c.$ of which the two first terms, i and k , are respectively equal to, $\frac{P^1 Q^1 R^1}{r}$ and $\frac{P^{11} Q^{11} R^{11}}{r^2}$, the two first terms of the series Z .

Then will G be $= \frac{g^g}{g-b}$; and Γ will be $= \frac{i^i}{i-k}$.

CCCCCLXXVI. These things being premised, it seems reasonable to conjecture that the geometrical progression $g + b + \frac{b^2}{g} + \frac{b^3}{g^2} + \frac{b^4}{g^3} + \frac{b^5}{g^4} + \&c.$ A conjectural supposition upon which this method of approximation is founded.

$\frac{b^6}{g^5} + \frac{b^7}{g^4} + \&c.$ *ad infinitum*, or G , will bear pretty nearly the same proportion to its kindred series S , (of which the two first terms, $\frac{Q^1 R^1}{r}$ and

$\frac{Q^{11} R^{11}}{r^2}$, are respectively equal to its two first terms g and b), as the geometrical progression $i + k + \frac{k^2}{i} + \frac{k^3}{i^2} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5} + \&c.$ *ad infinitum* bears to its kindred series Z , of which the two first terms, $\frac{P^1 Q^1 R^1}{r}$ and $\frac{P^{11} Q^{11} R^{11}}{r^2}$, are respectively equal to its two first terms,

i and k . And, if this conjecture is well-founded, we shall have Z nearly

$$\frac{\Gamma \times S}{G}, \text{ and consequently nearly } = \frac{\frac{i^i}{g-b} \times S}{\frac{g^g}{g-b}} = \frac{i^i}{i-k} \times S \times \frac{g-b}{g^g} =$$

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$$\begin{aligned} & \frac{g \times P_i \times g \times P_i}{g \times P_i - b \times P_{ii}} \times S \times \frac{g-b}{gg} = \frac{gg \times P_i \times P_i}{g \times P_i - b \times P_{ii}} \times S \times \frac{g-b}{gg} = \\ & \frac{P_i \times P_i}{g \times P_i - b \times P_{ii}} \times S \times \frac{g-b}{g} = \frac{P_i}{g \times P_i - b \times P_{ii}} \times S \times \frac{g-b}{g} \\ & \times P_i = \frac{P_i}{g \times P_i - b \times P_{ii}} \times S \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}} = P_i \times S \\ & \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}. \end{aligned}$$

But B is $= \frac{f_i}{2R} \times S$, and consequently S is $= B \times 2R$

Therefore $P_i \times S \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$ is $= P_i \times B \times 2R \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$; and consequently Σ is, nearly, $= P_i \times B \times 2R \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$.

Therefore C , or $\frac{f_i}{P \times 2R} \times \Sigma$, is nearly $= \frac{f_i}{P \times 2R} \times P_i \times B \times 2R \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}} = \frac{1}{P} \times P_i \times B \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}} = B \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$; that is, the value of an annuity of one pound a year for the joint continuance of all the three lives will be, nearly, equal to $B \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$; or may be derived from B , the value of the same annuity for the joint continuance of the two older lives, by multiplying it into the fraction $\frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$. Q.E.D.

CCCCCLXXVII. This expression $B \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$ will,

I believe, be almost always less than the true value of C . But in what de-

prob., or within what *limits*, it will differ from it, I do not know. But, that we may form some tolerable conjecture upon the subject, I will now proceed to apply this expression to the computation of the values of a few annuities for three joint lives, of which the learned Mr. Morgan has given us the true values to five places of figures, computed strictly by means of the

above-mentioned expression, $\frac{f_1}{P} \times$ the series $\frac{P^1 Q^1 R^1}{r} + \frac{P^{11} Q^{11} R^{11}}{r^2} + \frac{P^{111} Q^{111} R^{111}}{r^3} + \frac{P^{1111} Q^{1111} R^{1111}}{r^4} + \frac{P^{11111} Q^{11111} R^{11111}}{r^5} + \frac{P^{111111} Q^{111111} R^{111111}}{r^6} + \dots$ continued to the end of the table of the proba-

bilities of the duration of human life. These true values are contained in the seventh table of Mr. Morgan's treatise on the doctrine of annuities, page 273. They are the values of an annuity of one pound a year for three joint lives of equal ages, from the age of 60 years to the age of 99 years, inclusive of both. And they are computed from the Northampton table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4 per cent. This table is as fol-

TABLE

T A B L E XLIX.

Containing the true values of an annuity of one pound a year for the joint continuance of three lives of equal ages, from the age of 60 years to the age of 91 years, both included; when the interest of money is 4 per cent.

Computed from the Northampton table of the probabilities of the duration of human life.

Year, in the age of the first life.	Years in the age of the second life.	Years in the age of the third life.	Values of an annuity of 1l. a year for the joint continuance of all the three lives.	Years in the age of the first life.	Years in the age of the second life.	Years in the age of the third life.	Values of an annuity of 1l. a year for the joint continuance of all the three lives.
60	60	60	4.7826	76	76	76	1.9089
61	61	61	4.6115	77	77	77	1.7846
62	62	62	4.4382	78	78	78	1.5843
63	63	63	4.2626	79	79	79	1.3906
64	64	64	4.0849	80	80	80	1.2121
65	65	65	3.9050	81	81	81	1.0685
66	66	66	3.7230	82	82	82	1.0117
67	67	67	3.5390	83	83	83	0.9617
68	68	68	3.3533	84	84	84	0.8981
69	69	69	3.1662	85	85	85	0.7906
70	70	70	2.9780	86	86	86	0.7090
71	71	71	2.7895	87	87	87	0.7568
72	72	72	2.6015	88	88	88	0.5368
73	73	73	2.4160	89	89	89	0.3233
74	74	74	2.2352	90	90	90	0.1346
75	75	75	2.0636	91	91	91	0.1202

CCCCXXVIII. Mr. Morgan has also given us, in his valuable treatise on annuities before-mentioned, pages 74, 75, and 76, a complete table of the true values of an annuity of one pound a year for the joint continuance of two lives of equal ages, when the interest of money is 4 per cent. computed strictly from the aforesaid Northampton table of the probabilities of the duration of human life, for every age of life from the age of one year to that of 91 years, inclusively. This table is as follows.

T A B L E L.

Containing the true values of an annuity of one pound a year for the joint continuance of two lives of equal ages, from the age of one year to the age of 91 years, inclusively; when the interest of money is 4 per cent.

Computed from the Northampton table of the probabilities of the duration of human life.

Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of 1l. a year for the joint con- tinuance of both lives.	Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of 1l. a year for the joint con- tinuance of both lives.
1	1	10.5432	14	14	13.5638
2	2	11.3227	15	15	13.3673
3	3	12.5931	16	16	13.1616
4	4	13.1723	17	17	12.9461
5	5	13.6352	18	18	12.7201
6	6	13.9355	19	19	12.5346
7	7	14.1320	20	20	12.3928
8	8	14.2031	21	21	12.2985
9	9	14.2356	22	22	12.2038
10	10	14.1709	23	23	12.1089
11	11	14.0530	24	24	12.0139
12	12	13.9311	25	25	11.9191
13	13	13.7516	26	26	11.8242

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Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of 1l. a year for the joint con- tinuance of both lives.	Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of 1l. a year for the joint con- tinuance of both lives.
27	27	11.7292	60	60	6.2468
28	28	11.6351	61	61	6.0407
29	29	11.5422	62	62	5.8313
30	30	11.4500	63	63	5.6188
31	31	11.3007	64	64	5.4031
32	32	11.1462	65	65	5.1843
33	33	10.9880	66	66	4.9626
34	34	10.8240	67	67	4.7381
35	35	10.6538	68	68	4.5111
36	36	10.4782	69	69	4.2821
37	37	10.2962	70	70	4.0516
38	38	10.1073	71	71	3.8203
39	39	9.9110	72	72	3.5893
40	40	9.7065	73	73	3.3605
41	41	9.5532	74	74	3.1364
42	42	9.3960	75	75	2.9192
43	43	9.2358	76	76	2.7182
44	44	9.0723	77	77	2.5427
45	45	8.9054	78	78	2.2976
46	46	8.7350	79	79	2.0694
47	47	8.5608	80	80	1.8570
48	48	8.3829	81	81	1.6868
49	49	8.2008	82	82	1.5998
50	50	8.0779	83	83	1.5144
51	51	7.9599	84	84	1.4143
52	52	7.7818	85	85	1.244,62
53	53	7.6006	86	86	1.1213,51
54	54	7.4165	87	87	1.132,87
55	55	7.2294	88	88	0.840,92
56	56	7.0392	89	89	0.554,77
57	57	6.8459	90	90	0.298,169
58	58	6.6494	91	91	0.240,383
59	59	6.4497			

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CCCCXXIX. And the Northampton table of the probabilities of the duration of human life, from which the two foregoing tables of the values of an annuity of one pound for two and three joint lives were computed, is given us likewise in Mr. Morgan's afore said treatise on annuities, page 167, and is as follows.

T A B L E

Representing the probabilities of the duration of human life at the several ages therein mentioned, from the age of one year to the age of 92 years, inclusively; as deduced by Dr. Price from observations on the bills of mortality at Northampton.

Age.	Persons living.	Age.	Persons living.	Age.	Persons living.
1 year.	849	25	475	49	73
2 years.	722	26	467	50	74
3	672	27	459	51	75
4	646	28	451	52	76
5	625	29	443	53	77
6	609	30	435	54	78
7	596	31	428	55	79
8	586	32	421	56	80
9	577	33	414	57	81
10	570	34	407	58	82
11	564	35	400	59	83
12	558	36	393	60	84
13	553	37	386	61	85
14	548	38	379	62	86
15	543	39	372	63	87
16	538	40	365	64	88
17	533	41	357	65	89
18	528	42	349	66	90
19	522	43	341	67	91
20	515	44	333	68	92
21	507	45	325	69	93
22	499	46	317	70	
23	491	47	309	71	
24	483	48	301	72	

And

CCCCCLXXX. By the help of these two last tables we may apply the expression $B \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$ to the discovery of near values of the several annuities for three joint lives of equal ages, of which we have seen the true values above in Table XLIX, Art. cccclxxvii. This may be done in the manner following.

CCCCCLXXXI. When the three lives are all of the same age, $\mathcal{Q}$ and R will be equal to P , and $\mathcal{Q}^i$ and R^i will be equal to P^i , and $\mathcal{Q}^{ii}$ and R^{ii} will be equal to P^{ii} . Therefore in this case g , or $\frac{\mathcal{Q}^i R^i}{r}$, will be

$$\frac{P_i P_i}{r}, \text{ and } b, \text{ or } \frac{\mathcal{Q}^{ii} R^{ii}}{r^2}, \text{ will be } = \frac{P^{ii} P^{ii}}{r^2}; \text{ and consequently } g$$

$$P_i \text{ will be } = \frac{P_i P_i}{r} \times P_i = \frac{P_i \times P_i \times P_i}{r}; \text{ and } b \times P_i \text{ will be } =$$

$$\frac{P^{ii} P^{ii}}{r^2} \times P_i = \frac{P^{ii} \times P^{ii} \times P_i}{r^2}; \text{ and } b \times P^{ii} \text{ will be } = \frac{P^{ii} P^{ii}}{r^2}$$

$$P^{ii} = \frac{P^{ii} \times P^{ii} \times P^{ii}}{r^2}. \text{ Therefore } g \times P_i - b \times P_i \text{ will be } =$$

$$\frac{P_i \times P_i \times P_i}{r} - \frac{P^{ii} \times P^{ii} \times P_i}{r^2} = \frac{P_i \times P_i \times P_i \times r}{r^2}$$

$$\frac{P^{ii} \times P^{ii} \times P_i}{r^2} = \frac{P_i \times P_i \times P_i \times r - P^{ii} \times P^{ii} \times P_i}{r^2}; \text{ and } g \times P$$

$$- b \times P^{ii} \text{ will be } = \frac{P_i \times P_i \times P_i}{r} - \frac{P^{ii} \times P^{ii} \times P^{ii}}{r^2} = \frac{P_i \times P_i \times P_i}{r^2}$$

$$- \frac{P^{ii} \times P^{ii} \times P^{ii}}{r^2} = \frac{P_i \times P_i \times P_i \times r - P^{ii} \times P^{ii} \times P^{ii}}{r^2}; \text{ and con}$$

$$\text{sequently } \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P^{ii}} \text{ will be } = \frac{P_i \times P_i \times P_i \times r - P^{ii} \times P^{ii} \times P^{ii}}{P_i \times P_i \times P_i \times r - P^{ii} \times P^{ii} \times P^{ii}}$$

$$\text{will be } = \frac{P_i \times P_i \times P_i \times r - P^{ii} \times P^{ii} \times P^{ii}}{P_i \times P_i \times P_i \times r - P^{ii} \times P^{ii} \times P^{ii}}$$

$$= \frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}} \quad \text{Therefore } B \times \frac{P_i}{P} \times$$

$$\frac{\ell \times P_i - b \times P_i}{\ell \times P_i - b \times P_{ii}} \quad \text{or the near value of an annuity of one pound a year}$$

for the three joint lives, will, in this case of an equality of the three ages,

$$\text{be} = B \times \frac{P_i}{P} \times \frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}} \quad \text{which I}$$

take to be the most convenient form to which this expression can be reduced.

An example of the computation of the near value of an annuity of one pound a year for the joint continuance of three lives of equal ages, by means of the foregoing expression.

CCCCCLXXXII. Let it now be required, by means of this expression, $B \times \frac{P_i}{P} \times \frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}}$ to find a near value of an annuity of one pound a year for three joint lives all of the age of 60 years, when the interest of money is 4 per cent. according to the foregoing Northampton table of the probabilities of the duration of human life; of which annuity we have seen above, in Table XLIX, that the true value is £4.7826.

Now it appears from Table L, Art. cccclxxxviii, that B , or the value of an annuity of one pound a year for the joint continuance of two lives that are both of the same age of 60 years, when the interest of money is 4 per cent. and according to the Northampton table of the probabilities of the duration of human life, is £6.2468. And it appears by the said table of probabilities, that the number of persons therein supposed to be living at the several ages of 60 years, 61 years, and 62 years, are 203, 195, and 187. Therefore $P_i = 203$, $P_i = 195$, and $P_{ii} = 187$.

Also, since the interest of money is supposed to be 4 per cent. r will be = 1.04.

Therefore $P_i \times P_i \times P_i \times r$ will be (= $195 \times 195 \times 195 \times 1.04 = 7,711,470.00$; and $P_{ii} \times P_{ii} \times P_i$ will be (= $187 \times 187 \times 195 = 6,818,955$; and $P_{ii} \times P_{ii} \times P_{ii}$ will be (= $187 \times 187 \times 187 = 6,539,203$. Therefore $P_i \times$
E e e e

$P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i$ will be $(= 7,711,470 - 6,818,955) = 892,515$; and $P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}$ will be $(= 7,711,470 - 6,539,203) = 1,172,267$; and consequently $\frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}}$ will be $= \frac{892,515}{1,172,267} = 0.761,35$.

Therefore $\frac{P_i}{P} \times$ the fraction $\frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}}$

will be $(= \frac{P_i}{P} \times 0.761,35 = \frac{195}{203} \times 0.761,35 = \frac{148.46325}{203}) = 0.731,$

34; and $B \times \frac{P_i}{P} \times \frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}}$ will be $(=$

$B \times 0.731,34 = 6.2468 \times 0.731,34) = 4.5685$. Therefore 4.5685 is a near value of the proposed annuity of one pound a year for the joint continuance of three lives, all of the age of 60 years, when the interest of money is 4 per cent. according to the Northampton table of the probabilities of the duration of human life. Q E I.

This value of the proposed annuity is less than its true value £4.7826, (given above in Table XLIX,) by the difference £0.2141, which is about a 22d part of the said true value, £4.7826.

CCCCLXXXIII. If the values of an annuity of one pound a year for the joint continuance of three equal lives of the ages of 61 years, 62 years, 63 years, 64 years, &c. to the age of 91 years, inclusively, are computed in the same manner by means of the said expression, $B \times \frac{P_i}{P} \times \frac{P_i \times P_i - b \times P_i}{P_i \times P_i - b \times P_{ii}}$ or $B \times \frac{P_i}{P} \times \frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}}$ computed from the same expression, they will be found to be as follows; to wit.

for three equal joint lives of the age of 61 years, = 4.3992,
 of the age of 62 years, = 4.2281,
 of the age of 63 years, = 4.0555,
 of the age of 64 years, = 3.8814,
 of the age of 65 years, = 3.7059,

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of the age of 66 years, =	3.5293,
of the age of 67 years, =	3.3516,
of the age of 68 years, =	3.1730,
of the age of 69 years, =	2.9942,
of the age of 70 years, =	2.8153,
of the age of 71 years, =	2.6370,
of the age of 72 years, =	2.4600,
of the age of 73 years, =	2.2857,
of the age of 74 years, =	2.1156,
of the age of 75 years, =	1.9513,
of the age of 76 years, =	1.7961,
of the age of 77 years, =	1.6861,
of the age of 78 years, =	1.5067,
of the age of 79 years, =	1.3398,
of the age of 80 years, =	1.1842,
of the age of 81 years, =	1.0439,
of the age of 82 years, =	0.98201,
of the age of 83 years, =	0.92350,
of the age of 84 years, =	0.87917,
of the age of 85 years, =	0.72954,
of the age of 86 years, =	0.72759,
of the age of 87 years, =	0.69995,
of the age of 88 years, =	0.50505,
of the age of 89 years, =	0.31932,
of the age of 90 years, =	0.12872,
and of the age of 91 years, =	0.12019.

These near values, we may observe, are all less than the true values of the same annuities, given above in Table XLIX. And this will be found to be the case in most instances, except when some of the lives are very young; and then it sometimes happens that the near values of an annuity for three joint lives, obtained by the foregoing expression, is greater than its true value.

The differences of the foregoing near values of an annuity of one pound a year for three joint lives and the corresponding true values of the same annuity.

CCCCXXXIV. The differences of these near values of the said annuities for three equal joint lives from their true values contained above in Table XLIX, are as follows; to wit,

£.	—	£.	=	£.
4.6115	—	4.3992	=	0.2123 ;
4.4382	—	4.2281	=	0.2101 ;
4.2626	—	4.0555	=	0.2071 ;
4.0849	—	3.8814	=	0.2035 ;
3.9050	—	3.7059	=	0.1991 ;
3.7230	—	3.5293	=	0.1937 ;
3.5390	—	3.3516	=	0.1874 ;
3.3533	—	3.1730	=	0.1803 ;
3.1662	—	2.9942	=	0.1720 ;
2.9780	—	2.8153	=	0.1627 ;
2.7895	—	2.6370	=	0.1525 ;
2.6015	—	2.4600	=	0.1415 ;
2.4160	—	2.2857	=	0.1303 ;
2.2352	—	2.1156	=	0.1196 ;
2.0636	—	1.9513	=	0.1123 ;
1.9089	—	1.7961	=	0.1128 ;
1.7846	—	1.6861	=	0.0985 ;
1.5843	—	1.5067	=	0.0776 ;
1.3906	—	1.3398	=	0.0508 ;
1.2121	—	1.1842	=	0.0279 ;
1.0685	—	1.0439	=	0.0246 ;
1.0117	—	0.982,01	=	0.029,69 ;
0.9617	—	0.923,50	=	0.038,20 ;
0.8981	—	0.879,17	=	0.078,93 ;
0.7606	—	0.729,54	=	0.031,06 ;
0.7690	—	0.727,59	=	0.041,41 ;
0.7568	—	0.699,95	=	0.056,85 ;
0.5368	—	0.505,05	=	0.031,75 ;

$$\begin{aligned} 0.3233 & - 0.319,32 = 0.003,98; \\ 0.1346 & - 0.128,72 = 0.005,88; \\ 0.1202 & - 0.120,19 = 0.000,01. \end{aligned}$$

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CCCCXXXV. And the proportions of these differences to the said true values themselves are expressed by the following fractions, to wit,

$$\begin{array}{cccccccccccc} \frac{1}{11}, & \frac{1}{21}, & \frac{1}{20}, & \frac{1}{20}, & \frac{1}{19}, & \frac{1}{19}, & \frac{1}{18}, & \frac{1}{18}, & \frac{1}{18}, & \frac{1}{18}, & \frac{1}{18}, & \frac{1}{18}, \\ \frac{1}{18}, & \frac{1}{18}, & \frac{1}{20}, & \frac{1}{20}, & \frac{1}{27}, & \frac{1}{43}, & \frac{1}{43}, & \frac{1}{34}, & \frac{1}{25}, & \frac{1}{11}, & \frac{1}{24}, & \frac{1}{18}, \\ \frac{1}{13}, & \frac{1}{16}, & \frac{1}{81}, & \frac{1}{22}, & \text{and} & \frac{1}{12020}. \end{array}$$

The proportions of the foregoing differences to the said several true values, respectively.

These fractions (though they are not so small as might have been wished,) are sufficiently small to make the foregoing near values of an annuity of one pound a year for three equal joint lives of the ages of 60 years, 61 years, 62 years, &c. to the end of life, of very considerable use in practice. And therefore we may conclude from them that, in estimating annuities for three equal joint lives in the latter period of human life, after the age of 60 years, the expression

$$\frac{P_1}{P} \times \frac{P_1 \times P_1 \times P_1 \times r - P_{11} \times P_{11} \times P_1}{P_1 \times P_1 \times P_1 \times r - P_{11} \times P_{11} \times P_{11}}, \quad \text{or} \quad B \times \frac{P_1}{P} \times \frac{P_1}{P} \times \frac{P_1}{P}$$

A conclusion from the foregoing examples in favour of the usefulness of the foregoing expression, when the ages of the three lives are equal to each other and are not less than 60 years.

may safely be adopted, as a means of determining the values of such annuities to a tolerable degree of exactness.

CCCCCLXXXVI. But it may be doubted, perhaps, whether this expression will be equally useful in determining the values of annuities for three joint lives, when the ages of the lives are younger than 60 years, and when the three lives are not all of equal ages. Now, in order to form some judgment upon this matter, it will be necessary to try this approximation in some other instances, in which the three lives shall be much younger than 60 years of age, and likewise in some instances in which the three lives shall not be all of equal ages. And with this view I shall now present the reader

An inquiry into the usefulness of the said expression, in other cases.

reader with four more tables of values of annuities for three joint lives, of which the two former contain the accurate value of an annuity of one pound a year for three equal joint lives of the ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35, 40, 45, 50, 55, 60, 65, and 70 years, and for three lives of unequal ages that differ from each other by 10 years and 20 years, and of which the youngest lives are of the ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35, 40, 45, 50, 55, 60, 65, 70, and 75 years, and the two latter contain the near values of the annuities mentioned in the two former tables, which result from the expression $B \times \frac{P_1}{P} \times$

$$\frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}.$$

The two first tables have been communicated to me by Dr. Price, and the two latter have been computed by an able arithmetician, who was employed for that purpose by Mr. Morgan, of the Society for equitable Assurances near Black-friars Bridge. In all the four tables the interest of money is supposed to be 4 per cent. and the table of the probabilities of the duration of human life, upon which the calculations are founded, is not that of Monsieur de Parcieux, (so often mentioned in the course of this work,) nor yet the Northampton table of those probabilities given above in Art. cccclxxix, but a new table derived from the said Northampton table by Dr. Price, and called by him "*The new Northampton Table of those probabilities*," being an improvement on the said former Northampton table, and (as I am informed,) differing but little from it. These four tables are as follows.

TABLE

T A B L E LII.

Containing the true values of an annuity of one pound a year for the joint continuance of the lives of three persons all of the same age, at the several ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35 years, 40 years, 45 years, 50 years, 55 years, 60 years, 65 years, and 70 years; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life.

Years in the age of the first life.	Years in the age of the second life.	Years in the age of the third life.	Values of an annuity of one pound a year for the joint continuance of the three lives.
5	5	5	£. 11.1704
10	10	10	12.2006
15	15	15	11.2746
20	20	20	10.3429
25	25	25	9.796,42
30	30	30	9.221,10
35	35	35	8.585,22
40	40	40	7.865,05
45	45	45	7.126,40
50	50	50	6.317,17
55	55	55	5.550,60
60	60	60	4.755,03
65	65	65	3.914,00
70	70	70	2.995,84

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T A B L E LIII.

Containing the true values of an annuity of one pound a year for the joint continuance of the lives of three persons of unequal ages that differ from each other by 10 years and 20 years, when the age of the youngest life is either 5 years, or 10 years, or 15 years, or 20 years, or 25 years, or 30 years, or 35, or 40, 45, 50, 55, 60, 65, 70, or 75 years; upon a supposition that the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life.

Years in the age of the first, or youngest, life.	Years in the age of the second life.	Years in the age of the third, or oldest, life.	Values of an annuity of one pound a year for the joint continuance of the three lives.
5	15	25	£. 10.6551
10	20	30	10.4379
15	25	35	9.738,56
20	30	40	8.986,72
25	35	45	8.313,10
30	40	50	7.570,83
35	45	55	6.816,07
40	50	60	5.994,15
45	55	65	5.145,62
50	60	70	4.219,37
55	65	75	3.297,98
60	70	80	2.408,48
65	75	85	1.623,48
70	80	90	1.122,51
75	85	95	0.169,378

T A B L E LIV.

Containing approximations to the values of the annuities for three equal joint lives mentioned above in Table LII; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by means of the expression $B \times \frac{P_1}{P} \times \frac{P_1 - b \times P_1}{P_1 - b \times P_{11}} \text{ or } B \times \frac{P_1}{P} \times \frac{P_1 \times P_1 \times P_1 \times P_1 \times r - P_{11} \times P_{11} \times P_1}{P_1 \times P_1 \times P_1 \times P_1 \times r - P_{11} \times P_{11} \times P_{11}}$ given above in Art. CCCCLXXVI and CCCCLXXXI.

Years in the age of the first life.	Years in the age of the second life.	Years in the age of the third life.	Value of an annuity of one pound a year for the joint continuance of the three lives.
5	5	5	£. 10.4912
10	10	10	12.3942
15	15	15	11.4322
20	20	20	10.2377
25	25	25	9.667,62
30	30	30	9.073,29
35	35	35	8.426,20
40	40	40	7.691,08
45	45	45	6.941,47
50	50	50	6.118,36
55	55	55	5.342,53
60	60	60	4.543,77
65	65	65	3.718,59
70	70	70	2.839,99

T A B L E LV.

Containing approximations to the values of the annuities for three joint lives of different ages, mentioned above in Table LIII; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by means of the expression $B^f \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{iD}}$ given above in Art.

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Years in the age of the first, or youngest, life.	Years in the age of the second life.	Years in the age of the third, or eldest, life.	Value of an annuity of one pound a year for the joint continuance of the three lives.
5	15	25	£. 9.1325
10	20	30	10.5080
15	25	35	9.858,45
20	30	40	8.756,27
25	35	45	8.068,36
30	40	50	7.343,65
35	45	55	6.594,61
40	50	60	5.786,88
45	55	65	4.947,33
50	60	70	4.044,10
55	65	75	3.181,57
60	70	80	2.332,36
65	75	85	1.604,14
70	80	90	1.093,25
75	85	95	0.169,378

CCCCXXXVII. The differences between the true values of the annuities for three equal joint lives, mentioned in Table LII, and the near values of the same annuities which are contained in Table LIV, are as follows; to wit,

£.		£.	£.
11.1704	—	10.4912	= 0.6792;
— 12.2006	+	12.3940	= 0.1936;
— 11.2746	+	11.4322	= 0.1576;
10.3429	—	10.2377	= 0.1052;
9.796,42	—	9.667,62	= 0.128,80;
9.221,10	—	9.073,29	= 0.147,81;
8.585,22	—	8.426,20	= 0.159,02;
7.865,05	—	7.691,08	= 0.173,97;
7.126,40	—	6.941,47	= 0.184,93;
6.317,17	—	6.118,36	= 0.198,81;
5.550,60	—	5.342,53	= 0.208,07;
4.755,03	—	4.543,77	= 0.211,26;
3.914,00	—	3.718,59	= 0.195,41;
2.995,84	—	2.839,99	= 0.155,85.

The differences of the true values of the annuities for three equal joint lives in Table LII and the near values of the same annuities in Table LIV.

CCCCXXXVIII. It may be here observed that all the foregoing near values of annuities for three equal joint lives, contained in Table

A comparison of the said near values with the corresponding true values.

LIV, and which were obtained by means of the expression $B \times \frac{P_1}{P} \times$

$\frac{P_1 \times P_1 \times P_1 \times r - P_n \times P_{n+1} \times P_i}{P_1 \times P_1 \times P_1 \times r - P_{n+1} \times P_{n+2} \times P_{i+1}}$ or $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_2}{g \times P_1 - b \times P_{n+1}}$

A remark on the magnitude of the said differences with respect to one year's annuity.

CCCCCLXXXIX. It may also be observed that all the foregoing differences between the true values and the near values of those annuities, except the first difference £0.6792, (which relates to the age of 5 years, and is much greater than any of the others,) are less than £0.25, or a quarter of a year's annuity; which, with a view to practical purposes, can hardly be considered as a very important error.

The proportions of the said differences to the several corresponding true values of the said annuities.

CCCCXC. The proportions of the foregoing differences to their several corresponding true values are expressed by the following fractions; to wit,

$\frac{1}{16}$, $\frac{1}{63}$, $\frac{1}{71}$, $\frac{1}{98}$, $\frac{1}{76}$, $\frac{1}{62}$, $\frac{1}{53}$, $\frac{1}{45}$, $\frac{1}{38}$, $\frac{1}{31}$, $\frac{1}{26}$, $\frac{1}{22}$, $\frac{1}{20}$, and $\frac{1}{19}$. These fractions are, for the most part, consi-

derably less than the fractions in Art. cccclxxv, which express the proportions of the differences of the near values and true values of the former set of annuities for three equal joint lives, (where the ages were 60 years and upwards,) to their corresponding true values. We may therefore con-

clude that the expression $B \times \frac{P_1}{P} \times \frac{P_1 \times P_1 \times P_1 \times P_1 \times P_1 \times P_1}{P_1 \times P_1 \times P_1 \times P_1 \times P_1 \times P_1}$ or $B \times \frac{P_1}{P} \times \frac{P_1 \times P_1 - b \times P_1}{g \times P_1 - b \times P_1}$, will be as useful, or rather more so, in

computing the values of annuities for three equal joint lives, when the lives are under the age of 60 years as when they go beyond it.

The differences between the true values of the annuities for three joint lives of unequal ages, mentioned in Table LIII, and the near values of the same annuities in Table LV.

CCCCXCI. The differences between the true values of the annuities for three joint lives of unequal ages which differ from each other by 10 years and 20 years, given above in Table LIII, and the near values of the same annuities which are contained in Table LV, are as follows; to wit,

$\pounds$	$\pounds$	$\pounds$
10.6551	— 9.1325	= 1.5226;
— 10.4379	+ 10.5080	= 0.0701;
— 9.738,56	+ 9.858,45	= 0.119,89;

8.986,72

£.	£.	£.
8.986,72 —	8.756,27 =	0.230,45 ;
8.313,10 —	8.068,36 =	0.244,74 ;
7.570,83 —	7.343,65 =	0.227,18 ;
6.816,07 —	6.594,61 =	0.221,46 ;
5.994,15 —	5.786,88 =	0.207,27 ;
5.145,62 —	4.947,33 =	0.198,29 ;
4.219,37 —	4.044,10 =	0.175,27 ;
3.297,98 —	3.181,57 =	0.116,41 ;
2.408,48 —	2.332,36 =	0.076,12 ;
1.623,48 —	1.604,14 =	0.019,34 ;
1.122,51 —	1.093,25 =	0.029,26 ;
0.169,378 —	0.169,378 =	0.000,000.

CCCCXCII. In looking over the near values and true values contained in the foregoing article we may observe, that the near values are, for the most part, less than the corresponding true values ; but not constantly so : for the near values £10.5080 and £9.858,45, (the former of which relates to three lives of the ages of 10, 20, and 30 years, and the latter to three lives of the ages of 15, 25, and 35 years,) are greater than the corresponding true values, £10.4379 and £9.738,56 : which agrees pretty much with what was observed in Art. cccclxxviii concerning the near values and the true values contained in Art. cccclxxvii.

A comparison of the said near values with the corresponding true values.

CCCCXCIII. It may also be observed that all the foregoing differences between the near values in Table LV and the corresponding true values in Table LIII, contained in Art. cccxcxi, except the first difference, £1.5226, (which relates to three lives of the ages of 5 years, 15 years, and 25 years, and which is vastly greater than any of the others,) are less than £0.25, or a quarter of a year's annuity ; which (as was before observed) is no very important variation from the truth.

CCCCXCIV. And the proportions of the foregoing differences (contained in Art. cccxcxi) to their several corresponding true values are expressed in the following Table.

The proportions of the said differences to the several corresponding true values of the said annuities.

pressed by the following fractions, to wit, $\frac{1}{7}$, $\frac{1}{148}$, $\frac{1}{81}$, $\frac{1}{38}$, $\frac{1}{33}$,
 $\frac{1}{33}$, $\frac{1}{30}$, $\frac{1}{28}$, $\frac{1}{25}$, $\frac{1}{24}$, $\frac{1}{28}$, $\frac{1}{31}$, $\frac{1}{83}$, and $\frac{1}{38}$.

These fractions are for the most part somewhat greater than the fractions in Art. cccxc, but somewhat less than those in Art. cccclxxv, which express the proportions of the differences of the near values and true values of the former set of annuities for three equal joint lives (where the ages were 60 years and upwards,) to their corresponding true values. We may therefore conclude that the expression $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$ will be as useful, or rather more so, in computing the values of annuities for three joint lives whose ages differ from each other by 10 years and 20 years, throughout all the periods of life, or, at least, when the youngest life is not younger than 10 years, as in computing the values of annuities for three equal joint lives of the age of 60 years, or upwards.

CCCCXCV. And from all these trials, taken together, it seems reasonable to conjecture, that the expression $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$ will always give us a tolerable approximation to the true value of an annuity for three joint lives of any ages whatsoever.

$$B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$$

Another method of approximating to the value of an annuity for the joint continuance of three lives of given ages.

Of Mr. Thomas Simpson's method of finding a near value of an annuity for three joint lives.

CCCCXCVI. The late very learned mathematician, Mr. Thomas Simpson, of Woolwich Academy, in his book entitled, *Select Exercises in the Mathematics*, page 279, has given us another method of finding a near value of an annuity for the joint continuance of three lives of any given ages, which is exceedingly short and simple, and which also (as he informs us in the same book, page 312,) gives the quantity sought to a very considerable degree of exactness, so as seldom to differ from the true value of the annuity by

by more than an eighth part of a years annuity. This method may be described as follows.

CCCCXCVII. Let the proposed annuity, of which we are to find A description of the value, be an annuity of one pound a year, as in all the foregoing the said method. instances. And let it be supposed that we have tables of the values of an annuity of one pound a year for single lives of all ages, already computed to our hands; and likewise that we have tables of the values of an annuity of one pound a year for the joint continuance of two lives, computed to our hands, which contain a sufficient variety of different ages of the two lives to enable us to find, (by the help of the method of Interpolation above explained in Art. ccccxix, &c. — ccccxixi,) the value of such an annuity for any two proposed lives of any given ages whatsoever to a considerable degree of exactness.

These things being supposed, we must, in the first place, find (by the tables, or by the rules given for finding the values of annuities for two joint lives,) the value of the proposed annuity of one pound a year for the joint continuance of the two oldest of the three given lives. This value, it is evident, will be equal to the value of the same annuity of one pound a year for a single life that is still older than either of the said two oldest of the three given lives. Let this fourth life be found by means of a table of the values of an annuity of one pound a year for single lives, by looking along the column containing the values of such an annuity till we meet with one that is equal, or nearly equal, to the said value of the same annuity of one pound a year for the joint continuance of the said two oldest of the three given lives. For the age corresponding to this value will be the age of this fourth, or imaginary, life.

And, lastly, find the value of a like annuity of one pound a year for the joint continuance of this fourth, or imaginary, life, and the first, or youngest of the three given lives.

This last value will be nearly equal (as Mr. Simpson assures us) to the value of the proposed annuity of one pound a year during the joint continuance of the three given lives. Q E L

CCCCXCVIII. This may be expressed more concisely in the following manner. Call the youngest life *A*, the next *B*, and the third, or oldest, manner of expression. And let *D* be the fourth, or imaginary, life, an annuity for which is pressing the force equal going description.

equal in value to the same annuity for the joint continuance of the lives *B* and *C*.

Then, in the first place, we must find the value of an annuity of one pound a year for the joint continuance of the lives *B* and *C*. And, secondly, we must find the single life *D*, an annuity for which is equal in value to the same annuity for the two joint lives *B* and *C*. And, thirdly, we must find the value of an annuity of one pound a year for the joint continuance of the two lives *D* and *A*. This last value will be nearly equal to the value sought, or the value of an annuity of one pound a year for the joint continuance of all the three given lives. Q E I.

An example of the foregoing method of approximating to the value of an annuity for three joint lives.

CCCCXCIX Let us suppose the three given lives to be all of the same age of 60 years, as in the example given above in Art. cccclxxxiii; and let the annuity be, as before, an annuity of one pound a year. And let it be required to find the value of this annuity for the joint continuance of these three lives of the age of 60 years, according to the Northampton table of the probabilities of the duration of human life given above in Art. cccclxxix, Table XLI, and upon a supposition that the interest money is 4 per cent. To find this value by the foregoing method, we must proceed as follows.

The value of an annuity of one pound a year during the joint continuance of two of these lives, upon the suppositions here made, appears from Table L, Art. cccclxxviii, to be £6.2468. We must therefore look into Mr. Morgan's table of the values of an annuity of one pound a year for single lives in his treatise on the doctrine of annuities, pages 64, 65, 66, in order to find a value equal, or nearly equal, to the said value £6.2468, that table of Mr. Morgan having been formed from the aforesaid Northampton table of the probabilities of the duration of human life, and upon a supposition that the interest of money is 4 per cent. And we shall there find that the value that comes nearest to £6.2468 is £6.263,15, which answers to the age of 70 years, or is the value of an annuity of one pound a year for a single life of the age of 70 years. We must therefore, in the last place, seek for the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 60 years. Now the value of such an annuity appears by Mr. Morgan's sixth table (in the Appendix to his

his aforeſaid Treatiſe on Annuities, pages 271, 272) to be, nearly, £.4.858. Therefore £.4.858 will (according to Mr. Simpson's aſſertion,) be nearly equal to the value of an annuity of one pound a year for the joint continuance of the three given lives, all of the age of 60 years. Q E I.

CCCCC. This near value, £4.858, of the ſaid annuity of one pound a year for the ſaid three equal joint lives of the age of 60 years, is ſomewhat greater than its true value, which we have ſeen above, in Table XLIX, to be £.4.7826. But the difference between them is but trifling, being only £.0.0754, which is leſs than the 63d part of the ſaid true value £.4.7826, and leſs alſo than £.0.0833, or than the 12th part of a pound, or than the 12th part of a year's annuity, or than a month's annuity.

The difference between the foregoing near value of an annuity for three equal joint lives (obtained by Mr. Simpson's method,) and the true value of the ſame annuity.

CCCCCI. The former near value of this annuity for three equal joint lives of the age of 60 years, which was obtained in Art. CCCCLXXXII, by means of the expreſſion $B \times \frac{P_i}{P} \times \frac{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_i}{P_i \times P_i \times P_i \times r - P_{ii} \times P_{ii} \times P_{ii}}$ or $B \times \frac{P_i}{P} \times \frac{g \times P_i - b \times P_i}{g \times P_i - b \times P_{ii}}$ was £4.585; which is leſs than the true value £4.7826, and differs from it by the quantity £0.1976, which is greater than the difference £0.0754. Therefore in the preſent inſtance Mr. Simpson's method of approximating to the values of theſe annuities comes nearer to the truth than the other method. But whether or no it does ſo in general, can only be known by trying it in a variety of inſtances, and comparing the values reſulting from it with the true values of the ſame annuities. With this view I ſhall preſent the reader with the two following tables of near values of the two ſets of annuities, whoſe true values have been ſet down in Tables LI and LIH, computed according to the foregoing method of Mr. Simpson; which we may afterwards compare with the ſaid true values. Theſe tables of near values are as follows.

This near value (obtained by Mr. Simpson's method,) is more exact than the former near value of the ſame annuity, obtained in Art. CCCCLXXXII.

Two tables of near values of annuities for three joint lives, computed by Mr. Simpson's method of approximation.

T A B L E LVI.

Containing approximations to the values of the annuities for three equal joint lives mentioned above in Table LIII: when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by the foregoing method of Mr. Thomas Simpson, of Woolwich.

Years in the age of the first life.	Years in the age of the second life.	Years in the age of the third life.	Values of an annuity of one pound a year for the joint continuance of the three lives.
5	5	5	£. 11.2119
10	10	10	12.2447
15	15	15	11.3543
20	20	20	10.4653
25	25	25	9.902,20
30	30	30	9.321,74
35	35	35	8.718,71
40	40	40	7.939,79
45	45	45	7.274,62
50	50	50	6.403,16
55	55	55	5.656,00
60	60	60	4.900,62
65	65	65	3.969,56
70	70	70	3.099,22

T A B L E LVII.

Containing approximations to the values of the annuities for three joint lives of different ages, mentioned above in Table LIII; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by the foregoing method of Mr. Thomas Simpson, of Woolwich.

Years in the age of the first, or youngest, life.	Years in the age of the second life.	Years in the age of the third, or oldest, life.	Values of an annuity of one pound a year for the joint continuance of the three lives.
5	15	25	£. 10.5974
10	20	30	10.4853
15	25	35	9.872,64
20	30	40	9.037,79
25	35	45	8.426,81
30	40	50	7.622,29
35	45	55	6.944,67
40	50	60	6.003,39
45	55	65	5.243,74
50	60	70	4.285,36
55	65	75	3.397,35
60	70	80	2.362,16
65	75	85	1.575,23
70	80	90	1.043,73

G B B B 2

CCCCII. The

fractions; to wit, $\frac{1}{269}, \frac{1}{290}, \frac{1}{141}, \frac{1}{84}, \frac{1}{92}, \frac{1}{91}, \frac{1}{64}, \frac{1}{105},$
 $\frac{1}{48}, \frac{1}{73}, \frac{1}{52}, \frac{1}{32}, \frac{1}{70},$ and $\frac{1}{28}.$

CCCCCV. All these fractions, except the fourth, $\frac{1}{84},$ are confi-

derably less than the fractions in Art. ccccx, which express the proportions of the differences of the former near values of the same annuities

(which were obtained by means of the expression $B^f \times \frac{P^i}{P} \times$

$\frac{P^i \times P^i \times P^i \times r - P^{ii} \times P^{ii} \times P^i}{P^i \times P^i \times P^i \times r - P^{ii} \times P^{ii} \times P^{ii}},$ or $B^f \times \frac{P^i}{P} \times \frac{g \times P^i - b \times P^i}{g \times P^i - b \times P^{i+1}},$)

from their several true values respectively, to the said true values. Therefore in these instances, as well as in the example given in Art. CCCXCIX, Mr. Simpson's method of approximation seems to be preferable to the former method by means of the said expression.

All the foregoing near values of annuities for three equal joint lives (obtained by Mr. Simpson's method,) except one, are more exact than the near values obtained above in Art. CCCCLXXXVI, Table LIV.

CCCCCVI. The differences between the near values of annuities for three joint lives of different ages, contained above in Table LVII, and the true values of the same annuities in Table LIII, are as follows; to wit,

The differences between the true values of annuities for three joint lives of different ages, in Table LIII, and the near values of the same annuities (obtained by Mr. Simpson's method,) in Table LVII.

$\pounds.$		$\pounds.$	$\pounds.$
10.6551	—	10.5974	= 0.0577 ;
10.4379	+	10.4853	= 0.0474 ;
9.738,56	+	9.872,64	= 0.134,08 ;
8.986,72	+	9.037,79	= 0.051,05 ;
8.313,10	+	8.426,81	= 0.113,71 ;
7.570,83	+	7.622,29	= 0.051,46 ;
6.816,07	+	6.994,67	= 0.128,60 ;
5.994,15	+	6.003,39	= 0.009,24 ;
5.145,62	+	5.243,74	= 0.098,12 ;
4.219,37	+	4.285,36	= 0.065,99 ;
3.297,98	+	3.397,35	= 0.099,37 ;
			+ 2.408,48

$$\begin{array}{rcl}
 + & 2.408,48 & - & 2.362,16 & = & 0.046,323 \\
 + & 1.623,48 & - & 1.575,23 & = & 0.048,253 \\
 + & 1.122,51 & - & 1.043,73 & = & 0.078,78.
 \end{array}$$

All the foregoing near values of annuities for three joint lives, of different ages, (which were obtained by Mr. Simpson's method,) except the first and the three last, are greater than the true values of the same annuities, respectively.

CCCCCVII. It is remarkable that all the near values set down in Table LVII and in the foregoing article, (and which were obtained by Mr. Simpson's method of approximation,) except the first and the three last, are greater than the several true values of the same annuities, respectively; as was observed in Art. ccccciii concerning all the near values, without any exception, that are set down in Table LVI and in Art. cccccii, and which were likewise obtained by Mr. Simpson's method of approximation; whereas the near values of these annuities obtained above by means of the expression $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$ and set down in Table LV and Art. ccccxci, are, for the most part, less than the said true values, respectively.

The proportions of the foregoing differences to the corresponding true values.

CCCCCVIII. The proportions of the foregoing differences in Art. cccccvi to their corresponding true values are expressed by the following fractions: to wit, $\frac{1}{184}$, $\frac{1}{220}$, $\frac{1}{72}$, $\frac{1}{176}$, $\frac{1}{73}$, $\frac{1}{147}$, $\frac{1}{53}$, $\frac{1}{648}$, $\frac{1}{52}$, $\frac{1}{63}$, $\frac{1}{33}$, $\frac{1}{51}$, $\frac{1}{33}$, and $\frac{1}{14}$.

All the foregoing near values of annuities for three joint lives of different ages, (obtained by Mr. Simpson's method,) except the first and the two last, are more exact than the near values of the same annuities obtained above in Art. CCCCLXXXVI, Table LV.

CCCCCIX. All these fractions, except the third, to wit, $\frac{1}{72}$, and the two last, to wit, $\frac{1}{33}$ and $\frac{1}{14}$, are considerably less than the fractions in Art. cccccxiv, which express the proportions of the differences of the former near values of the same annuities (which were obtained by means of the expression $B \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$) from their several true values respectively to their said true values. Therefore in the greater part of these instances, as well as in those of the annuities for three equal joint lives contained

ained in Table VI, and in the example given in Art. ccccxix, this method of Mr. Simpson seems to be preferable, in point of exactness, to the former method of approximation by means of the expression $B \times \frac{P_1}{P} \times \frac{P_1}{P}$. And it is certainly much shorter and easier in practice than that other method, which (as we have seen above in Art. cccclxxxii) requires a good deal of calculation. And therefore I think it must be considered, upon the whole, as the better method of the two. Yet, as there are now and then some instances in which the other method comes nearer to the truth than this, I think it is convenient to be possessed of both methods, to the end that in doubtful cases we may resort to one of them as a kind of confirmation of the result obtained by the other to a moderate degree of exactness.

CCCCCX. The only thing that seems wanting to make Mr. Simpson's method satisfactory, is a demonstration of its truth, or an investigation of it in some way or other. But this is what Mr. Simpson has not given us. For he only says (in his *Select Exercises*, page 312,) "That the reasonableness of this method of proceeding is evident from the nature of the subject, without calling in the assistance of any kind of computation; and that, in a number of examples respecting lives of different ages, he scarce ever found the error to exceed an eighth part of a year's purchase." And this account of the degree of exactness to which this method of Mr. Simpson gives the values of these annuities is confirmed by the foregoing trials; since of all the fourteen differences in Art. ccccxix (which relate to annuities for three equal joint lives) only the seventh difference, to wit £.o.133,49, the ninth, to wit, £.o.148,22, and the twelfth, to wit, £.o.145,59, are greater than £.o.125,00, or an eighth part of a year's annuity; and of all the fourteen differences in Art. ccccxvi, (which relate to annuities for three joint lives of different ages, which differ from each other by 10 years and 20 years,) only the third difference, to wit, £.o.134,08, and the seventh difference, to wit, £.o.128,60, are greater than £.o.125,00, or an eighth part of a year's annuity. It seems probable, therefore, that the differences of the near values of annuities for three joint lives, that would be obtained by this method of Mr. Simpson in any other instances, (in which the ages of the lives were different from those above-supposed,) from the true values of the same annuities, respectively, would seldom be greater than £.o.125,00, or one eighth part of a year's annuity; and consequently, that this method of Mr. Simpson will, in all those

Of the degree of exactness to which it may reasonably be conjectured that Mr. Simpson's method of approximation will give, in most cases, the value of an annuity for three joint lives.

those cases be a very useful method of approximating to the values of such annuities. And with this I shall conclude what I had to offer concerning the valuation of annuities for the joint continuance of three lives of given ages.

[End of the doctrine of the valuation of annuities for the joint continuance of three lives.]

Of the values of annuities that depend on the continuance of the longest of two, or more, lives of given ages.

CCCCCXI. Having now gone through the doctrine of the valuation of annuities depending on the *joint* continuance of two, or more, lives of given ages, it remains that I shew how to estimate the values of annuities that depend on the continuance of *any one* out of two, or more, lives of given ages.

The principles upon which the determination of these values is founded, are explained above in Prob. IV. and its corollaries, Art. LVIII, LIX, LX, &c. — LXXVI, pages 58, 59, 60, 60, &c. — 90. It is there shewn in Coroll. IV, pages 63, 64, 65, 66, “That the value of an annuity of one pound a year for the lives of two persons of given ages, and the life of the longer liver of them, is equal to the excess of the sum of two separate annuities of one pound a year each, for the single lives of the same persons, above the value of a like annuity of one pound a year for their joint lives.” And it is shewn in Coroll. XI. Art. LXXVI, pages 83, 84, 85, &c. — 90, “That the value of an annuity of one pound a year for the lives of three persons of given ages, and the life of the longest liver of them, is equal to the excess of the sum of the four following values, to wit, 1st. the value of an annuity of one pound a year for the life of the first of the said three persons; 2dly, the value of a like annuity for the life of the second of them; 3dly, the value of a like annuity for the life of the third of them; and, 4thly, the value of a like annuity for the joint lives of all the three, above the sum of the three following values, to wit, 1st. the value of a like annuity of one pound a year during the joint lives of the first and second persons; 2dly, the va-

"the value of the like annuity during the joint lives of the first and third persons; and, 3dly, the value of a like annuity during the joint lives of the second and third persons." Therefore, if A be put for the value of an annuity of one pound a year for the life of the younger of two persons of given ages, and B for the value of a like annuity of one pound a year for the life of the older of the said two persons, and AB be put for the value of a like annuity of one pound a year for the joint continuance of both lives; I say, if these are supposed to be the values of A , B , and AB , the value of a like annuity of one pound a year for the lives of both the said persons and the life of the longer liver of them, will be equal to $A + B - AB$. And, if C be put for the value of a like annuity of one pound a year for the life of a third person that is older than either of the two former persons, and AC be put for the value of an annuity of one pound a year for the joint continuance of the first and third lives, and BC be put for the value of a like annuity of one pound a year for the joint continuance of the second and third lives, and ABC be put for the value of a like annuity of one pound a year for the joint continuance of all the three lives; I say, if these are supposed to be the values of A , B , C , AB , AC , BC , and ABC , the value of an annuity of one pound a year for the life of the longer liver of the said three persons will be equal to $A + B + C + ABC - AB - AC - BC$, or $A + B + C - AB - AC - BC + ABC$. And, in like manner the value of an annuity for the longest of four, or more, lives may be deduced from the values of the like annuities for the same single lives and for their joint continuance, by the principles laid down above in Prob. IV. and its corollaries, pages 58, 59, 60, &c. ————— 90, whatever the number of lives may be. But it is seldom thought necessary, in treating of this subject of life-annuities, to suppose the lives to be more than three.

An example of the computation of the value of an annuity of one pound a year for the longest of two lives of given ages, by means of the expression $A + B - AB$.

CCCCCXII. Let the younger life be supposed to be of the age of 20 years, and the older of the age of 30 years. And let the interest of money be supposed to be $3\frac{1}{4}$ per cent. and the probabilities of the duration of human life to be such as they are represented to be in Monsieur de Parcieux's table.

Then will A , or the value of an annuity of one pound a year for the first, or younger life, be = £19,440,616; and B , or the value of a like annuity of one pound a year for the second, or older, life, will be = £18,068,798; and AB , or the value of a like annuity of one pound a year for the joint continuance of both lives, will be = £15,298,763; as appears by Tables XV and XXXI, in pages 224 and 494. Therefore

H h h h

$A + B -$

$A + B - AB$, or the value of an annuity of one pound a year for the longer of the two given lives of the ages of 20 years and 30 years, will be $= £19,440,616 + £18,068,798 - £15,298,750 (= £37,509,414 - £15,298,750) = £22,210,664$. Q. E. I.

An example of the computation of the value of an annuity of one pound a year for the longest of three lives of given ages, by means of the expression $A + B + C - AB - AC - BC + ABC$.

CCCCCXIII. Let the youngest life be supposed (as in the last example) to be of the age of 20 years, and the second life to be of the age of 30 years; and let the third, or oldest, life be of the age of 40 years. And let the interest of money be supposed (as in the last example) to be $3\frac{1}{4}$ per cent. and the probabilities of the duration of human life to be such as they are represented to be in Monsieur de Parcieux's table of them.

Then, in the first place, it appears from Table XV, page 224, that A , or the value of an annuity of one pound a year for the first, or youngest, life (which is of the age of 20 years) is $= £19,440,616$; and that B , or the value of an annuity of one pound a year for the second life (which is of the age of 30 years) is $= £18,068,798$; and that C , or the value of an annuity of one pound a year for the third, or oldest, life (which is of the age of 40 years) is $= £16,104,542$. Therefore $A + B + C$ will be $(= £19,440,616 + £18,068,798 + £16,104,542) = £53,613,956$.

In the second place, it appears from Table XXXI, page 494, that, when the interest of money is $3\frac{1}{4}$ per cent. the value of an annuity of one pound a year for two joint lives of the ages of 20 years and 30 years is $£15,298,75$; and the value of a like annuity for two joint lives of the ages of 30 years and 40 years is $£13,709,61$. And it appears from Table XXXII, page 495, that the value of an annuity of one pound a year for two joint lives of the ages of 20 years and 40 years (when the interest of money is at the same rate of $3\frac{1}{4}$ per cent.) is $£14,028,29$. Therefore AB is $= £15,298,75$, and BC is $= £13,709,61$, and AC is $= £14,028,29$; and consequently $AB + BC + AC$ is $(= £15,298,75 + £13,709,61 + £14,028,29) = £43,036,65$. Therefore $A + B + C - AB - BC - AC$, or $A + B + C - AB - AC - BC$, is $(= A + B + C - £43,036,65 = £53,613,956 - 43,036,65) = £10,577,306$.

In the last place we must find the value of ABC , or of an annuity of one pound a year for the joint continuance of the three given lives of the ages of 20 years, 30 years, and 40 years. Now this may be done to a moderate degree of exactness by means of Mr. Simpson's method of approximation, (above explained in Art. ccccxvii, ccccxviii, pages 591, 592) in the manner following.

The

— $AC - BC + £12.246,578$, or to $£10.577,306 + £12.246,578$, or) $£22.823,884$; that is, the value of an annuity of one pound a year for the longest of three lives of the ages of 20 years, 30 years, and 40 years, when the interest of money is $3\frac{1}{4}$ per cent. will be, nearly, equal to $£22.823,884$.

C O N C L U S I O N.

CCCCCXIV. I have now gone through every thing that seemed to me to be necessary to compleat the object of this treatise, which was to explain in a clear and familiar manner *the principles* of the doctrine of life-annuities. Much more, indeed, might be added to it concerning *the application* of this doctrine to a variety of useful questions concerning life-annuities, that may often occur in the course of men's dealings with each other; such as the methods of finding the values of an annuity for one or more lives of given ages, that shall take place in case of the failure of one or more other lives of given ages, or in case of the failure, of one, or more, other lives of given ages, before a third set of lives of given ages, or that shall depend on a variety of other contingencies. But for the solution of questions of this kind I shall refer my reader to Mr. Thomas Simpson's *Doctrine of Life-Annuities*, and to his *Select Exercises in the Mathematicks*, and to Dr. Price's *Observations on Reverſionary Payments*, and Mr. Morgan's *Doctrine of Annuities and Assurances on Lives and Survivorſhips*, and to various other learned and useful tracts on this subject. I am contented with having explained the fundamental principles of the doctrine, and with having presented the reader with a compleat set of tables of the values of life-annuities for single lives at the several different rates of the interest of money at which Mr. Smart has given us tables of the values of annuities for terms of years, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. all fairly computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, without having recourse to Mr. de Moivre's Hypothesis, or any other inaccurate supposition, in order to facilitate the computation; and likewise with a considerable number of tables of the values of annuities for two joint lives at the two different rates of interest of $3\frac{1}{4}$ per cent and $4\frac{1}{2}$ per cent all fairly computed likewise from the said table of Monsieur de Parcieux; and with having furnished the reader with rules for finding tolerably near values of other annuities, for two joint lives, and likewise of annuities for *three* joint lives, of any ages whatsoever, at the same, or any other, rates of interest; by all which, I flatter myself, the foregoing treatise will be justly intitled to be considered as a *useful appendix*, or *supplement*, to Mr. John Smart's very valuable *tables of interest*.

F I N I S.

A P P E N D I X.

As the tables of the values of remote life-annuities that were computed for the bill mentioned above in the scholium, page 34, (which was brought into the House of Commons by the late Mr. Dowdeswell in the year 1773,) have never been published, I presume they will be thought to make no improper addition to those which have been inserted in the preceding work. Nor can I suppose that a copy of that bill itself, to which the said tables of the values of remote life-annuities were annexed, will be unwelcome to such of my readers as shall approve the scope and view of it, which was "to encourage the poor to industry and frugality" by accommodating them with a safe and convenient method of "laying out what little money they could save out of the earnings of their labour." I shall therefore now proceed to add to the foregoing sheets an exact copy of this bill, in its last form, as it passed the House of Commons, after a variety of amendments and improvements made in it, with great care and pains, by the gentlemen who were concerned for its success, and also a copy of the tables of the values of remote life-annuities which were annexed to the said bill, and were considered as a part of it. This bill was as follows,

A BILL,

PRINTED

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BILL,

INTITULED

An ACT for the better Support of Poor Persons in certain Circumstances, by enabling Parishes to grant them Annuities for Life, upon Purchase, and under certain Restrictions.

WH^{EREAS} it often happens that persons engaged as journey-men in manufactures and handicraft trades, and likewise household servants, labourers, and divers other persons, get more money, as the wages of their labour and service, than is sufficient for their present maintenance, and might easily, if they were so minded, lay by, out of their said gettings, a sufficient sum to provide for their support in their old age :

The preamble.

And whereas it would be highly useful, both to the said persons themselves and to the nation in general, that they should endeavour to make such provision for their support, in the latter periods of their lives, as they would thereby become more sober and virtuous in their ordinary course of life, and more industrious in the prosecution of their several callings and employments,

employments, which would tend to the increase of the riches and manufactures of this kingdom:

And whereas it is probable that many of the said persons might be induced to lay up some part of their earnings in their youth and middle age, in order to make such provision for their old age, if a convenient opportunity were offered them of employing the money they should so lay up in a safe and advantageous manner:

And whereas the most safe and advantageous way, in which the said frugal and industrious persons can employ the several sums of money, which they may be able to save out of their wages, for the support of their old age, seems to be to purchase therewith annuities for their lives, which should commence at some remote period, when their strength and ability to work will be considerably impaired; and the poor's rates of the several parishes in England and Wales seem likely both to be, and to be thought by the said industrious and frugal persons, a sufficient and convenient fund to secure, at all events, the payment of such life-annuities as aforesaid, to the several persons who shall have purchased them, in case any deficiency should happen in the fund created for the payment of the said annuities, by the monies that shall have been paid for the purchase of them: ~~be it therefore enacted~~ by the king's most excellent majesty, by and with the advice and consent of the lords spiritual and temporal, and commons, in this present parliament assembled, and by the authority of the same, That, from and after the fifth day of July, one thousand seven hundred and seventy-three, the plan and method herein-after prescribed, for granting annuities to such industrious and frugal persons for their lives, shall take place and be carried into execution, in all parishes and townships which maintain their own poor, within England and Wales, and the town of Berwick upon Tweed, where there shall be two churchwardens, and three or more overseers of the poor, or one churchwarden, and three or more overseers of the poor, and where the same shall have been approved by a majority of the inhabitants of the said parishes or townships, who are liable to be charged to the poor's rates of the same, assembled at two different parish meetings, held in the churches of the said parishes or townships, after due notice given thereof, and at the distance of, at least, twenty-one days one from the other, subject to the rules and regulations herein-after prescribed; such majority in number being also charged, in the last rate made for the relief of the poor of the said parish or township, in a sum greater than that which was assessed in the said rate upon the rest of the inhabitants assembled at the said meeting; and such notice shall express the times of both the said meetings, and shall be published in the respective parish churches, on a Sunday immediately after divine service, and affixed upon the door of the said parish church,

After July 5, 1773, the following plan shall take place in all parishes and townships in England, in which the majority of the inhabitants, both in number and value, shall adopt it in two different parish-meetings held for the purpose after due public notice.

church, by the clerk of the said parish, or the person acting as such, who is and are hereby required to publish and affix the same, upon the request of any person desirous of purchasing any such annuity; and the rector, vicar, or perpetual curate of any such parish or township, together with the churchwardens and overseers of the poor in the same, shall be, and they are hereby, authorised and required to receive, on the behalf of all the said inhabitants of such parish or township so liable, as is aforesaid, to be charged to the poor's rate of the same, from any person to whom the said inhabitants shall have agreed to grant a like annuity in manner herein-after mentioned, any sum of money that such person shall have agreed to pay them for the purchase of the same; and to grant to such person, in the name and on the part of the said rateable inhabitants of the same parish or township, an annuity for the life of the grantee, equivalent to the purchase-money, to commence at such future period of such grantee's life, as shall have been settled between the said inhabitants and the said purchaser, subject to the restrictions herein-after mentioned; and the poor's rate of such parish or township shall be, and they are hereby declared to be, subject by such grant to the payment of the annuity thereby granted, from time to time, as the same shall become due and payable; and, if it shall happen that the rector, vicar, or perpetual curate of any such parish or township, shall be absent from such parish or township, when the grant of any such annuity shall be demanded, and shall continue to be absent from the same for the space of thirty days from the same time, it shall and may be lawful for the churchwardens and overseers of the poor of the said parish or township, and they are hereby empowered and required, after the expiration of the said thirty days from such demand being made, notwithstanding the absence of such rector, vicar, or perpetual curate, to make grants of such annuities, without the concurrence of such rector, vicar, or perpetual curate; and such grants shall be, and are hereby declared to be, as good, valid, and effectual, as if such rector, vicar, or perpetual curate, had been present, and concurred in the making thereof.

The rector, or vicar, or perpetual curate, of the parish, together with the churchwardens and overseers thereof, shall have power to make grants of life-annuities to any inhabitants thereof for certain reasonable prices to be paid by the said grantees, under certain restrictions.

And the poor's rates of such parishes shall become liable, in consequence of these grants, to the payment of such annuities, when they become due.

And in case of the absence of the rector, vicar, or perpetual curate, for the space of 30 days together, the said grants may be made by the churchwardens and overseers of the poor without them.

And for obviating all doubts that may arise touching the value of such life-annuities, and the money to be paid for the purchase thereof, be it further enacted by the authority aforesaid, That in all parishes and townships within the cities of London and Westminster, or of Southwark, the said annuities shall be granted according to the respective rates herein-after mentioned and expressed in the set of tables hereunto annexed, No. I. and in all other parishes and townships within England, Wales, and the town of Berwick upon Tweed, the said annuities shall be granted according to the respective prices herein-after mentioned and expressed in the set of tables hereunto annexed, No. II; and if any such life-annuity shall be granted to

I i i

The prices to be paid for such annuities shall be those which are contained in the tables hereunto annexed; of which the first set, called No. I, shall relate to annuities granted in the cities of London, Westminster, and the borough of Southwark; and the second set, called No. II, shall relate to those granted in all other parts of England and Wales.

any person under the age of fifteen years, the same price shall be paid for such annuity as if the grantee thereof was fifteen years of age.

No sum less than 5*l*. shall be employed in the purchase of any of these annuities. And no annuity shall be greater than 20*l*. *per annum*.

Annuities granted to men shall not take place till they shall be 50 years old, nor to women till they shall be 35 years old.

Provided always, That no smaller sum than five pounds of lawful money of Great Britain shall be laid out in the purchase of any of the said annuities, nor shall any person be allowed to purchase any annuity or annuities from any one parish or township, to a greater amount than twenty pounds per annum; nor shall any of the said annuities commence after the grantee thereof shall have attained his age of seventy-five years; nor shall it commence for the life of any man, before he shall have attained the age of fifty years, nor for the life of any woman before she shall have attained the age of thirty-five years; nor shall any such annuity be granted to any person for any other life than that of the grantee.

These annuities shall not be granted without the consent of such a majority of the parishioners present at the meetings in which they are granted, as pays more to the poor's rate than the other parishioners there present.

And be it further enacted, That no such annuity shall be granted, in any parish or township, where the number of inhabitants assessed towards the relief of the poor shall exceed the number of eighteen, unless twelve, at the least, of such inhabitants shall be present in a vestry, or other public meeting to be held for that purpose; and in parishes or townships where the whole number of inhabitants, liable to be assessed as aforesaid, shall be less than nineteen, no such annuity shall be granted, unless at a vestry or other public meeting, where two-thirds of such inhabitants shall be present; nor shall any such annuity be granted, at any of the said meetings, unless the major part of the inhabitants so assembled, being also charged in the last poor's rate, as aforesaid, in a sum greater than what is assessed in the same rate upon the rest of the inhabitants there assembled, shall agree to grant the same; and no such meeting shall be held for granting such annuities, unless previous notice of holding a meeting for that purpose shall be published in the parish church upon two Sundays immediately after divine service, and a written copy of such notice affixed upon the door of such church previous to the first publication thereof, as aforesaid; and the clerk of the said parish, or the person acting as such, is and are hereby required to publish and affix the same, at the request of any person desirous of purchasing such annuity; and no such annuity shall be granted to any person who shall not (in the opinion of the majority of such inhabitants so assembled) appear to have a legal settlement within the parish or township upon which the same shall be charged.

Nor shall they be granted to any persons but such as are supposed to have a legal settlement in the parish.

Yet, if, through mistake, they are granted in the manner directed in this act, to inhabitants of the parish who have not a legal settlement in it, they shall nevertheless be valid and binding.

Provided nevertheless, That if any annuity shall have been granted in the manner directed by this act, such grant shall be valid and binding upon the parish, or township, granting the same; and the annuity thereby granted shall be regularly paid, according to the purport and true meaning of such grant,

grant, notwithstanding it shall afterwards be discovered that, at the time of making such grant, the grantee thereof was not legally settled in such parish or township.

And be it further enacted, That the said annuities shall be granted by deeds of grant, fairly written or printed on parchment, and signed and sealed by the churchwardens and overseers of the poor, and by the rector, vicar, or perpetual curate (unless he shall be absent as aforesaid) of the parish, or township, in behalf of which such annuities shall be granted, and may be made in the form, or to the effect, following:

The deeds of grant shall be made out on parchment.

" A. T a public meeting of the inhabitants of the parish or township of _____ in the county of _____

The form of the deeds by which these annuities shall be granted.

" holden in the vestry of the church of the said parish or township, on the _____ day of _____ in the _____ year of the reign

" of our sovereign lord King George the Third, and in the year of our Lord Christ _____

" notice of the said meeting being first given. _____ after due

" A. B. rector (or vicar, or perpetual curate, as it may happen to be) of the parish or township of _____ aforesaid, in the county

" aforesaid, C. D. and E. F. churchwardens of the said parish or township,

" G. H. I. K. and L. M. overseers of the poor of the said parish or township; To all to whom this present writing shall come, send greeting:

" Whereas N. O. of the said parish or township aforesaid, Bricklayer, (or

" household servant to P. Q. Esquire, of the said parish or township, or day-

" labourer or otherwise, according to his proper addition or employment)

" appearing to us to be a person lawfully settled in the said parish or town-

" ship, and intitled to be relieved by the poor rate raised in the same, in

" case he should become poor and helpless, and now of the the age of _____

" years, hath paid unto the hands of us the rector, or vicar, or

" perpetual curate, churchwardens, and overseers of the poor of the parish

" or township, aforesaid, the sum of _____ pounds of lawful

" money of Great Britain, as the price of an annuity for his life, that shall

" begin when he is _____ years of age, that is, at the end

" of _____ years from this present time, and in the year of our

" our Lord Christ _____ to be paid

" to him, or his certain attorney, by the rector, churchwardens, and over-

" seers of the poor of the said parish or township for the time being, by

" equal quarterly payments to be made at the four following feast days;

" to wit, the feast day of the annunciation of the Blessed Virgin Mary, the

" feast day of Saint John the Baptist, the feast day of Saint Michael the

“ Archangel, and the feast day of the Nativity of our Lord Christ, or
 “ within seven days after each of the said feast days, according to the statute
 “ of the thirteenth year of the reign of King George the Third, in that
 “ behalf made and provided: NOW know ye, that we the rector, (vicar, or
 “ perpetual curate,) churchwardens, and overseers of the poor of the said
 “ parish or township, in consideration of the said sum of pounds,
 “ to us in hand paid, (the receipt whereof is hereby acknowledged,) and
 “ in pursuance of the statute aforesaid, do by this present writing, in the
 “ behalf of the inhabitants of the said parish or township, of shillings
 “ give and grant unto the said N. O. an annuity or yearly pension of
 “ of of lawful money of Great Britain, which shall
 “ commence on the feast day of that shall be in
 “ the year of our Lord Christ
 “ and continue to be paid to the said N. O. from thence forwards during
 “ his life, by equal quarterly payments, at the four feast days above men-
 “ tioned, or within seven days after each of them respectively: and fur-
 “ ther, by virtue of the statute aforesaid, we do bind and engage the rats,
 “ of the said parish or township, that shall hereafter be raised therein for
 “ the relief of the poor thereof, for the full and due payment of the said
 “ annuity of to the said N. O. by equal quarterly
 “ payments, on the four feast days above-mentioned, or within seven days,
 “ after each of them respectively, from the said feast day of
 “ to the feast day or day of payment that shall happen,
 “ next before the death of the said N. O. including both the said days.
 “ In witness whereof, we have hereunto set our hands, and have fixed our
 “ common seal, this day of in the
 “ year of the reign of our sovereign lord George the Third, and in the
 “ year of our Lord Christ

A. B. Rector, or Vicar, or perpetual Curate,
 of the said Parish or Township.

C. D. } Churchwardens.

E. F. }

G. H. }

I. K. }

L. M. }

Overseers of the Poor.

The Common
 Seal of the Parish
 of in the
 County of

And in case of the absence of the rector, vicar, or perpetual curate as afore-
 said, such grant shall be made in the names of the churchwardens and over-
 seers of the poor only; and such absence of the rector, vicar, or perpetual
 curate, shall be specified in the same, and the charges and expences attend-
 ing

ing the making out every such grant shall be defrayed by the person to whom such annuity shall be granted, who shall pay to the said parish officers the sum of two shillings and sixpence for the same, and no more.

And be it further enacted, That, in every such deed of grant, the pounds, shillings, and pence, thereby granted, and the date of the year of our Lord Christ in which such annuity is to commence, shall be written in words at length, and not in figures; and such deeds of grant, immediately after the same shall be signed and sealed as aforesaid, shall be delivered to the grantee of the annuity thereby granted, to be kept by him or her, as the proof of his or her right to such annuity.

And be it further enacted by the authority aforesaid, That the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor of every such parish or township, shall, and they are hereby required to cause a copy of every such deed of grant to be entered in a book, to be kept in the parish chest for that purpose, and to seal and sign the said copy in the same manner as the original deed is hereby directed to be signed and sealed, (both which shall be executed at a public meeting of the inhabitants of the parish or township, to be holden for that purpose;) and the grantee of the said annuity shall, at the same time sign and seal an acknowledgment in writing, which shall be put at the bottom of the said copy, in the words or to the effect following:

I N. O. of the parish or township of _____ in the county of _____ do acknowledge, that the above is a true copy of the deed of grant of a certain life-annuity, which has been this day granted to me by the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor of the said parish or township, or by the churchwardens and overseers of the poor, in case of the absence of the rector, vicar, or perpetual curate, as aforesaid.

N. O.

Which said copy shall be made at the expence of the inhabitants of the said parish or township, and be paid for out of the money received as the price of the said annuity, from the said grantee thereof, without any new expence to the said grantee: and if it shall at any time happen, that any original deed of grant, delivered as aforesaid, shall be lost or destroyed, the copy thereof so entered as aforesaid shall be deemed sufficient evidence that such grant had been made, and shall intitle such grantee to receive his annuity according to the purport of such copy. And, if such grantee shall desire to have a copy of such deed of grant made from the said parish copy, instead of the original deed so lost or destroyed, the rector, vicar, or perpetual curate,

And the parish-officers shall give the grantee a new copy of the lost deed of grant of his annuity from such parish-copy of it, for the sum of 2s. 6d.

And the said new copy of the said deed of grant shall be sufficient evidence of its having been made, although the copy in the parish-book should afterwards be lost.

curate, churchwardens, and overseers of the poor of the said parish or township for the time being, shall cause such copy to be made out upon parchment, and shall sign the same, in attestation of its being a true copy from the said copy in the parish book, and shall deliver such attested copy to such grantee, without requiring any new consideration-money for the same, or any fee or reward, except the necessary expence of making out such new copy; for which they shall take the sum of two shillings and sixpence, and no more: and the said new copy, so attested, shall be sufficient evidence that such grant had been made, and shall intitile the grantee to receive his annuity from the said parish or township, according to the purport of such attested copy, notwithstanding the copy in the parish book should afterwards happen to be lost or destroyed.

The overseers of the poor shall enter all proceedings relating to the execution of this act, in proper books to be provided for that purpose.

And be it further enacted by the authority aforesaid, That, previous to the holding of any meeting in pursuance of this act, the overseers of the poor for the parish or township where such meeting shall be held, shall cause proper books to be provided and kept in the public chest, belonging to such parish or township; in which books all orders, proceedings, and accounts, relating to the execution of this act, shall be fairly entered at length, by some person to be appointed by the respective churchwardens and overseers of the poor; and the names of all the persons who shall give a vote for, or against, the making of any order, or the coming to any resolution relating to the execution of this act, shall be entered in one of the said books, and the persons agreeing to, or concurring in, such order or resolution shall sign the same; and an account of all monies received or payed in pursuance of this act shall also be entered in one of the said books, expressing particularly the times of all such receipts and payments, the person to, or from, whom the same were paid or received, and for what purposes; which accounts shall be balanced and closed once in every year, as near as conveniently may be to the time for holding the petty sessions for appointing overseers of the poor for such respective parish or township, and (being signed by the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor, for each such parish or township,) shall be laid before the justices of the peace at such petty sessions, who shall examine such accounts with the vouchers thereto; and, if the same shall appear to such justices to be just and true accounts, they shall certify the same in writing upon such accounts; and a duplicate of every such account shall be transmitted by the respective overseers of the poor to the clerk of the peace for the county, riding, or division, wherein such parish or township is situated, (whether the same be a town-corporate, having exclusive jurisdiction or not,) before the next general quarter sessions of the peace; and each and every clerk of the peace is hereby required to cause every such account to be filed amongst the records of his office.

And a duplicate of every such account shall be transmitted to the clerk of the peace for the county, before the next general quarter sessions of the peace.

And be it further enacted by the authority aforesaid, That, previous to the holding of any meeting in pursuance of this act, the overseers of the poor for the parish or township where such meeting shall be held, shall cause proper books to be provided and kept in the public chest, belonging to such parish or township; in which books all orders, proceedings, and accounts, relating to the execution of this act, shall be fairly entered at length, by some person to be appointed by the respective churchwardens and overseers of the poor; and the names of all the persons who shall give a vote for, or against, the making of any order, or the coming to any resolution relating to the execution of this act, shall be entered in one of the said books, and the persons agreeing to, or concurring in, such order or resolution shall sign the same; and an account of all monies received or payed in pursuance of this act shall also be entered in one of the said books, expressing particularly the times of all such receipts and payments, the person to, or from, whom the same were paid or received, and for what purposes; which accounts shall be balanced and closed once in every year, as near as conveniently may be to the time for holding the petty sessions for appointing overseers of the poor for such respective parish or township, and (being signed by the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor, for each such parish or township,) shall be laid before the justices of the peace at such petty sessions, who shall examine such accounts with the vouchers thereto; and, if the same shall appear to such justices to be just and true accounts, they shall certify the same in writing upon such accounts; and a duplicate of every such account shall be transmitted by the respective overseers of the poor to the clerk of the peace for the county, riding, or division, wherein such parish or township is situated, (whether the same be a town-corporate, having exclusive jurisdiction or not,) before the next general quarter sessions of the peace; and each and every clerk of the peace is hereby required to cause every such account to be filed amongst the records of his office.

And

And be it further enacted, That no deed of grant, or any of the copies thereof herein before directed to be made, nor any power of attorney for accepting and transferring stock, and for the receiving the dividends due thereon, shall be charged or chargeable with any stamp-duty whatsoever, but shall be good, valid, and effectual, to all intents and purposes, without any stamp being impressed thereon.

No stamp-duty shall be paid for the said deeds of grant, or the copies of them.

And be it further enacted, That if any grantee of any such life-annuity shall consent that the same may be made unalienable, and thereupon a clause for that purpose, expressing his consent that it should be made so, shall be inserted in the deed of grant delivered to him, and in the copy thereof kept in the parish register of the said grants of life-annuities, and which he shall have acknowledged to be a true copy of his said grant in the manner above directed, in such case the said annuity shall be payable to the said grantee alone, or his certain attorney, during his life, without any power in him to alienate, or assign, it to any person, or in any manner, whatsoever; and every assignment of such annuity that shall afterwards be made by the said grantee to any other person, shall be totally void in law and equity, and of no effect or operation whatsoever.

The said annuities may, with the consent of the grantees thereof expressed in the deeds of grant, be made unalienable.

And be it further enacted, That if any grantee of any of the said annuities that shall not have consented as above to make his said annuity unalienable, shall be desirous of selling or disposing of any such annuity to any other person, he shall, in the first place, make an offer to sell the same to the rector, vicar, or perpetual curate, churchwardens and overseers of the poor, and other inhabitants chargeable to the poor rates of the said parish or township where the same was bought, at the price which such annuity shall be then worth, according to the rates herein-after mentioned, or at any lower price; which offer shall be made at a vestry or other publick meeting of the inhabitants of the said parish or township, notice of which meeting shall be given on two different Sundays, in the same manner as herein-before directed, concerning meetings for granting the said annuities; and, if, upon such offer being made, the major part of the rateable inhabitants so assembled, being also charged in the last poor's rate as aforesaid, in a sum greater than what is assessed in the same rate upon the rest of the inhabitants there assembled, shall think proper to purchase the same, the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor of such parish or township, or the churchwardens and overseers of the poor only, in the absence of such rector, vicar, or perpetual curate as aforesaid, shall be, and are hereby, authorised and required to buy up the said annuity, at the said price, or at any lower price as shall be agreed for, and to defray the expense of such purchase out of the fund created by the monies received from the grantees of the said life-annuities; and in case the major part of the

And, when they are not made unalienable, they shall not be alienated before an offer has been made of them to the parish by which they have been granted, at the values set down in the tables hereunto annexed.

But, if the parish refuse to buy them in, the grantees shall be at liberty during the space of six months to sell them to any other persons.

And

After any such annuity shall have been sold, or assigned once, it may be assigned again without making any previous offer of it to the parish.

inhabitants, so assembled as aforesaid, shall not agree with the said grantee for the purchase of the said annuity, the grantee thereof shall, for the space of six months from the time of such refusal or non-agreement, be at liberty to sell and assign his said annuity to any person whatsoever, by a deed duly sealed and delivered in the presence of two credible witnesses, and signed by the said witnesses, in attestation of their having seen the same so signed and delivered: and after the expiration of the said six months, the said grantee shall not have power to sell or assign such annuity to any person whatsoever, unless he shall again make an offer thereof, in the manner before prescribed, to the rector, vicar, or perpetual curate, and churchwardens and overseers of the poor, and other rateable inhabitants of the said parish or township; but in case of their refusal or non-agreement to purchase the same, such grantee shall again be at liberty, during the space of six months from such refusal or non-agreement, to assign such annuity to any person whatsoever: and so, in like manner, as often as he shall be desirous of selling the said annuity, he shall acquire a right so to do, for the space of six months after making such offer, and the refusal of the same, or non-agreement, as aforesaid. And all assignments made of such annuities by the grantees thereof, otherwise than according to the rules herein prescribed, shall be utterly void and of no effect. But, when any of the said annuities shall have been assigned by the grantee thereof to any other person, the said person, to whom it shall have been so assigned, shall be at liberty to assign it to whom he shall think fit, and in whatever time and manner he shall think fit, agreeable to the usual rules of law, without being obliged to make a previous offer of it to the parish officers and rateable inhabitants of the parish, or township, in which it was granted, as is required of the grantee thereof; and every subsequent assignee thereof shall have the same liberty.

If any person shall make any fraudulent alteration in any of the said parish deeds of grant, or, knowing any such fraudulent alteration to have been made, shall receive any money upon the same, he shall thereby become guilty of felony without benefit of clergy. And, upon his conviction the annuity shall cease.

And be it further enacted, That, if any person shall at any time make, or cause to be made, any alteration in any deed of grant of any such annuity, or any copy thereof which shall be made out as aforesaid, so as to make the annual sum thereby granted appear to be greater than it really was, or the time of the commencement of such annuity appear to be earlier than that which was appointed by the said deed, or shall know that any such alteration has been made in such deed or copy, and having made, or caused to be made, such alteration, or knowing the same to have been made therein, shall demand and receive of the parish officers of the parish or township in which such annuity was granted, any part of such annuity, according to the purport of such fraudulent alteration, every person so offending, and being convicted thereof, shall be deemed and adjudged guilty of felony, and shall suffer death as a felon, without benefit of clergy; and, upon such conviction, such annuity shall cease and determine.

And be it further enacted, That if any person shall forge or frame, or cause to be forged or framed, any deed of grant of any such parish annuity as is above mentioned, with an intent to defraud the inhabitants of the parish or township, in which such deed shall purport such annuity to have been granted, and shall for that purpose counterfeit, or cause to be counterfeited, the names of the parish officers which are herein required to be subscribed to every such deed of grant, and, by means of such forged deed, shall obtain from the said officers of the parish, or township, in which such forged deed shall purport the said annuity to have been granted, any sum of money whatsoever, as part of such pretended annuity, every person so offending, and being convicted thereof, shall be deemed and adjudged guilty of felony, and shall suffer death as a felon, without benefit of clergy.

If any person shall forge, or cause to be forged, any such deed of grant, and shall, by means of such forged deed, obtain from the officers of any parish any sum of money, he shall thereby become guilty of felony without benefit of clergy.

And be it further enacted, That if any person shall, by any means whatsoever, get possession of any real deed of grant of any such annuity, or any copy thereof, given by the parish, or township, wherein such annuity is granted, as is above mentioned, without having a right to the annuity thereby granted, either as the grantee thereof, or the lawful assignee thereof, by one or more assignment or assignments thereof, in the manner above prescribed, and shall falsely pretend to be the person lawfully intitled to such annuity, and under such pretence shall produce any such deed of grant to the parish officers of the parish or township in which such annuity was granted, and in consequence thereof shall demand and obtain, from the said officers, any sum of money whatsoever, as a part of such annuity, every person so offending, and being convicted thereof, shall be deemed and adjudged to be guilty of felony.

And, if any person shall, by any means, get possession of any real deed of grant of such an annuity, belonging to another person, and shall, by falsely pretending that he is the person to whom the said annuity belongs, obtain payment of any of the money due upon it, he shall thereby become guilty of felony.

And be it further enacted, That the several persons intitled to the said annuities shall apply to the said officers of the parishes, or townships, in which the same have been respectively granted, for the payment thereof, as soon as they conveniently can, after the several quarterly feast days in which they are made payable; and if any such annuitant shall neglect to apply (either by himself or his lawful attorney) for the payment of his annuity, to the said officers of the parish or township in which the same was granted, for more than the space of one year and a quarter, so that five or more quarterly payments thereof shall be in arrear, he shall not be entitled to receive more than the four last quarterly payments of such annuity, and shall forfeit his right to all the former payments so in arrear.

Any grantee of any one of these annuities who shall neglect to apply for payment of it for more than five quarters of a year, shall forfeit all the arrears due to him, except those of the last four quarters of a year.

Provided always, That nothing in this clause shall extend, or be construed to extend, to any grantee, who, from being beyond the seas, or any unavoidable incapacity, shall have been disabled from making his demand within

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A provisoe in favour of persons beyond the seas, or who shall have been under any unavoidable incapacity of making application for the payment of their said annuities in due time.

the time so limited, and shall make such demand within six months after his return from abroad, or the removal of such incapacity.

The money paid for the purchase of these parish-annuities shall be the property of the inhabitants of the parish that are chargeable to the poor rates; and shall be invested in the three per cent. bank annuities in their name.

And be it further enacted, That the money to be received by the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor of every parish or township, in which any such annuities shall be granted, shall be the property of all the inhabitants of such parish or township, chargeable to the poor rates. And the respective rectors, vicars, or perpetual curates, churchwardens, and overseers of the poor shall, and they are hereby required as soon as conveniently may be, after the receipt of any such purchase-money, to lay out the same in the purchase of some of the public annuities, established by authority of parliament, and payable at the bank of England, after the rate of three *per centum per annum*: which money so laid out, shall stand in the books of the governor and company of the said bank, in the name of the rector, vicar, or perpetual curate, churchwardens, overseers of the poor, and inhabitants of such parish or township, chargeable to the poor rates of the same; who shall be, and are hereby declared to be, a body politic and corporate, having perpetual succession for the several purposes of this act, and shall be known by the name of *The Rector, Vicar, or perpetual Curate, Churchwardens, Overseers of the Poor, and Inhabitants of such Parish or Township, chargeable to the Poor Rates of the same*, and shall have and use a common seal, on which seal shall be engraven the name of such parish or township, and the county, city, or town in which the same is situated, and such seal shall be carefully kept in the chest of such parish or township. And the rector, vicar, or perpetual curate of such parish or township, and the churchwardens and overseers of the poor thereof, or a majority of them, shall be, and are hereby constituted and declared to be, *the acting members* of such body corporate, and shall have power to purchase the said bank annuities, in the name of the whole body, and to receive the dividends of interest that shall become due thereupon, and to sell and transfer the said annuities whensoever they shall think fit.

And the rector, vicar, or perpetual curate, of the parish, and the churchwardens and overseers of the poor, or a majority of them, shall be *the acting members* of every such body corporate.

The said corporations may appoint agents, or attornies, to transact their business at the Bank of England.

And, for the more easy transacting the business relating to the said bank annuities, be it further enacted, That the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor of every such parish or township, or the majority of them, shall, and they are hereby empowered, with the concurrence of a majority of the rateable inhabitants assembled at a parish meeting, (such majority in number being also charged in the last rate made for the relief of the poor of the said parish or township, in a sum greater than that which was assessed in the same rate upon the rest of the inhabitants assembled at the said meeting,) upon notice given as aforesaid, to constitute and appoint any person or persons that they shall think proper, to

be their agent or attorney, agents or attornies, to purchase, sell, or transfer, in their stead and place, and in the behalf of the said body politick and corporate, the said bank annuities, or to receive the dividends of interest due thereupon; which appointment shall be by a letter, or power, of attorney for that purpose, in writing, sealed and delivered by a majority of them, the said rector, vicar, or perpetual curate, churchwardens and overseers of the poor in manner herein-after directed, and likewise signed by them in presence of two credible witnesses, who shall likewise sign their names thereto, in attestation of their having seen the said power so signed, sealed, and delivered; and shall continue in force until it shall be expressly revoked by another instrument in writing, made and executed in the same manner, by such rector, vicar, or perpetual curate, churchwardens and overseers of the poor, or the major part of them, as aforesaid, and attested by the same number of subscribing witnesses, as the said letter, or power, of attorney is hereby directed to be attested by. And every such letter, or power, of attorney shall (during the time it shall continue in force) be sufficient to empower the person to whom it was given, (if it shall purport so to empower him) to purchase and accept any stock in the said bank-annuities, for, and in behalf of, the said body politick and corporate; and likewise to receive, in the same behalf, all the dividends of interest that shall become due on the stock of such body; and likewise to sell and transfer all, or any part of, the stock which shall belong to such body corporate, either at the time of executing such letter of attorney, or at any time after, while such letter of attorney shall continue in force.

And be it further enacted, That all the money which shall be paid for the purchase of any of the said life-annuities, and which is herein before directed to be invested in the three *per centum* bank annuities, shall be kept and used as a fund, for the payment of the said life-annuities, as they shall become due, and shall not be applied to any other use, or purpose, whatsoever; and the respective rectors, vicars, or perpetual curates, and churchwardens and overseers of the poor of every parish or township, in which such life-annuities shall be granted, shall, and they are hereby required, either by themselves or their lawful attorney, empowered in the manner before-mentioned, to receive the dividends of interest that shall become due upon the said fund or stock of the said bodies politick and corporate respectively, in the said bank annuities, as often and as soon as they shall become due and payable, and to invest the money, arising by such dividends, immediately in the purchase of new stock in the said bank-annuities, so as thereby to encrease such parish fund continually, and enable it to furnish the payments of the several life-annuities charged thereupon, and to pay out of the said dividends such annuities as shall be then due to the persons entitled to receive the same, by equal quarterly payments in every year, at the four feast-days before-mentioned,

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or

The aforesaid money so invested in the three per cent. bank-annuities shall be kept and used as a fund for the payment of the said parish life-annuities, and shall be applied to no other use or purpose whatever.

or within seven days after each of the said feast days respectively; and in case such dividends shall not be sufficient for that purpose, to pay the same out of the fund so vested in bank-annuities; and for that purpose to sell and transfer, from time to time, whenever it shall be necessary, so much of the said fund or stock of the said parishes or townships respectively, in the said bank annuities, as shall be sufficient to enable them to make the said payments.

Excepting only the necessary charges of managing this fund; which may be defrayed out of it.

Provided always, and be it hereby further enacted, That it shall and may be lawful for the said rector, vicar, or perpetual curate, and churchwardens and overseers of the poor in every such parish or township, to defray the necessary charges and expences of investing the money received from the purchasers of the said life-annuities in the said bank-annuities, and of receiving the dividends thereof, and investing them in the purchase of fresh stock in the same, and of selling and transferring the same, and of preparing the said letters of attorney for the transacion of this business, and of procuring the aforesaid books for entering and registering copies of the grants of the said annuities, or any other books necessary for carrying this act into execution, and of every other thing necessary to be done by any other person than themselves in the execution of this act, out of the monies received by them from time to time from the purchasers of such life-annuities, or out of the dividends payable out of the said stock or fund, they keeping an exact account of the particulars of the said charges and expences; any thing to the contrary hereof above-mentioned in any-wise notwithstanding.

Sums of money given by charitable persons in aid of the funds to be established by this act shall be employed in the purchase of three percent bank-annuities in the same manner as the money paid for the purchase of the said life-annuities.

And be it further enacted, That if any sum or sums of money shall be given by charitable persons, or otherwise, for the purpose of enlarging the said original fund of any parish or township, and enabling it with more certainty to furnish the several payments of the life-annuities charged thereupon, without any more particular directions from the donor or donors, as to the manner of applying the same, the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor of such parish or township, shall, and they are hereby authorized and required to, invest the money arising by such gifts, or otherwise, in the said bank-annuities, in the same manner as the money contributed by the purchasers of the said life-annuities is herein-before directed to be invested, and to employ and use such money as an additional fund for the payment of the said life-annuities, in the same manner as they are herein-before authorized and required to employ the aforesaid original fund, contributed by the purchasers of the said life-annuities.

And, for securing the payment of the said life-annuities, be it further enacted, That if it shall at any time happen in any parish or township, where such

If these funds should prove insufficient for the payment of the life-annuities that shall have been granted by virtue of this act, the deficiencies shall be made good out of the poor's rate.

such life-annuities shall have been granted, that the aforesaid original fund arising from the contributions of the purchasers of the said life-annuities, together with the said additional fund, shall not be sufficient to furnish the quarterly payments of the said life-annuities, the churchwardens and overseers of the poor shall, and they are hereby required to, make the said payments out of any other money in their hands, raised for the relief of the poor of such parish or township; and if such money in their hands be not sufficient for that purpose, they shall, and are hereby required to, make such an addition to the rates, which they are empowered to make, by a statute made in the forty-third year of the reign of Queen Elizabeth, intituled, *An Act for the Relief of the Poor*, as shall be sufficient to supply the deficiencies of the said funds, and to enable them to compleat the said quarterly payments of the said life-annuities, and to defray the necessary charges and expences of executing this act; which said additional rate shall be made, levied, and recovered, under the same regulations and restrictions, and by the same ways and means, as the rates made for the relief of the poor are, by the said last-mentioned statute, or any subsequent act or acts of parliament, directed to be made, levied, and recovered.

And be it further enacted by the authority aforesaid, That, if any of the capital stock of any parish or township, in the said bank annuities, shall remain, after all the annuities shall become extinct, the interest of such stock shall be applied in aid of the rate to be raised and levied for the relief of the poor of the said parish or township, from time to time, as such interest shall become due; and it shall and may be lawful for the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor of such parish or township, with the advice and consent of the inhabitants of such parish or township, rateable towards the relief of the poor, or the major part of them, assembled in vestry, or other publick meeting for that purpose, (due notice of the said meeting being given in manner herein-before directed,) to sell and dispose of such capital stock, and to apply the money arising by such sale, either in repairing, amending, or new-building any alms-houses, in such parish or township, or furnishing them in a more convenient manner, or in building or furnishing new alms-houses, or in any other manner for the benefit of the poor of such parish or township, as the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor, and inhabitants assembled, shall order and direct.

And be it further enacted by the authority aforesaid, That every such rector, vicar, or perpetual curate, concerned in the execution of this act, who shall remove from any such parish, and every churchwarden, or overseer of the poor, at the time of going out of his office, and the executors or administrators

And, on the other hand, if after the extinction of all the life-annuities that have been granted in any parish, there shall remain any part of the said parish-fund in the said three per cent. bank-annuities, such fund shall be employed in aid of the poor rate in the said parish.

The acting members of these corporations who shall have any of the money belonging to the funds in their hands at the time of their removal from the parish or the expiration of their office, and their executors or administrators shall do the same after their deaths.

And, in case of neglect or refusal so to do, any two justices of the peace for the county in which the offender shall reside, may inquire into the matter in a summary way, and compel him to do justice.

ministrators of every such rector, vicar, or perpetual curate, churchwarden, or overseer of the poor, who shall happen to die, during the time he shall be concerned in the execution of this act, shall deliver in a true and perfect account of, and shall pay, or cause to be paid, all monies remaining in his or their hands, for the purposes of this act, to the rector, vicar, or perpetual curate, churchwardens, or overseers of the poor, for such parish or township, within one month after any such removal, going out of office, or death; and in case of neglect or refusal so to do, it shall and may be lawful for any two justices of the peace for the county or place where the offender shall reside, to make inquiry concerning the same, in a summary way, either by confession of the party, or by the testimony of any credible witness or witnesses upon oath, (which oath such justices are hereby empowered to administer;) and to cause the money remaining in the hands of such rector, vicar, or perpetual curate, churchwarden, overseer of the poor, executor, or administrator respectively, to be recovered by distress and sale of his goods and chattels, rendering the overplus to the owner of such goods and chattels, after deducting the charges attending such distress and sale; and such justices are hereby empowered to cause the books and accounts before-mentioned to be brought, and such witnesses to be summoned to appear before them as they shall think necessary for their information: and if any person shall refuse to appear, or to give evidence, or to produce such books and accounts, as aforesaid, to such justices, it shall and may be lawful for such justices to impose any fine or fines upon such person and persons so offending as they shall think fit, (so as no such fine shall exceed the sum of ten pounds upon any one person for one offence,) and to levy such fines by distress and sale of the offenders goods and chattels; and, if no such distress can be found, it shall and may be lawful for such justices to commit the offender to the common gaol of the county or place, for any time not exceeding six months, unless such fine shall be sooner paid.

And, in case the churchwardens and overseers of the poor shall neglect, or refuse, to pay any annuity for the space of seven days after it shall have become due and been demanded, any one justice of the peace of the county in which the parish is situated, may inquire into the matter in a summary way, and compel them to do justice.

And be it further enacted by the authority aforesaid, That, if any such annuity shall be behind and unpaid for the space of seven days after the same shall become due, and have been demanded, it shall and may be lawful for any one justice of the peace, for the county or place where the parish or township from which such annuity shall be so due and unpaid shall be situate, to make enquiry concerning the same, in a summary way, either by confession of the party, or by the testimony of any credible witness or witnesses upon oath, (which oath such justice is hereby empowered to administer,) and to cause the money so due to be levied by warrant under his hand and seal, by distress and sale of the goods and chattels of any one of the churchwarden or overseers of the poor of such parish, rendering the overplus to the owner of such goods and chattels, after deducting the charges attending such distress and sale.

And whereas, by sundry acts of parliament, several parishes and townships have been united into several corporations, and the management, maintenance, and regulation of the poor of such parishes and townships is thereby vested in the governors, guardians, and acting members of such corporations respectively; *be it enacted* by the authority aforesaid, That in all places where such corporations exist, and where the plan and method herein-before prescribed for granting annuities to industrious and frugal persons, shall have been approved by the majority of a general court, to be called and held for that purpose, the power of granting life-annuities in pursuance of this act to persons appearing to be legally settled in any parish or township, so incorporated as aforesaid, shall be vested in the governors, guardians, and acting members of the corporation, to which such parish or township does respectively belong, and not in the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor, and rateable inhabitants, of such parish or township; and the governors, guardians, and acting members of such corporation, assembled in a general court, or the major part of them so assembled, are hereby empowered to agree, according to the respective prices expressed in the set of tables hereunto annexed, No. 2, for the sale of any annuity or annuities for life, to any person or persons, appearing to them to have a lawful settlement in any parish or township, within the limits of the jurisdiction of the said corporation, the purchase-money not being less, nor the annuity or annuities greater, than the sums before limited, nor the commencement of any such annuity at any other age than as before limited for other parishes and townships; and upon such agreement made, the said governors, guardians, and acting members, are hereby empowered and required to receive the purchase-money for, and to do every act for granting and securing such life-annuity or annuities by deeds, to be executed by them in the same form and manner as other deeds are executed by them, for vesting the purchase-money received by them in the purchase of three per cent. bank annuities, in the name of the said corporation, for receiving the dividends, and transferring the said stock, so purchased by them, for paying, or causing to be paid, the life-annuities so granted by them, and for re-purchasing the same in the manner before directed, which the several rectors, vicars, or perpetual curates, churchwardens, and overseers of the poor of other parishes and townships, are by this act empowered or required to do; for all and every the purposes before mentioned, and in the same form and manner, as near as the institution of such corporation, and the several regulations touching its manner of acting and proceeding, will admit. And in case at any time the fund or funds for securing such life-annuities shall prove deficient, the governors, guardians, and acting members of every such corporation, are hereby respectively empowered and required to pay the same out of any money in their hands, applicable to the maintenance:

The foregoing regulations shall extend to such parishes and townships as have been united together, by acts of parliament, into several corporations for the purposes of maintaining and managing their poor.

And they shall be carried into execution by the governors, guardians, and acting members of such corporations.

nance of the poor of the parish or township in which such annuitant appeared to have been lawfully settled at the time of granting such annuity, or to make, or cause to be made, a sufficient rate upon such inhabitants within the limits of the jurisdiction of the said corporation, as would have been liable to contribute to the relief of the poor of such parish or township; and the said governors, guardians, and acting members of the said corporations, are hereby further empowered and required to compel the several persons concerned in the receipt and payment of money, upon account of such life-annuities, to produce and pass their accounts of all monies received and paid by them, in such manner, and by such means, as the several accountants are compellable to account for other monies received and paid by them, by the said several acts of parliament; and no consent of the rateable inhabitants of the said several parishes and townships shall be requisite to make valid, or confirm any of the said proceedings of the governors, guardians, and acting members of such corporations.

And the aforesaid penal clauses against persons guilty of frauds with respect to annuities granted by single parishes shall extend to persons guilty of the like frauds with respect to annuities granted by the said corporations consisting of several parishes united together.

And be it further enacted by the authority aforesaid, That every clause, provision, and regulation herein contained, as well with respect to persons who shall alter, forge, frame, in manner before mentioned, or, not being the owner thereof, shall get possession of, and demand and obtain money for, any grant, or copy of any grant, of any annuity, to be granted by virtue of this act, by rectors, vicars, or perpetual curates, churchwardens, and overseers of the poor, as with respect to every other matter or thing directed to be done by them in the execution hereof, shall, in every case where, and as far as it can be made applicable, extend, and take place, also with respect to such grants, or copies of grants, of annuities, which, in pursuance hereof, shall be granted by the said corporations.

And be it further enacted by the authority aforesaid, That in all parishes and townships, which have not been united into several corporations, for the management, maintenance, and regulation of their poor, the following form shall be observed in all powers of attorney to be executed by the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor of such parishes or townships, for the acceptance of stock, and the receipt of dividends due thereupon; that is to say,

The Parish of in the County of

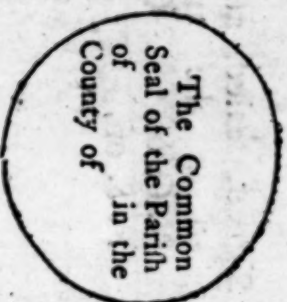
The form of the powers of attorney which shall be granted by single parishes for the acceptance of stock in the three per cent. bank-annuities and the receipt of dividends due thereupon.

“ **K** NOW all men by these presents, That we, the rector, (vicar, or perpetual curate, as the case may happen,) churchwardens and overseers of the poor, and rateable inhabitants of the parish of in the county of in vestry assembled, by virtue

“ of an act of Parliament of the thirteenth year of the reign of his Majesty
 “ King George the Third, (intituled, *An Act for the better Support of*
 “ *Poor Persons in certain Circumstances, by enabling Parishes to grant them*
 “ *Annuities for Life upon Purchase, and under certain Restrictions,*) do make,
 “ constitute, and appoint, *A. B.* of the parish of _____
 “ in the county of _____ Gentleman, our true and
 “ lawful attorney, for us, in our names, and on our behalf, to accept all
 “ transfers that are, or may, at any time or times, be made unto us, the
 “ said rector, (*vicar, or perpetual curate, as the case may happen,*) church-
 “ warden, overseers of the poor, and rateable inhabitants of the parish
 “ of _____ in the county of _____ and
 “ our successors, of any interest or shares in the capital, or joint, stock of
 “ three *per cent.* reduced, (*or consolidated,*) bank annuities, (*as the case may*
 “ *happen*); and also, on our behalf, to receive, and give receipts for, all divi-
 “ dends and interest that shall grow due and payable, on our interest or
 “ share in the above-said capital, or joint-stock, or on any part thereof, for
 “ the time being, and to do all lawful acts requisite for effecting the pre-
 “ mises, hereby ratifying and confirming all that our said attorney shall do
 “ therein by virtue hereof. In witness whereof, we, the acting members
 “ of the said body corporate, have hereunto set our hands, and affixed
 “ our common seal, the _____ day of _____
 “ in the year of our Lord _____

Signed, sealed, and deliver- } *D. E.* Rector of the
 ed in the presence of } Parish of _____ in
N. O. of } the County of _____
R. S. of } Yeoman.

J. R. } Churchwardens of
S. M. } the said Parish.
T. N. } Overseers of the
E. A. } Poor of the said
F. M. } Parish.



And in all such parishes or townships the following form shall be ob-
 served in all powers of attorney, for the transfer of all such stock as shall
 have been purchased by them; that is to say,

L I I I

K N O W

The Parish of in the County of

The form of the powers of attorney which shall be granted by single parish for the selling and transferring of stock in the said 3 per cent. bank annuities.

“ **K** NOW all men by these presents, That we, the rector, (vicar, or perpetual curate, as the case may happen,) churchwardens, overseers of the poor, and rateable inhabitants of the parish of in the county of act of parliament, of the thirteenth year of the reign of his Majesty King George the Third, (intituled, *An Act for the better Support of Poor Persons in certain Circumstances, by enabling Parishes to grant them Annuities for Life, upon Purchase, and under certain Restrictions,*) do make, constitute, and appoint A. B. of the parish of in the county of Gentleman, our true and lawful attorney, for us, in our names, and on our behalf, to sell, assign, and transfer or any part thereof, being part of [or all] the interest or share in the capital, or joint stock, of three per cent. reduced (or consolidated) bank-annuities, (as the case may happen) standing in the names of us the rector, (vicar, or perpetual curate, as the case may happen,) churchwardens, overseers of the poor, and rateable inhabitants of the said parish of in the county of and to receive the consideration-money, and to give discharges and receipts for the same, and to do all lawful acts requisite for effecting the premises, hereby ratifying and confirming all that our said attorney shall do herein by virtue hereof. In witness whereof, we, the acting members of the said body corporate, have hereunto set our hands, and affixed our common seal, the day of in the year of our Lord

Signed, sealed, and delivered in the presence of
N. O. of, Gentleman,
R. S. of, Yeoman.

D. E. Rector of the Parish of in the County of

J. R. } Churchwardens of
S. M. } the said Parish.

T. N. } Overseers of the
E. A. } Poor of the said
F. M. } Parish.

The Common
Seal of the Parish
of in the
County of

T A B L E S

SHEWING THE VALUES,

IN A SINGLE PRESENT PAYMENT,

OF AN

ANNUITY OF ONE POUND,

PAYABLE QUARTERLY,

For the Lives of Persons of all Ages, from 15 to 73;

Such Annuity being supposed to commence at any Age, not
younger than 35 nor older than 75 Years.

FLBA 2

SHAW AND SONS

ИЗДАНИЕ ПЕРВОЕ. БУДАНОВ.

OLVA

UNITED STATES OF AMERICA

БѢЛЫЕ. ОНЪ КЛЕВЕТЪ.

THE HISTORY OF THE

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FOR USING THE FOLLOWING

T A B L E S.

Drawn up by the Rev. Dr. Richard Price, of Newington-Green,

near Mlington, in the County of Middlesex.

THESE tables give the present payments that ought to be made (reckoning the interest at three per cent.) by persons of all ages, from 15 to 73, for a life-annuity of one pound, payable quarterly, (that is, five shillings each quarter,) to commence at any given age not less than 35, nor exceeding 75, having only one child living.

For instance, Table the first, in the second set of tables, shews, that the payment due from a person whose age is between 14 $\frac{1}{2}$ and 15 $\frac{1}{2}$ is £4 os. 1d. for a life-annuity of one pound, payable quarterly, to commence when he attains to 14 $\frac{1}{2}$ years of age; or (more exactly) at the end of 28 years and a quarter from the day on which the payment is made. In like manner the same table shews, that the present payment due from a person of the same age, for the same annuity, is *one pound*, supposing the first quarterly payment of the annuity to be made at the end of forty-five years and a quarter, or not till he attains to 60 years of age; and *six shillings*, if he chuses to wait for the first quarterly payment 55 years and a quarter, or till he attains to 70 years of age.

A life-annuity therefore of £2. to commence at these different periods, must be worth double these sums; and a life-annuity of £10. must be worth ten times these sums.

Universally,

Universally therefore. In order to find from the following tables the present money that ought to be given for any annuity payable quarterly to purchasers of given ages, after attaining any other given ages—Multiply the value in the table for the given age, by the number of pounds in the proposed annuity, and the product will be the answer.

Example. From table 7th, in the second set of tables, (calculated for the country,) it will appear by inspection, that the value to a purchaser aged 21, of an annuity of £1. for life, payable quarterly, to commence at 50 years of age, is £3. 15. 2d. If therefore the annuity is £5. its value will be this sum multiplied by 5, or £15. 5s. 10d. If the annuity is £20. its value will be the same multiplied by 20, or £61. 3s. 4d.

Second Example. Suppose a person whose age is 25, to apply for an annuity (payable quarterly) of £10. for his life, after attaining his 55th year. From table 11, in the second set of tables, it appears, that, if the annuity had been one pound, its present value would have been £2. 9s. therefore the annuity being £10. its value is £2. 9s. multiplied by 10, or £24. 10s.

The value of the same annuity, according to table 11, in the first set of tables, calculated for London, is £1. 14s. 3d. multiplied by 10, or £17. 2s. 6d.

Again, Suppose a person of a given age to desire to be informed what annuity, as a provision for old age, he can purchase with a given sum of money, of which he is in possession. In order to discover the answer, look over the table for the given age, and find there the given sum 3 or, if it be not found exactly, find the sum nearest to it; and the correspondent age will shew, that with the given sum he may purchase an annuity of one pound for life, to commence at that age. In like manner, the ages corresponding to half, a third, a fourth, &c. of the sum will shew, that it will purchase for him an annuity of £2. £3. £4. &c. to commence at those ages respectively.

Example. A poor person, aged 22, has saved by his industry the sum of £5. and with this in his hands, he applies for such an annuity, to commence in some future year of his life, as he can purchase for him.

By looking over the table for the age of 22, or table 8 in the second set of tables, it will appear, that £4. 19s. 11d. or, very nearly the sum he offers, will purchase for him an annuity of one pound for his life,

to commence at the age of 44 years; and it appears also from the same table, that £2. 10s. 3d. (or about half his money,) will purchase the same annuity, to commence when he is 53; and that £1. 4s. 2d. (or about a quarter of his money,) will purchase the same annuity, to commence when he is 61; and that 19s. 7d. (or about a fifth part of his money,) will purchase the same annuity, to commence when he is 63. If, therefore, he thinks an annuity of one pound for his life, to commence at 44, too little, he may be offered for £5. os. 6d. £4. 16s. 8d. £4. 17s. 11d. (that is for sums nearly equal to £5.) an annuity of £2. for life, to commence at 53, or of £4. for life, to commence at 61, or of £5. for life, to commence at 63. From table 8, in the first set of tables, it will in the same way appear, that in London £5. would intitle such a person to a life-annuity of either £2. to commence at 49, or of £4. to commence at 57, or of £5. to commence at 59, or four years earlier than by the second set of tables, which are calculated for the country.

Observation 1st. The payments of persons who happen to die before the age agreed on for the commencement of their annuities, are in these tables supposed to be an advantage shared amongst survivors, without which the money advanced would be insufficient to bear the expence of the annuities.

These tables also suppose, that annuitants will be intitled to nothing for any part of that quarter of the year in which they shall happen to die.

Observation 2^d. It should be remembered, that the first of the three columns in these tables is intended to be an explanation of the column next to it, and to express with more precision the time at which the calculations suppose the annuity to commence. — Thus, if it were only expressed in table 1st, that a person, whose age is exactly 14½, would be entitled, for a present payment of £2. 7s. 11d. to an annuity, payable quarterly, of £1. for life, to commence at the age of 50, it would only appear that the annuity was to commence some time or other after he had attained to that age, or entered his 51st year. But the first column removes this uncertainty, by specifying, that the first quarterly payment is to be made at the end of 35 years and a quarter after purchasing, or exactly upon his attaining the age of 50. In like manner; supposing his age 15, 15 and a quarter, or 15 and a half, the same column, by specifying that the annuity was to commence at the end of 35 years and ⅔, would shew, that the first payment was to be made when he came to be a quarter of a year, half a year, or three quarters of a year, turned of 50.

Observations

Observation 3d. When there is any uncertainty with respect to the precise age of a purchaser, a younger age should always be taken, rather than an older, in order to guard against the losses to parishes, that would arise from intitling persons to higher annuities than are adequate to their payments. Much, however, will not depend on determining the age of any purchaser to greater exactness than half a year, or a year.

Observation 4th. A considerable difference will be found at all ages under 45 or 50 years between the values in the following tables for London, and for the country. The reason is, that the inhabitants of London, and of great towns in general, are much more short-lived than the inhabitants of small towns, and country parishes and villages. This appears from undeniable observations, and has created the necessity of calculating distinct tables for London and the country.

It may be proper to add, for the satisfaction of those who may wish to examine the following tables, that they have been calculated in the method explained and demonstrated, by Mr. Maseres, in a pamphlet intitled, *A Proposal for establishing Life-Annuities in Parishes, for the Benefit of the industrious Poor*, or by a rule in Dr. Price's Treatise on Annuities, Quest. 6th, page 17th.—In calculating the second set of tables, the probabilities of life at Northampton, as given in Table 4th, page 323, of the said Treatise of Dr. Price, have been combined with the values of lives in Table 6th, page 325. And in calculating the first set of tables, the probabilities and values of lives have been taken from Mr. Simpson's tables given in pages 332, and 334, of the same Treatise, or in page 254, and 260, of Mr. Simpson's Select Exercises, with no other than the following difference.—The tables which have been mentioned give the values of lives in yearly payments only. An annuity, payable quarterly, is worth three eighths of a years purchase, or 7s. 6d. more than an annuity payable yearly. Three eighths, therefore, or, in decimals, .375, have been always added to the values of lives taken from these tables, in order to obtain from thence the values in the following tables.

FIRST

FIRST SET OF TABLES.

Intended for the Use of LONDON.

NOTED BY THE ACTING SURGEON-GENERAL

T A B L E I.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 14½ to 15½.

To commence at the end of Years after purchasing.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	Value of an annuity of £1. in one present payment.
Or at Age 35	Or at Age 56		
20½	41½	5	1
21½	42½	5 10	0
22½	43½	5 3	0 18
23½	44½	4 16	0 16
24½	45½	4 9	0 14
25½	46½	3 3	0 13
26½	47½	3 17	0 12
27½	48½	3 11	0 10
28½	49½	3 6	0 9
29½	50½	3 1	0 8
30½	51½	2 16	0 7
31½	52½	2 12	0 6
32½	53½	2 8	0 5
33½	54½	2 4	0 4
34½	55½	2 0	0 4
35½	56½	1 17	0 3
36½	57½	1 14	0 3
37½	58½	1 11	0 3
38½	59½	1 9	0 2
39½	60½	1 6	0 2
40½	61½	1 4	0 1
41½	62½	1 2	0 1
42½	63½	0 10	0 1
43½	64½	0 9	0 1
44½	65½	0 8	0 1
45½	66½	0 7	0 1
46½	67½	0 6	0 1
47½	68½	0 5	0 1
48½	69½	0 4	0 1
49½	70½	0 4	0 1
50½	71½	0 3	0 1
51½	72½	0 3	0 1
52½	73½	0 3	0 1
53½	74½	0 2	0 1
54½	75½	0 2	0 1
55½	76½	0 1	0 1

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T A B L E

TABLE II.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $15\frac{1}{4}$ to $16\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
19 $\frac{1}{4}$, or at Age 35	40 $\frac{1}{4}$, or at Age 56	£. 5. 15	£. 5. 11
20 $\frac{1}{4}$ —	41 $\frac{1}{4}$ —	5 7 11	0 18 11
21 $\frac{1}{4}$ —	42 $\frac{1}{4}$ —	5 0 6	0 17 2
22 $\frac{1}{4}$ —	43 $\frac{1}{4}$ —	4 13 8	0 15 6
23 $\frac{1}{4}$ —	44 $\frac{1}{4}$ —	4 6 11	0 14 0
24 $\frac{1}{4}$ —	45 $\frac{1}{4}$ —	4 0 9	0 12 7
25 $\frac{1}{4}$ —	46 $\frac{1}{4}$ —	3 14 7	0 11 4
26 $\frac{1}{4}$ —	47 $\frac{1}{4}$ —	3 9 2	0 10 2
27 $\frac{1}{4}$ —	48 $\frac{1}{4}$ —	3 3 10	0 9 1
28 $\frac{1}{4}$ —	49 $\frac{1}{4}$ —	2 19 1	0 8 0
29 $\frac{1}{4}$ —	50 $\frac{1}{4}$ —	2 14 5	0 7 2
30 $\frac{1}{4}$ —	51 $\frac{1}{4}$ —	2 10 4	0 6 3
31 $\frac{1}{4}$ —	52 $\frac{1}{4}$ —	2 6 3	0 5 6
32 $\frac{1}{4}$ —	53 $\frac{1}{4}$ —	2 2 9	0 4 9
33 $\frac{1}{4}$ —	54 $\frac{1}{4}$ —	1 19 3	0 4 2
34 $\frac{1}{4}$ —	55 $\frac{1}{4}$ —	1 16 2	0 3 8
35 $\frac{1}{4}$ —	56 $\frac{1}{4}$ —	1 13 2	0 3 2
36 $\frac{1}{4}$ —	57 $\frac{1}{4}$ —	1 10 8	0 2 8
37 $\frac{1}{4}$ —	58 $\frac{1}{4}$ —	1 7 3	0 2 3
38 $\frac{1}{4}$ —	59 $\frac{1}{4}$ —	1 5 11	0 1 11
39 $\frac{1}{4}$ —			

TABLE

T A B L E III.

For the Life of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 16 $\frac{1}{2}$ to 17 $\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
18 $\frac{1}{2}$ or at Age 35	£. 6 s. 0 d.	39 $\frac{1}{2}$ or at Age 56	£. 1 s. 1 d.
19 $\frac{1}{2}$ — 36	5 12 7	40 $\frac{1}{2}$ — 57	0 19 9
20 $\frac{1}{2}$ — 37	5 5 1	41 $\frac{1}{2}$ — 58	0 17 9
21 $\frac{1}{2}$ — 38	4 17 7	42 $\frac{1}{2}$ — 59	0 16 2
22 $\frac{1}{2}$ — 39	4 10 11	43 $\frac{1}{2}$ — 60	0 14 7
23 $\frac{1}{2}$ — 40	4 4 2	44 $\frac{1}{2}$ — 61	0 13 2
24 $\frac{1}{2}$ — 41	3 18 0	45 $\frac{1}{2}$ — 62	0 11 10
25 $\frac{1}{2}$ — 42	3 11 11	46 $\frac{1}{2}$ — 63	0 10 8
26 $\frac{1}{2}$ — 43	3 6 9	47 $\frac{1}{2}$ — 64	0 9 6
27 $\frac{1}{2}$ — 44	3 1 7	48 $\frac{1}{2}$ — 65	0 8 5
28 $\frac{1}{2}$ — 45	2 17 17	49 $\frac{1}{2}$ — 66	0 7 4
29 $\frac{1}{2}$ — 46	2 12 4	50 $\frac{1}{2}$ — 67	0 6 8
30 $\frac{1}{2}$ — 47	2 8 5	51 $\frac{1}{2}$ — 68	0 5 8
31 $\frac{1}{2}$ — 48	2 4 4	52 $\frac{1}{2}$ — 69	0 5 0
32 $\frac{1}{2}$ — 49	2 1 1	53 $\frac{1}{2}$ — 70	0 4 4
33 $\frac{1}{2}$ — 50	1 17 9	54 $\frac{1}{2}$ — 71	0 3 10
34 $\frac{1}{2}$ — 51	1 14 9	55 $\frac{1}{2}$ — 72	0 3 4
35 $\frac{1}{2}$ — 52	1 11 9	56 $\frac{1}{2}$ — 73	0 2 11
36 $\frac{1}{2}$ — 53	1 9 0	57 $\frac{1}{2}$ — 74	0 2 5
37 $\frac{1}{2}$ — 54	1 6 3	58 $\frac{1}{2}$ — 75	0 2 0
38 $\frac{1}{2}$ — 55	1 4 4		

TABLE IV.

For the Use of LONDON.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $17\frac{1}{2}$ to $18\frac{1}{2}$.

To commence at the end of	Value of an annuity of £1. in one present payment.	To commence at the end of	Value of an annuity of £1. in one present payment.
Years after purchasing.	£. s. d.	Years after purchasing.	£. s. d.
17 $\frac{1}{2}$, or at Age 35	6 5 8	38 $\frac{1}{2}$, or at Age 56	1 2 9
18 $\frac{1}{2}$	5 17 7	39 $\frac{1}{2}$	1 0 6
19 $\frac{1}{2}$	5 9 6	40 $\frac{1}{2}$	0 18 8
20 $\frac{1}{2}$	5 2 1	41 $\frac{1}{2}$	0 16 10
21 $\frac{1}{2}$	4 14 8	42 $\frac{1}{2}$	0 15 3
22 $\frac{1}{2}$	4 8 0	43 $\frac{1}{2}$	0 13 9
23 $\frac{1}{2}$	4 1 3	44 $\frac{1}{2}$	0 12 4
24 $\frac{1}{2}$	3 15 5	45 $\frac{1}{2}$	0 11 1
25 $\frac{1}{2}$	3 9 7	46 $\frac{1}{2}$	0 9 11
26 $\frac{1}{2}$	3 4 5	47 $\frac{1}{2}$	0 8 9
27 $\frac{1}{2}$	2 19 3	48 $\frac{1}{2}$	0 7 9
28 $\frac{1}{2}$	2 14 10	49 $\frac{1}{2}$	0 6 9
29 $\frac{1}{2}$	2 10 5	50 $\frac{1}{2}$	0 6 0
30 $\frac{1}{2}$	2 6 7	51 $\frac{1}{2}$	0 5 3
31 $\frac{1}{2}$	2 2 10	52 $\frac{1}{2}$	0 4 7
32 $\frac{1}{2}$	1 19 6	53 $\frac{1}{2}$	0 4 0
33 $\frac{1}{2}$	1 16 2	54 $\frac{1}{2}$	0 3 6
34 $\frac{1}{2}$	1 13 2	55 $\frac{1}{2}$	0 3 0
35 $\frac{1}{2}$	1 10 2	56 $\frac{1}{2}$	0 2 7
36 $\frac{1}{2}$	1 7 7	57 $\frac{1}{2}$	0 2 2
37 $\frac{1}{2}$	1 5 0		

TABLE

T A B L E V.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $18\frac{1}{2}$ to $19\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
£. s. d.	£. s. d.	£. s. d.	£. s. d.
16 $\frac{1}{4}$, or at Age 35	37 $\frac{1}{4}$, or at Age 56	6 11 6	1 3 8
17 $\frac{1}{4}$, —	38 $\frac{1}{4}$, —	6 2 9	1 1 6
18 $\frac{1}{4}$, —	39 $\frac{1}{4}$, —	5 14 6	0 19 4
19 $\frac{1}{4}$, —	40 $\frac{1}{4}$, —	5 6 5	0 17 8
20 $\frac{1}{4}$, —	41 $\frac{1}{4}$, —	4 19 1	0 16 0
21 $\frac{1}{4}$, —	42 $\frac{1}{4}$, —	4 11 9	0 14 5
22 $\frac{1}{4}$, —	43 $\frac{1}{4}$, —	4 5 1	0 12 11
23 $\frac{1}{4}$, —	44 $\frac{1}{4}$, —	3 18 5	0 11 7
24 $\frac{1}{4}$, —	45 $\frac{1}{4}$, —	3 12 9	0 10 4
25 $\frac{1}{4}$, —	46 $\frac{1}{4}$, —	3 7 2	0 9 2
26 $\frac{1}{4}$, —	47 $\frac{1}{4}$, —	3 2 1	0 8 1
27 $\frac{1}{4}$, —	48 $\frac{1}{4}$, —	2 17 0	0 7 2
28 $\frac{1}{4}$, —	49 $\frac{1}{4}$, —	2 12 9	0 6 6
29 $\frac{1}{4}$, —	50 $\frac{1}{4}$, —	2 8 6	0 5 9
30 $\frac{1}{4}$, —	51 $\frac{1}{4}$, —	2 4 10	0 4 2
31 $\frac{1}{4}$, —	52 $\frac{1}{4}$, —	2 1 1	0 4 2
32 $\frac{1}{4}$, —	53 $\frac{1}{4}$, —	1 17 10	0 3 7
33 $\frac{1}{4}$, —	54 $\frac{1}{4}$, —	1 14 7	0 3 1
34 $\frac{1}{4}$, —	55 $\frac{1}{4}$, —	1 11 7	0 2 7
35 $\frac{1}{4}$, —	56 $\frac{1}{4}$, —	1 8 7	0 2 3
36 $\frac{1}{4}$, —	57 $\frac{1}{4}$, —	1 6 2	

T A B L E

TABLE VI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $19\frac{1}{2}$ to $20\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
15 $\frac{1}{2}$, or at Age 35	£. 6 s. 17 d.	36 $\frac{1}{2}$, or at Age 56	£. 1 s. 4 d.
16 $\frac{1}{2}$ —	6 17 8	37 $\frac{1}{2}$ —	1 4 10
17 $\frac{1}{2}$ —	5 19 3	38 $\frac{1}{2}$ —	1 2 5
18 $\frac{1}{2}$ —	5 11 4	39 $\frac{1}{2}$ —	1 0 5
19 $\frac{1}{2}$ —	5 3 3	40 $\frac{1}{2}$ —	0 18 8
20 $\frac{1}{2}$ —	4 15 11	41 $\frac{1}{2}$ —	0 16 0
21 $\frac{1}{2}$ —	4 8 8	42 $\frac{1}{2}$ —	0 15 6
22 $\frac{1}{2}$ —	4 2 3	43 $\frac{1}{2}$ —	0 13 1
23 $\frac{1}{2}$ —	3 15 10	44 $\frac{1}{2}$ —	0 12 10
24 $\frac{1}{2}$ —	3 10 3	45 $\frac{1}{2}$ —	0 10 7
25 $\frac{1}{2}$ —	3 4 8	46 $\frac{1}{2}$ —	0 9 6
26 $\frac{1}{2}$ —	2 19 10	47 $\frac{1}{2}$ —	0 8 5
27 $\frac{1}{2}$ —	2 14 11	48 $\frac{1}{2}$ —	0 7 7
28 $\frac{1}{2}$ —	2 10 8	49 $\frac{1}{2}$ —	0 6 8
29 $\frac{1}{2}$ —	2 6 8	50 $\frac{1}{2}$ —	0 5 0
30 $\frac{1}{2}$ —	2 3 0	51 $\frac{1}{2}$ —	0 4 4
31 $\frac{1}{2}$ —	1 19 5	52 $\frac{1}{2}$ —	0 3 10
32 $\frac{1}{2}$ —	1 16 2	53 $\frac{1}{2}$ —	0 3 3
33 $\frac{1}{2}$ —	1 12 11	54 $\frac{1}{2}$ —	0 2 10
34 $\frac{1}{2}$ —	1 10 0	55 $\frac{1}{2}$ —	0 2 5
35 $\frac{1}{2}$ —	1 7 2		

TABLE

T A B L E VII.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from 20 $\frac{1}{2}$ to 21 $\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
14 $\frac{1}{2}$, Or at Age 35	£. 7. 3. d. 5	35 $\frac{1}{2}$, Or at Age 56	£. 5. 10. d. 10
15 $\frac{1}{2}$, —	6 13 11	36 $\frac{1}{2}$, —	1 3 6
16 $\frac{1}{2}$, —	6 5 0	37 $\frac{1}{2}$, —	1 1 1
17 $\frac{1}{2}$, —	5 16 1	38 $\frac{1}{2}$, —	0 19 3
18 $\frac{1}{2}$, —	5 8 1	39 $\frac{1}{2}$, —	0 17 5
19 $\frac{1}{2}$, —	5 0 1	40 $\frac{1}{2}$, —	0 15 8
20 $\frac{1}{2}$, —	4 12 10	41 $\frac{1}{2}$, —	0 14 0
21 $\frac{1}{2}$, —	4 5 7	42 $\frac{1}{2}$, —	0 12 7
22 $\frac{1}{2}$, —	3 19 5	43 $\frac{1}{2}$, —	0 11 3
23 $\frac{1}{2}$, —	3 13 3	44 $\frac{1}{2}$, —	0 10 0
24 $\frac{1}{2}$, —	3 7 2	45 $\frac{1}{2}$, —	0 8 10
25 $\frac{1}{2}$, —	2 17 7	46 $\frac{1}{2}$, —	0 7 10
26 $\frac{1}{2}$, —	2 12 11	47 $\frac{1}{2}$, —	0 6 10
27 $\frac{1}{2}$, —	2 8 10	48 $\frac{1}{2}$, —	0 6 0
28 $\frac{1}{2}$, —	2 4 10	49 $\frac{1}{2}$, —	0 5 2
29 $\frac{1}{2}$, —	2 1 3	50 $\frac{1}{2}$, —	0 4 7
30 $\frac{1}{2}$, —	1 17 11	51 $\frac{1}{2}$, —	0 3 11
31 $\frac{1}{2}$, —	1 14 6	52 $\frac{1}{2}$, —	0 3 5
32 $\frac{1}{2}$, —	1 11 2	53 $\frac{1}{2}$, —	0 2 15
33 $\frac{1}{2}$, —	1 8 6	54 $\frac{1}{2}$, —	0 2 7
34 $\frac{1}{2}$, —	—		
35	—		

T A B L E

TABLE VIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 21 $\frac{1}{4}$ to 22 $\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
13 $\frac{1}{4}$, or at Age 35	34 $\frac{1}{4}$, or at Age 56	£. 5. 4.	£. 5. 4.
14 $\frac{1}{4}$, —	35 $\frac{1}{4}$, —	7 7 1	7 7 1
15 $\frac{1}{4}$, —	36 $\frac{1}{4}$, —	4 4 5	4 4 5
16 $\frac{1}{4}$, —	37 $\frac{1}{4}$, —	2 2 3	2 2 3
17 $\frac{1}{4}$, —	38 $\frac{1}{4}$, —	0 0 0	0 0 0
18 $\frac{1}{4}$, —	39 $\frac{1}{4}$, —	18 18 2	18 18 2
19 $\frac{1}{4}$, —	40 $\frac{1}{4}$, —	16 16 4	16 16 4
20 $\frac{1}{4}$, —	41 $\frac{1}{4}$, —	14 14 9	14 14 9
21 $\frac{1}{4}$, —	42 $\frac{1}{4}$, —	13 13 1	13 13 1
22 $\frac{1}{4}$, —	43 $\frac{1}{4}$, —	11 11 9	11 11 9
23 $\frac{1}{4}$, —	44 $\frac{1}{4}$, —	10 10 5	10 10 5
24 $\frac{1}{4}$, —	45 $\frac{1}{4}$, —	9 9 3	9 9 3
25 $\frac{1}{4}$, —	46 $\frac{1}{4}$, —	8 8 1	8 8 1
26 $\frac{1}{4}$, —	47 $\frac{1}{4}$, —	7 7 2	7 7 2
27 $\frac{1}{4}$, —	48 $\frac{1}{4}$, —	6 6 3	6 6 3
28 $\frac{1}{4}$, —	49 $\frac{1}{4}$, —	5 5 6	5 5 6
29 $\frac{1}{4}$, —	50 $\frac{1}{4}$, —	4 4 9	4 4 9
30 $\frac{1}{4}$, —	51 $\frac{1}{4}$, —	4 4 2	4 4 2
31 $\frac{1}{4}$, —	52 $\frac{1}{4}$, —	3 3 6	3 3 6
32 $\frac{1}{4}$, —	53 $\frac{1}{4}$, —	3 3 0	3 3 0
33 $\frac{1}{4}$, —	54 $\frac{1}{4}$, —	2 2 7	2 2 7
	55 $\frac{1}{4}$, —		

TABLE

T A B L E IX.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $22\frac{1}{2}$ to $23\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
12 $\frac{1}{2}$, or at Age 35	7 16 6	33 $\frac{1}{2}$, or at Age 56	1 8 2
13 $\frac{1}{2}$	7 6 2	34 $\frac{1}{2}$	1 5 8
14 $\frac{1}{2}$	6 16 5	35 $\frac{1}{2}$	1 3 1
15 $\frac{1}{2}$	6 6 9	36 $\frac{1}{2}$	1 1 0
16 $\frac{1}{2}$	5 18 0	37 $\frac{1}{2}$	0 19 0
17 $\frac{1}{2}$	5 9 2	38 $\frac{1}{2}$	0 17 2
18 $\frac{1}{2}$	5 1 3	39 $\frac{1}{2}$	0 15 4
19 $\frac{1}{2}$	4 13 4	40 $\frac{1}{2}$	0 13 10
20 $\frac{1}{2}$	4 4 8	41 $\frac{1}{2}$	0 12 3
21 $\frac{1}{2}$	4 0 0	42 $\frac{1}{2}$	0 10 11
22 $\frac{1}{2}$	3 13 11	43 $\frac{1}{2}$	0 9 7
23 $\frac{1}{2}$	3 7 11	44 $\frac{1}{2}$	0 8 6
24 $\frac{1}{2}$	3 3 10	45 $\frac{1}{2}$	0 7 5
25 $\frac{1}{2}$	2 17 9	46 $\frac{1}{2}$	0 6 6
26 $\frac{1}{2}$	2 13 4	47 $\frac{1}{2}$	0 5 8
27 $\frac{1}{2}$	2 9 0	48 $\frac{1}{2}$	0 5 0
28 $\frac{1}{2}$	2 5 1	49 $\frac{1}{2}$	0 4 4
29 $\frac{1}{2}$	2 1 2	50 $\frac{1}{2}$	0 3 9
30 $\frac{1}{2}$	1 17 8	51 $\frac{1}{2}$	0 3 2
31 $\frac{1}{2}$	1 14 1	52 $\frac{1}{2}$	0 2 8
32 $\frac{1}{2}$	1 11 2		

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TABLE

TABLE X.

For the Life of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $23\frac{1}{2}$ to $24\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
11 $\frac{1}{4}$, or at Age 35	£. 8 s. 4 d.	32 $\frac{1}{4}$, or at Age 56	£. 5 s. 4 d.
12 $\frac{1}{4}$	7 13 9	33 $\frac{1}{4}$	1 9 8
13 $\frac{1}{4}$	7 2 3	34 $\frac{1}{4}$	1 6 9
14 $\frac{1}{4}$	6 13 0	35 $\frac{1}{4}$	1 4 4
15 $\frac{1}{4}$	6 3 5	36 $\frac{1}{4}$	1 2 0
16 $\frac{1}{4}$	5 14 8	37 $\frac{1}{4}$	0 19 11
17 $\frac{1}{4}$	5 5 11	38 $\frac{1}{4}$	0 17 11
18 $\frac{1}{4}$	4 18 3	39 $\frac{1}{4}$	0 16 2
19 $\frac{1}{4}$	4 10 7	40 $\frac{1}{4}$	0 14 5
20 $\frac{1}{4}$	4 4 0	41 $\frac{1}{4}$	0 12 11
21 $\frac{1}{4}$	3 17 4	42 $\frac{1}{4}$	0 11 4
22 $\frac{1}{4}$	3 11 6	43 $\frac{1}{4}$	0 10 10
23 $\frac{1}{4}$	3 5 8	44 $\frac{1}{4}$	0 8 10
24 $\frac{1}{4}$	3 0 9	45 $\frac{1}{4}$	0 7 10
25 $\frac{1}{4}$	2 15 9	46 $\frac{1}{4}$	0 6 10
26 $\frac{1}{4}$	2 11 5	47 $\frac{1}{4}$	0 5 2
27 $\frac{1}{4}$	2 7 1	48 $\frac{1}{4}$	0 4 6
28 $\frac{1}{4}$	2 3 2	49 $\frac{1}{4}$	0 3 11
29 $\frac{1}{4}$	1 19 4	50 $\frac{1}{4}$	0 3 4
30 $\frac{1}{4}$	1 16 0	51 $\frac{1}{4}$	0 2 10
31 $\frac{1}{4}$	1 12 6		

TABLE

T A B L E X I.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 24 $\frac{1}{2}$ to 25 $\frac{1}{2}$.

To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.	
		£. s. d.				£. s. d.	
10 $\frac{1}{2}$	35	8	12	31 $\frac{1}{2}$	56	1	11
11 $\frac{1}{2}$	36	8	0	32 $\frac{1}{2}$	57	1	8
12 $\frac{1}{2}$	37	7	10	33 $\frac{1}{2}$	58	1	5
13 $\frac{1}{2}$	38	6	19	34 $\frac{1}{2}$	59	1	3
14 $\frac{1}{2}$	39	6	9	35 $\frac{1}{2}$	60	1	0
15 $\frac{1}{2}$	40	6	0	36 $\frac{1}{2}$	61	0	18
16 $\frac{1}{2}$	41	5	11	37 $\frac{1}{2}$	62	0	16
17 $\frac{1}{2}$	42	5	2	38 $\frac{1}{2}$	63	0	15
18 $\frac{1}{2}$	43	4	15	39 $\frac{1}{2}$	64	0	13
19 $\frac{1}{2}$	44	4	7	40 $\frac{1}{2}$	65	0	12
20 $\frac{1}{2}$	45	4	1	41 $\frac{1}{2}$	66	0	10
21 $\frac{1}{2}$	46	3	14	42 $\frac{1}{2}$	67	0	9
22 $\frac{1}{2}$	47	3	9	43 $\frac{1}{2}$	68	0	8
23 $\frac{1}{2}$	48	3	3	44 $\frac{1}{2}$	69	0	7
24 $\frac{1}{2}$	49	2	18	45 $\frac{1}{2}$	70	0	6
25 $\frac{1}{2}$	50	2	13	46 $\frac{1}{2}$	71	0	5
26 $\frac{1}{2}$	51	2	9	47 $\frac{1}{2}$	72	0	4
27 $\frac{1}{2}$	52	2	5	48 $\frac{1}{2}$	73	0	4
28 $\frac{1}{2}$	53	2	1	49 $\frac{1}{2}$	74	0	3
29 $\frac{1}{2}$	54	1	17	50 $\frac{1}{2}$	75	0	3
30 $\frac{1}{2}$	55	1	14				

T A B L E XII.

For the Use of L O N D O N.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from $25\frac{1}{4}$ to $26\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	f. s. d.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	f. s. d.
$9\frac{1}{4}$, or at Age 35	36	9 0 2	$30\frac{1}{4}$, or at Age 56	1 12 7	
$10\frac{1}{4}$	37	8 0 7	$31\frac{1}{4}$	1 9 5	
$11\frac{1}{4}$	38	7 17 0	$32\frac{1}{4}$	1 6 9	
$12\frac{1}{4}$	39	7 6 4	$33\frac{1}{4}$	1 4 2	
$13\frac{1}{4}$	40	6 15 6	$34\frac{1}{4}$	1 1 11	
$14\frac{1}{4}$	41	5 16 1	$35\frac{1}{4}$	0 19 8	
$15\frac{1}{4}$	42	5 8 6	$36\frac{1}{4}$	0 17 9	
$16\frac{1}{4}$	43	4 19 1	$37\frac{1}{4}$	0 15 10	
$17\frac{1}{4}$	44	4 12 8	$38\frac{1}{4}$	0 14 2	
$18\frac{1}{4}$	45	4 5 4	$39\frac{1}{4}$	0 12 6	
$19\frac{1}{4}$	46	3 18 7	$40\frac{1}{4}$	0 11 1	
$20\frac{1}{4}$	47	3 12 3	$41\frac{1}{4}$	0 9 2	
$21\frac{1}{4}$	48	3 6 9	$42\frac{1}{4}$	0 8 7	
$22\frac{1}{4}$	49	3 1 4	$43\frac{1}{4}$	0 7 6	
$23\frac{1}{4}$	50	2 16 7	$44\frac{1}{4}$	0 6 7	
$24\frac{1}{4}$	51	2 11 10	$45\frac{1}{4}$	0 5 8	
$25\frac{1}{4}$	52	2 7 6	$46\frac{1}{4}$	0 5 0	
$26\frac{1}{4}$	53	2 3 3	$47\frac{1}{4}$	0 4 3	
$27\frac{1}{4}$	54	1 19 6	$48\frac{1}{4}$	0 3 8	
$28\frac{1}{4}$	55	1 15 10	$49\frac{1}{4}$	0 3 1	

TABLE

T A B L E XII.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $26\frac{1}{4}$ to $27\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
8 $\frac{1}{4}$, or at Age 35	29 $\frac{1}{4}$, or at Age 56	£. s. d.	£. s. d.
9 $\frac{1}{4}$, —	30 $\frac{1}{4}$, —	9 9 4	1 14 2
10 $\frac{1}{4}$, —	31 $\frac{1}{4}$, —	8 16 10	1 11 0
11 $\frac{1}{4}$, —	32 $\frac{1}{4}$, —	7 5 1	1 7 11
12 $\frac{1}{4}$, —	33 $\frac{1}{4}$, —	7 13 3	1 5 5
13 $\frac{1}{4}$, —	34 $\frac{1}{4}$, —	6 2 8	1 3 0
14 $\frac{1}{4}$, —	35 $\frac{1}{4}$, —	6 12 1	1 0 9
15 $\frac{1}{4}$, —	36 $\frac{1}{4}$, —	5 13 0	0 18 6
16 $\frac{1}{4}$, —	37 $\frac{1}{4}$, —	5 4 11	0 16 8
17 $\frac{1}{4}$, —	38 $\frac{1}{4}$, —	4 16 9	0 14 11
18 $\frac{1}{4}$, —	39 $\frac{1}{4}$, —	4 9 6	0 13 3
19 $\frac{1}{4}$, —	40 $\frac{1}{4}$, —	4 2 2	0 11 7
20 $\frac{1}{4}$, —	41 $\frac{1}{4}$, —	3 16 0	0 10 3
21 $\frac{1}{4}$, —	42 $\frac{1}{4}$, —	3 9 11	0 9 0
22 $\frac{1}{4}$, —	43 $\frac{1}{4}$, —	3 4 6	0 7 11
23 $\frac{1}{4}$, —	44 $\frac{1}{4}$, —	3 19 2	0 6 11
24 $\frac{1}{4}$, —	45 $\frac{1}{4}$, —	2 14 6	0 6 6
25 $\frac{1}{4}$, —	46 $\frac{1}{4}$, —	2 9 11	0 5 2
26 $\frac{1}{4}$, —	47 $\frac{1}{4}$, —	2 5 7	0 4 6
27 $\frac{1}{4}$, —	48 $\frac{1}{4}$, —	2 1 3	0 3 10
28 $\frac{1}{4}$, —		1 17 8	0 3 3

T A B L E

TABLE XIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $27\frac{1}{2}$ to $28\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
or at Age 35	or at Age 56	£. s. d.	£. s. d.
7 $\frac{1}{2}$	28 $\frac{1}{2}$	9 18 9	1 16 0
8 $\frac{1}{2}$	29 $\frac{1}{2}$	9 5 11	1 12 6
9 $\frac{1}{2}$	30 $\frac{1}{2}$	8 13 2	1 9 7
10 $\frac{1}{2}$	31 $\frac{1}{2}$	8 1 5	1 6 8
11 $\frac{1}{2}$	32 $\frac{1}{2}$	7 9 8	1 4 2
12 $\frac{1}{2}$	33 $\frac{1}{2}$	6 19 1	1 1 9
13 $\frac{1}{2}$	34 $\frac{1}{2}$	6 8 6	0 19 7
14 $\frac{1}{2}$	35 $\frac{1}{2}$	5 19 3	0 17 6
15 $\frac{1}{2}$	36 $\frac{1}{2}$	5 10 0	0 15 8
16 $\frac{1}{2}$	37 $\frac{1}{2}$	4 13 11	0 13 10
17 $\frac{1}{2}$	38 $\frac{1}{2}$	4 6 9	0 12 3
18 $\frac{1}{2}$	39 $\frac{1}{2}$	3 19 6	0 10 9
19 $\frac{1}{2}$	40 $\frac{1}{2}$	3 8 8	0 9 6
20 $\frac{1}{2}$	41 $\frac{1}{2}$	3 13 8	0 8 3
21 $\frac{1}{2}$	42 $\frac{1}{2}$	3 7 2	0 7 3
22 $\frac{1}{2}$	43 $\frac{1}{2}$	2 17 5	0 6 4
23 $\frac{1}{2}$	44 $\frac{1}{2}$	2 12 5	0 5 8
24 $\frac{1}{2}$	45 $\frac{1}{2}$	2 7 9	0 4 8
25 $\frac{1}{2}$	46 $\frac{1}{2}$	2 3 7	0 4 0
26 $\frac{1}{2}$	47 $\frac{1}{2}$	1 19 6	0 3 5

TABLE

T A B L E XV.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $28\frac{1}{2}$ to $29\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
6 $\frac{1}{2}$, or at Age 35	27 $\frac{1}{2}$, or at Age 56	£. 10	£. 1
7 $\frac{1}{2}$ — 36	28 $\frac{1}{2}$ — 57	9 9	17 9
8 $\frac{1}{2}$ — 37	29 $\frac{1}{2}$ — 58	9 2	14 3
9 $\frac{1}{2}$ — 38	30 $\frac{1}{2}$ — 59	8 9	10 11
10 $\frac{1}{2}$ — 39	31 $\frac{1}{2}$ — 60	7 7	8 2
11 $\frac{1}{2}$ — 40	32 $\frac{1}{2}$ — 61	6 17	5 5
12 $\frac{1}{2}$ — 41	33 $\frac{1}{2}$ — 62	6 15	2 2
13 $\frac{1}{2}$ — 42	34 $\frac{1}{2}$ — 63	5 4	0 6
14 $\frac{1}{2}$ — 43	35 $\frac{1}{2}$ — 64	5 15	18 6
15 $\frac{1}{2}$ — 44	36 $\frac{1}{2}$ — 65	4 7	16 8
16 $\frac{1}{2}$ — 45	37 $\frac{1}{2}$ — 66	4 18	14 10
17 $\frac{1}{2}$ — 46	38 $\frac{1}{2}$ — 67	4 10	11 4
18 $\frac{1}{2}$ — 47	39 $\frac{1}{2}$ — 68	3 4	9 11
19 $\frac{1}{2}$ — 48	40 $\frac{1}{2}$ — 69	3 17	8 9
20 $\frac{1}{2}$ — 49	41 $\frac{1}{2}$ — 70	3 11	7 7
21 $\frac{1}{2}$ — 50	42 $\frac{1}{2}$ — 71	3 5	6 8
22 $\frac{1}{2}$ — 51	43 $\frac{1}{2}$ — 72	3 0	5 10
23 $\frac{1}{2}$ — 52	44 $\frac{1}{2}$ — 73	2 15	5 0
24 $\frac{1}{2}$ — 53	45 $\frac{1}{2}$ — 74	2 10	4 2
25 $\frac{1}{2}$ — 54	46 $\frac{1}{2}$ — 75	2 5	3 7
26 $\frac{1}{2}$ — 55		2 1	

T A B L E

TABLE XVI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $29\frac{1}{4}$ to $30\frac{1}{2}$.

To commence at the end of Years after purchasing.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	Value of an annuity of £1. in one present payment.
or at Age	or at Age	£. s. d.	£. s. d.
35	56	11 0 3	1 19 11
36	57	10 6 1	1 16 0
37	58	9 11 11	1 12 9
38	59	8 18 11	1 9 7
39	60	8 5 11	1 6 10
40	61	7 14 2	1 4 1
41	62	7 2 5	1 1 9
42	63	6 12 2	0 19 4
43	64	6 1 11	0 17 4
44	65	5 12 11	0 15 4
45	66	5 4 0	0 13 8
46	67	4 16 2	0 11 11
47	68	4 8 4	0 10 7
48	69	4 1 8	0 9 2
49	70	3 15 0	0 8 1
50	71	3 9 2	0 7 0
51	72	3 3 4	0 6 1
52	73	2 18 2	0 5 3
53	74	2 12 11	0 4 6
54	75	2 8 4	0 3 9
55		2 3 10	

TABLE

T A B L E XVII.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $30\frac{1}{2}$ to $31\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
4 $\frac{1}{2}$, Or at Age 35	£. 11 12 9	25 $\frac{1}{2}$, Or at Age 56	£. 2 2 0
5 $\frac{1}{2}$ —	10 17 4	26 $\frac{1}{2}$ —	1 18 1
6 $\frac{1}{2}$ —	10 2 10	27 $\frac{1}{2}$ —	1 14 3
7 $\frac{1}{2}$ —	9 8 4	28 $\frac{1}{2}$ —	1 11 3
8 $\frac{1}{2}$ —	8 15 4	29 $\frac{1}{2}$ —	1 8 3
9 $\frac{1}{2}$ —	8 2 4	30 $\frac{1}{2}$ —	1 5 6
10 $\frac{1}{2}$ —	7 10 7	31 $\frac{1}{2}$ —	1 2 10
11 $\frac{1}{2}$ —	6 18 10	32 $\frac{1}{2}$ —	1 0 7
12 $\frac{1}{2}$ —	6 8 10	33 $\frac{1}{2}$ —	0 18 3
13 $\frac{1}{2}$ —	5 18 11	34 $\frac{1}{2}$ —	0 16 3
14 $\frac{1}{2}$ —	5 9 11	35 $\frac{1}{2}$ —	0 14 3
15 $\frac{1}{2}$ —	5 1 0	36 $\frac{1}{2}$ —	0 12 8
16 $\frac{1}{2}$ —	4 13 5	37 $\frac{1}{2}$ —	0 11 0
17 $\frac{1}{2}$ —	4 5 11	38 $\frac{1}{2}$ —	0 9 6
18 $\frac{1}{2}$ —	4 19 4	39 $\frac{1}{2}$ —	0 8 5
19 $\frac{1}{2}$ —	3 12 10	40 $\frac{1}{2}$ —	0 7 5
20 $\frac{1}{2}$ —	3 7 0	41 $\frac{1}{2}$ —	0 6 5
21 $\frac{1}{2}$ —	3 1 3	42 $\frac{1}{2}$ —	0 5 7
22 $\frac{1}{2}$ —	2 16 0	43 $\frac{1}{2}$ —	0 4 8
23 $\frac{1}{2}$ —	2 10 8	44 $\frac{1}{2}$ —	0 4 0
24 $\frac{1}{2}$ —	2 6 4		

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T A B L E

TABLE XVIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from $31\frac{1}{2}$ to $32\frac{1}{2}$.

To commence at the end of	Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.	To commence at the end of	Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
35	35	12	5	1	24	56	2	4	5		
36	36	11	9	4	25	57	2	0	1		
37	37	10	13	7	26	58	1	16	6		
38	38	9	19	1	27	59	1	12	11		
39	39	9	4	7	28	60	1	9	10		
40	40	8	11	7	29	61	1	6	10		
41	41	7	18	6	30	62	1	4	2		
42	42	7	7	1	31	63	1	1	7		
43	43	6	15	8	32	64	0	19	4		
44	44	6	5	8	33	65	0	17	1		
45	45	5	15	8	34	66	0	15	2		
46	46	5	7	0	35	67	0	13	3		
47	47	4	18	4	36	68	0	11	9		
48	48	4	10	4	37	69	0	10	2		
49	49	4	3	6	38	70	0	9	0		
50	50	3	17	0	39	71	0	7	10		
51	51	3	10	6	40	72	0	6	10		
52	52	3	4	8	41	73	0	5	10		
53	53	2	18	10	42	74	0	5	9		
54	54	2	13	9	43	75	0	4	3		
55	55	2	8	8							

TABLE

T A B L E XIX.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $32\frac{1}{2}$ to $33\frac{1}{2}$.

To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.	
		£. s. d.				£. s. d.	
2 $\frac{1}{2}$	35	12	19	23 $\frac{1}{2}$	56	2	6
3 $\frac{1}{2}$	36	12	2	24 $\frac{1}{2}$	57	2	2
4 $\frac{1}{2}$	37	11	5	25 $\frac{1}{2}$	58	1	18
5 $\frac{1}{2}$	38	10	9	26 $\frac{1}{2}$	59	1	14
6 $\frac{1}{2}$	39	9	15	27 $\frac{1}{2}$	60	1	11
7 $\frac{1}{2}$	40	8	0	28 $\frac{1}{2}$	61	1	8
8 $\frac{1}{2}$	41	7	7	29 $\frac{1}{2}$	62	1	5
9 $\frac{1}{2}$	42	7	14	30 $\frac{1}{2}$	63	1	2
10 $\frac{1}{2}$	43	6	3	31 $\frac{1}{2}$	64	1	0
11 $\frac{1}{2}$	44	6	12	32 $\frac{1}{2}$	65	0	18
12 $\frac{1}{2}$	45	5	2	33 $\frac{1}{2}$	66	0	15
13 $\frac{1}{2}$	46	5	12	34 $\frac{1}{2}$	67	0	14
14 $\frac{1}{2}$	47	4	4	35 $\frac{1}{2}$	68	0	12
15 $\frac{1}{2}$	48	4	15	36 $\frac{1}{2}$	69	0	10
16 $\frac{1}{2}$	49	4	8	37 $\frac{1}{2}$	70	0	9
17 $\frac{1}{2}$	50	4	1	38 $\frac{1}{2}$	71	0	8
18 $\frac{1}{2}$	51	3	14	39 $\frac{1}{2}$	72	0	7
19 $\frac{1}{2}$	52	3	8	40 $\frac{1}{2}$	73	0	6
20 $\frac{1}{2}$	53	3	2	41 $\frac{1}{2}$	74	0	5
21 $\frac{1}{2}$	54	2	16	42 $\frac{1}{2}$	75	0	4
22 $\frac{1}{2}$	55	2	11				

T A B L E XX.

For the Use of L O N D O N.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $33\frac{1}{2}$ to $34\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
1 $\frac{1}{2}$, or at Age 35	£. 13 s. 4 d.	22 $\frac{1}{2}$, or at Age 56	£. 2 s. 9 d.
2 $\frac{1}{2}$, — 36	12 13 10	23 $\frac{1}{2}$, — 57	2 2 4
3 $\frac{1}{2}$, — 37	11 15 18	24 $\frac{1}{2}$, — 58	2 4 0
4 $\frac{1}{2}$, — 38	11 18 2	25 $\frac{1}{2}$, — 59	1 16 8
5 $\frac{1}{2}$, — 39	10 2 11	26 $\frac{1}{2}$, — 60	1 13 3
6 $\frac{1}{2}$, — 40	9 11 4	27 $\frac{1}{2}$, — 61	1 9 11
7 $\frac{1}{2}$, — 41	8 16 9	28 $\frac{1}{2}$, — 62	1 7 0
8 $\frac{1}{2}$, — 42	8 4 0	29 $\frac{1}{2}$, — 63	1 4 0
9 $\frac{1}{2}$, — 43	7 11 3	30 $\frac{1}{2}$, — 64	1 1 6
10 $\frac{1}{2}$, — 44	7 0 2	31 $\frac{1}{2}$, — 65	0 19 0
11 $\frac{1}{2}$, — 45	6 9 0	32 $\frac{1}{2}$, — 66	0 17 0
12 $\frac{1}{2}$, — 46	5 19 4	33 $\frac{1}{2}$, — 67	0 14 10
13 $\frac{1}{2}$, — 47	5 9 8	34 $\frac{1}{2}$, — 68	0 13 1
14 $\frac{1}{2}$, — 48	5 1 4	35 $\frac{1}{2}$, — 69	0 11 4
15 $\frac{1}{2}$, — 49	4 13 1	36 $\frac{1}{2}$, — 70	0 10 0
16 $\frac{1}{2}$, — 50	4 5 10	37 $\frac{1}{2}$, — 71	0 8 8
17 $\frac{1}{2}$, — 51	3 18 7	38 $\frac{1}{2}$, — 72	0 7 7
18 $\frac{1}{2}$, — 52	3 12 2	39 $\frac{1}{2}$, — 73	0 6 6
19 $\frac{1}{2}$, — 53	3 5 8	40 $\frac{1}{2}$, — 74	0 5 7
20 $\frac{1}{2}$, — 54	3 0 0	41 $\frac{1}{2}$, — 75	0 4 9
21 $\frac{1}{2}$, — 55	2 14 4		

T A B L E

T A B L E XXI.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $34\frac{1}{2}$ to $35\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
1, or at Age 35	1, or at Age 56	£. s. d.	£. s. d.
1, —	21, —	14 9 6	2 12 2
2, —	22, —	13 10 4	2 7 5
3, —	23, —	12 12 4	2 2 8
4, —	24, —	11 14 3	1 18 11
5, —	25, —	10 18 2	1 15 2
6, —	26, —	9 2 0	1 11 9
7, —	27, —	8 7 4	1 8 4
8, —	28, —	7 12 0	1 5 7
9, —	29, —	6 7 3	1 2 9
10, —	30, —	5 16 11	0 17 3
11, —	31, —	4 16 9	0 15 9
12, —	32, —	3 10 2	0 13 8
13, —	33, —	2 16 10	0 12 1
14, —	34, —	1 18 8	0 10 6
15, —	35, —	1 10 7	0 9 3
16, —	36, —	0 18 4	0 8 0
17, —	37, —	0 16 2	0 6 11
18, —	38, —	0 14 0	0 5 10
19, —	39, —	0 11 8	0 4 5
20, —	40, —	0 9 8	0 3 0

T A B L E

TABLE XXII.

For the Life of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $35\frac{1}{2}$ to $36\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.	To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
$1\frac{1}{2}$	36	14	5	6	20 $\frac{1}{2}$	56	2	15	4		
$1\frac{3}{4}$	37	13	6	4	21 $\frac{1}{2}$	57	2	10	0		
$2\frac{1}{4}$	38	12	8	3	22 $\frac{1}{2}$	58	2	5	6		
$2\frac{3}{4}$	39	11	10	2	23 $\frac{1}{2}$	59	2	1	0		
$3\frac{1}{4}$	40	10	14	0	24 $\frac{1}{2}$	60	1	17	2		
$3\frac{3}{4}$	41	9	17	8	25 $\frac{1}{2}$	61	1	13	5		
$4\frac{1}{4}$	42	9	3	5	26 $\frac{1}{2}$	62	1	10	2		
$4\frac{3}{4}$	43	8	9	2	27 $\frac{1}{2}$	63	1	6	11		
$5\frac{1}{4}$	44	7	16	8	28 $\frac{1}{2}$	64	1	4	1		
$5\frac{3}{4}$	45	7	4	3	29 $\frac{1}{2}$	65	1	1	4		
$6\frac{1}{4}$	46	6	13	5	30 $\frac{1}{2}$	66	0	18	11		
$6\frac{3}{4}$	47	6	2	7	31 $\frac{1}{2}$	67	0	16	6		
$7\frac{1}{4}$	48	5	13	4	32 $\frac{1}{2}$	68	0	14	8		
$7\frac{3}{4}$	49	5	4	1	33 $\frac{1}{2}$	69	0	12	9		
$8\frac{1}{4}$	50	4	16	0	34 $\frac{1}{2}$	70	0	11	2		
$8\frac{3}{4}$	51	4	8	0	35 $\frac{1}{2}$	71	0	9	8		
$9\frac{1}{4}$	52	4	0	8	36 $\frac{1}{2}$	72	0	8	6		
$9\frac{3}{4}$	53	3	13	5	37 $\frac{1}{2}$	73	0	7	3		
$10\frac{1}{4}$	54	3	7	1	38 $\frac{1}{2}$	74	0	6	3		
$10\frac{3}{4}$	55	3	0	9	39 $\frac{1}{2}$	75	0	5	3		

TABLE

T A B L E XXIII.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from $36\frac{1}{2}$ to $37\frac{1}{4}$.

To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.		To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	
		£. s. d.				£. s. d.	
$1\frac{1}{4}$	37	14	1	6	57	2	13
$1\frac{1}{2}$	38	13	2	5	58	2	7
$2\frac{1}{4}$	39	12	4	3	59	2	3
$2\frac{1}{2}$	40	11	6	2	60	1	19
$3\frac{1}{4}$	41	10	9	9	61	1	15
$3\frac{1}{2}$	42	9	13	4	62	1	11
$4\frac{1}{4}$	43	8	19	8	63	1	8
$4\frac{1}{2}$	44	7	5	2	64	1	5
$5\frac{1}{4}$	45	7	13	0	65	0	19
$5\frac{1}{2}$	46	6	7	0	66	0	17
$6\frac{1}{4}$	47	6	10	1	67	0	15
$6\frac{1}{2}$	48	5	19	7	68	0	13
$7\frac{1}{4}$	49	5	10	6	69	0	11
$7\frac{1}{2}$	50	5	1	4	70	0	10
$8\frac{1}{4}$	51	4	13	4	71	0	10
$8\frac{1}{2}$	52	4	5	0	72	0	8
$9\frac{1}{4}$	53	3	18	0	73	0	7
$9\frac{1}{2}$	54	3	10	8	74	0	6
$10\frac{1}{4}$	55	3	4	6	75	0	5
$10\frac{1}{2}$	56	2	18	5			

T A B L E

TABLE XXIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $37\frac{1}{4}$ to $38\frac{1}{2}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.	£. s. d.
1 $\frac{1}{4}$	38	13	17 6
1 $\frac{1}{2}$	39	12	18 9
2 $\frac{1}{4}$	40	12	0 6
2 $\frac{1}{2}$	41	11	2 2
3 $\frac{1}{4}$	42	10	6 1
3 $\frac{1}{2}$	43	9	10 1
4 $\frac{1}{4}$	44	8	16 1
4 $\frac{1}{2}$	45	7	10 2
5 $\frac{1}{4}$	46	6	17 0
5 $\frac{1}{2}$	47	6	17 9
6 $\frac{1}{4}$	48	5	17 4
6 $\frac{1}{2}$	49	5	17 0
7 $\frac{1}{4}$	50	5	17 11
7 $\frac{1}{2}$	51	4	18 9
8 $\frac{1}{4}$	52	4	10 8
8 $\frac{1}{2}$	53	4	2 6
9 $\frac{1}{4}$	54	3	15 5
9 $\frac{1}{2}$	55	3	8 3
10 $\frac{1}{4}$	56	3	2 2
10 $\frac{1}{2}$			
11 $\frac{1}{4}$			
11 $\frac{1}{2}$			
12 $\frac{1}{4}$			
12 $\frac{1}{2}$			
13 $\frac{1}{4}$			
13 $\frac{1}{2}$			
14 $\frac{1}{4}$			
14 $\frac{1}{2}$			
15 $\frac{1}{4}$			
15 $\frac{1}{2}$			
16 $\frac{1}{4}$			
16 $\frac{1}{2}$			
17 $\frac{1}{4}$			
17 $\frac{1}{2}$			
18 $\frac{1}{4}$			

T A B L E XXV.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 38 $\frac{1}{2}$ to 39 $\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
1 $\frac{1}{2}$, or at Age 39	£. 13 s. 6 d.	19 $\frac{1}{2}$, or at Age 58	£. 2 s. 13 d.
1 $\frac{3}{4}$ —	12 13 2	20 $\frac{1}{2}$ —	2 9 11
2 $\frac{1}{4}$ —	11 15 8	21 $\frac{1}{2}$ —	2 4 2
2 $\frac{3}{4}$ —	10 16 2	22 $\frac{1}{2}$ —	2 0 1
3 $\frac{1}{4}$ —	10 18 6	23 $\frac{1}{2}$ —	1 15 10
3 $\frac{3}{4}$ —	9 2 11	24 $\frac{1}{2}$ —	1 12 3
4 $\frac{1}{4}$ —	9 6 9	25 $\frac{1}{2}$ —	1 8 9
4 $\frac{3}{4}$ —	8 12 8	26 $\frac{1}{2}$ —	1 5 7
5 $\frac{1}{4}$ —	8 18 10	27 $\frac{1}{2}$ —	1 2 5
5 $\frac{3}{4}$ —	7 7 11	28 $\frac{1}{2}$ —	0 19 10
6 $\frac{1}{4}$ —	7 14 8	29 $\frac{1}{2}$ —	0 17 3
6 $\frac{3}{4}$ —	6 14 4	30 $\frac{1}{2}$ —	0 15 3
7 $\frac{1}{4}$ —	6 4 4	31 $\frac{1}{2}$ —	0 13 8
7 $\frac{3}{4}$ —	5 5 4	32 $\frac{1}{2}$ —	0 11 1
8 $\frac{1}{4}$ —	5 16 3	33 $\frac{1}{2}$ —	0 10 8
8 $\frac{3}{4}$ —	4 4 8	34 $\frac{1}{2}$ —	0 8 4
9 $\frac{1}{4}$ —	4 8 10	35 $\frac{1}{2}$ —	0 7 3
9 $\frac{3}{4}$ —	3 12 11	36 $\frac{1}{2}$ —	0 6 3
10 $\frac{1}{4}$ —	3 5 19		
10 $\frac{3}{4}$ —	2 19 11		
11 $\frac{1}{4}$ —			
11 $\frac{3}{4}$ —			
12 $\frac{1}{4}$ —			
12 $\frac{3}{4}$ —			
13 $\frac{1}{4}$ —			
13 $\frac{3}{4}$ —			
14 $\frac{1}{4}$ —			
14 $\frac{3}{4}$ —			
15 $\frac{1}{4}$ —			
15 $\frac{3}{4}$ —			
16 $\frac{1}{4}$ —			
16 $\frac{3}{4}$ —			
17 $\frac{1}{4}$ —			
17 $\frac{3}{4}$ —			
18 $\frac{1}{4}$ —			
18 $\frac{3}{4}$ —			

P P P P

T A B L E

T A B L E XXVI.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $39\frac{1}{4}$ to $40\frac{1}{4}$.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.			To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.		
		£.	s.	d.			£.	s.	d.
$\frac{1}{4}$	40	13	11	6	$18\frac{1}{4}$	58	2	17	9
$1\frac{1}{4}$	41	12	11	1	$19\frac{1}{4}$	59	2	12	1
$2\frac{1}{4}$	42	11	13	0	$20\frac{1}{4}$	60	2	7	3
$3\frac{1}{4}$	43	10	14	10	$21\frac{1}{4}$	61	2	2	6
$4\frac{1}{4}$	44	9	19	1	$22\frac{1}{4}$	62	1	18	4
$5\frac{1}{4}$	45	9	3	3	$23\frac{1}{4}$	63	1	14	2
$6\frac{1}{4}$	46	8	9	6	$24\frac{1}{4}$	64	1	10	7
$7\frac{1}{4}$	47	7	15	9	$25\frac{1}{4}$	65	1	7	1
$8\frac{1}{4}$	48	7	4	0	$26\frac{1}{4}$	66	1	4	0
$9\frac{1}{4}$	49	6	12	3	$27\frac{1}{4}$	67	1	1	0
$10\frac{1}{4}$	50	6	2	0	$28\frac{1}{4}$	68	0	18	7
$11\frac{1}{4}$	51	5	11	8	$29\frac{1}{4}$	69	0	16	2
$12\frac{1}{4}$	52	5	2	6	$30\frac{1}{4}$	70	0	14	3
$13\frac{1}{4}$	53	4	13	3	$31\frac{1}{4}$	71	0	12	4
$14\frac{1}{4}$	54	4	5	3	$32\frac{1}{4}$	72	0	10	9
$15\frac{1}{4}$	55	3	17	2	$33\frac{1}{4}$	73	0	9	2
$16\frac{1}{4}$	56	3	10	4	$34\frac{1}{4}$	74	0	7	10
$17\frac{1}{4}$	57	3	3	5	$35\frac{1}{4}$	75	0	6	8

T A B L E

TABLE XXVII.
For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $40\frac{1}{2}$ to $41\frac{1}{2}$.

To commence at the end of	Years after purchasing.	Value of an annuity of £1. in one present payment.			To commence at the end of	Years after purchasing.	Value of an annuity of £1. in one present payment.		
		£.	s.	d.			£.	s.	d.
1 $\frac{1}{2}$ or at Age 41	1 $\frac{1}{2}$	13	7	6	18 $\frac{1}{2}$ or at Age 59	18 $\frac{1}{2}$	2	15	8
1 $\frac{1}{2}$	1 $\frac{1}{2}$	12	7	1	19 $\frac{1}{2}$	19 $\frac{1}{2}$	2	10	3
2 $\frac{1}{2}$	2 $\frac{1}{2}$	11	9	4	20 $\frac{1}{2}$	20 $\frac{1}{2}$	2	5	5
3 $\frac{1}{2}$	3 $\frac{1}{2}$	10	11	7	21 $\frac{1}{2}$	21 $\frac{1}{2}$	2	0	7
4 $\frac{1}{2}$	4 $\frac{1}{2}$	9	15	8	22 $\frac{1}{2}$	22 $\frac{1}{2}$	1	16	7
5 $\frac{1}{2}$	5 $\frac{1}{2}$	8	19	8	23 $\frac{1}{2}$	23 $\frac{1}{2}$	1	12	7
6 $\frac{1}{2}$	6 $\frac{1}{2}$	8	6	3	24 $\frac{1}{2}$	24 $\frac{1}{2}$	1	9	0
7 $\frac{1}{2}$	7 $\frac{1}{2}$	7	12	10	25 $\frac{1}{2}$	25 $\frac{1}{2}$	1	5	5
8 $\frac{1}{2}$	8 $\frac{1}{2}$	7	1	2	26 $\frac{1}{2}$	26 $\frac{1}{2}$	1	2	6
9 $\frac{1}{2}$	9 $\frac{1}{2}$	6	9	7	27 $\frac{1}{2}$	27 $\frac{1}{2}$	0	19	7
10 $\frac{1}{2}$	10 $\frac{1}{2}$	5	19	4	28 $\frac{1}{2}$	28 $\frac{1}{2}$	0	17	4
11 $\frac{1}{2}$	11 $\frac{1}{2}$	5	9	1	29 $\frac{1}{2}$	29 $\frac{1}{2}$	0	15	1
12 $\frac{1}{2}$	12 $\frac{1}{2}$	4	19	8	30 $\frac{1}{2}$	30 $\frac{1}{2}$	0	13	3
13 $\frac{1}{2}$	13 $\frac{1}{2}$	4	10	3	31 $\frac{1}{2}$	31 $\frac{1}{2}$	0	11	5
14 $\frac{1}{2}$	14 $\frac{1}{2}$	4	2	6	32 $\frac{1}{2}$	32 $\frac{1}{2}$	0	9	11
15 $\frac{1}{2}$	15 $\frac{1}{2}$	3	14	8	33 $\frac{1}{2}$	33 $\frac{1}{2}$	0	8	4
16 $\frac{1}{2}$	16 $\frac{1}{2}$	3	7	10	34 $\frac{1}{2}$	34 $\frac{1}{2}$	0	7	1
17 $\frac{1}{2}$	17 $\frac{1}{2}$	3	1	0					

T A B L E XXVIII.

For the Use of L O N D O N.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from $41\frac{1}{2}$ to $42\frac{1}{2}$.

To commence at the end of	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of	Value of an an- nuity of £1. in one present pay- ment.
Years after pur- chasing.		Years after pur- chasing.	
Or at Age		Or at Age	
42	£. 13 s. 6 d.	59	£. 2 s. 19 d.
43	12 3 6	60	2 13 3
44	11 4 6	61	2 13 9
45	10 8 6	62	2 8 3
46	9 12 10	63	2 3 7
47	8 17 2	64	1 18 10
48	8 3 10	65	1 14 10
49	7 10 5	66	1 10 9
50	6 18 9	67	1 7 4
51	6 7 1	68	1 3 11
52	5 16 7	69	1 1 2
53	5 6 1	70	0 18 4
54	4 17 0	71	0 16 3
55	4 7 9	72	0 14 1
56	4 0 3	73	0 12 3
57	3 12 3	74	0 10 5
58	3 5 9	75	0 9 7

T A B L E

T A B L E XXIX.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from 42 $\frac{1}{2}$ to 43 $\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.	To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1 $\frac{1}{2}$	43	12	19	6	17 $\frac{1}{2}$	60	2	17	6		
2 $\frac{1}{2}$	44	12	2	0	18 $\frac{1}{2}$	61	2	11	11		
3 $\frac{1}{2}$	45	11	3	8	19 $\frac{1}{2}$	62	2	6	4		
4 $\frac{1}{2}$	46	10	5	5	20 $\frac{1}{2}$	63	2	1	9		
5 $\frac{1}{2}$	47	9	10	1	21 $\frac{1}{2}$	64	1	17	2		
6 $\frac{1}{2}$	48	8	14	8	22 $\frac{1}{2}$	65	1	13	1		
7 $\frac{1}{2}$	49	8	1	5	23 $\frac{1}{2}$	66	1	9	0		
8 $\frac{1}{2}$	50	7	8	1	24 $\frac{1}{2}$	67	1	5	9		
9 $\frac{1}{2}$	51	6	16	4	25 $\frac{1}{2}$	68	1	2	5		
10 $\frac{1}{2}$	52	6	4	8	26 $\frac{1}{2}$	69	0	19	10		
11 $\frac{1}{2}$	53	5	13	11	27 $\frac{1}{2}$	70	0	17	2		
12 $\frac{1}{2}$	54	5	3	2	28 $\frac{1}{2}$	71	0	15	1		
13 $\frac{1}{2}$	55	4	14	3	29 $\frac{1}{2}$	72	0	13	1		
14 $\frac{1}{2}$	56	4	5	4	30 $\frac{1}{2}$	73	0	11	3		
15 $\frac{1}{2}$	57	3	17	7	31 $\frac{1}{2}$	74	0	9	6		
16 $\frac{1}{2}$	58	3	9	10	32 $\frac{1}{2}$	75	0	8	2		
17 $\frac{1}{2}$	59	3	3	8							

T A B L E

TABLE XXX.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $43\frac{1}{2}$ to $44\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
$43\frac{1}{2}$, or at Age 44	12 17 0	$43\frac{1}{2}$, or at Age 60	3 1 4
1 1 1	11 18 0	17 1 1	2 15 1
2 1 1	11 0 0	18 1 1	2 9 9
3 1 1	10 2 2	19 1 1	2 4 4
4 1 1	9 11 1	20 1 1	1 19 8
5 1 1	8 11 7	21 1 1	1 15 1
6 1 1	7 18 4	22 1 1	1 11 2
7 1 1	7 5 0	23 1 1	1 7 3
8 1 1	6 13 1	24 1 1	1 4 4
9 1 1	6 1 1	25 1 1	1 1 1
10 1 1	5 10 7	26 1 1	0 18 6
11 1 1	5 0 2	27 1 1	0 16 0
12 1 1	4 11 3	28 1 1	0 14 0
13 1 1	4 2 4	29 1 1	0 12 0
14 1 1	3 15 8	30 1 1	0 10 3
15 1 1	3 7 8	31 1 1	0 8 8

TABLE

TABLE XXXI.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 44 $\frac{1}{2}$ to 45 $\frac{1}{2}$.

To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.	To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.
1 $\frac{1}{4}$	45	12	16 $\frac{1}{4}$	61	2
2 $\frac{1}{4}$	46	11	17 $\frac{1}{4}$	62	19
3 $\frac{1}{4}$	47	10	18 $\frac{1}{4}$	63	12
4 $\frac{1}{4}$	48	9	19 $\frac{1}{4}$	64	7
5 $\frac{1}{4}$	49	8	20 $\frac{1}{4}$	65	2
6 $\frac{1}{4}$	50	7	21 $\frac{1}{4}$	66	17
7 $\frac{1}{4}$	51	6	22 $\frac{1}{4}$	67	13
8 $\frac{1}{4}$	52	5	23 $\frac{1}{4}$	68	9
9 $\frac{1}{4}$	53	4	24 $\frac{1}{4}$	69	5
10 $\frac{1}{4}$	54	3	25 $\frac{1}{4}$	70	2
11 $\frac{1}{4}$	55	2	26 $\frac{1}{4}$	71	19
12 $\frac{1}{4}$	56	1	27 $\frac{1}{4}$	72	17
13 $\frac{1}{4}$	57	1	28 $\frac{1}{4}$	73	14
14 $\frac{1}{4}$	58	1	29 $\frac{1}{4}$	74	12
15 $\frac{1}{4}$	59	1	30 $\frac{1}{4}$	75	10
	60	3			9

T A B L E XXXII.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from $45\frac{1}{2}$ to $46\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
$\frac{1}{4}$, or at Age 46	£. 12 s. 6 d.	$15\frac{1}{4}$, or at Age 61	£. 3 s. 2 d.
$1\frac{1}{4}$	11 10 9	$16\frac{1}{4}$	2 16 11
$2\frac{1}{4}$	10 13 4	$17\frac{1}{4}$	2 10 9
$3\frac{1}{4}$	9 15 11	$18\frac{1}{4}$	2 5 7
$4\frac{1}{4}$	8 0 8	$19\frac{1}{4}$	2 0 4
$5\frac{1}{4}$	7 5 6	$20\frac{1}{4}$	1 15 7
$6\frac{1}{4}$	6 11 10	$21\frac{1}{4}$	1 11 1
$7\frac{1}{4}$	6 18 2	$22\frac{1}{4}$	1 7 6
$8\frac{1}{4}$	6 6 3	$23\frac{1}{4}$	1 4 0
$9\frac{1}{4}$	5 14 4	$24\frac{1}{4}$	1 1 2
$10\frac{1}{4}$	5 4 2	$25\frac{1}{4}$	0 18 3
$11\frac{1}{4}$	4 14 0	$26\frac{1}{4}$	0 16 0
$12\frac{1}{4}$	4 5 7	$27\frac{1}{4}$	0 13 8
$13\frac{1}{4}$	3 17 2	$28\frac{1}{4}$	0 11 9
$14\frac{1}{4}$	3 10 0	$29\frac{1}{4}$	0 9 11

T A B L E

T A B L E XXXIII.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $46\frac{1}{2}$ to $47\frac{1}{2}$.

To commence at the end of Years after purchasing.	A, or at Age	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	B, or at Age	Value of an annuity of £1. in one present payment.	
		£. s.	d.			£. s.	d.
1 $\frac{1}{4}$	47	12	5	6	62	3	0
2 $\frac{1}{4}$	48	11	7	7	63	2	14
3 $\frac{1}{4}$	49	10	10	8	64	2	8
4 $\frac{1}{4}$	50	9	12	9	65	2	3
5 $\frac{1}{4}$	51	8	17	10	66	1	17
6 $\frac{1}{4}$	52	8	2	11	67	1	13
7 $\frac{1}{4}$	53	7	8	12	68	1	9
8 $\frac{1}{4}$	54	6	14	13	69	1	5
9 $\frac{1}{4}$	55	6	2	14	70	0	19
10 $\frac{1}{4}$	56	5	11	15	71	0	17
11 $\frac{1}{4}$	57	5	1	16	72	0	14
12 $\frac{1}{4}$	58	4	10	17	73	0	12
13 $\frac{1}{4}$	59	4	2	18	74	0	10
14 $\frac{1}{4}$	60	3	14	19	75	0	10
	61	3	7	20			

Qqqq

T A B L E

TABLE XXXIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 47½ to 48½.

To commence at the end of Years after pur- chasing.	½, or at Age 48	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1 ½	49	12	3	6	
1 ¾	49	11	4	2	
2 0	50	10	6	9	
2 ¼	51	9	9	5	
2 ½	51	8	13	9	
2 ¾	52	7	18	1	
3 0	53	6	10	6	
3 ¼	54	5	19	3	
3 ½	55	5	17	7	
3 ¾	56	4	18	6	
4 0	57	4	18	6	
4 ¼	58	4	18	6	
4 ½	59	4	18	6	
4 ¾	60	4	18	6	
5 0	61	4	18	6	
5 ¼	62	3	17	11	
5 ½	63	3	17	11	
5 ¾	64	2	17	11	
6 0	65	2	17	11	
6 ¼	66	2	17	11	
6 ½	67	2	17	11	
6 ¾	68	2	17	11	
7 0	69	1	17	11	
7 ¼	70	1	17	11	
7 ½	71	1	17	11	
7 ¾	72	1	17	11	
8 0	73	1	17	11	
8 ¼	74	1	17	11	
8 ½	75	1	17	11	

TABLE XXXV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchaser from 48½ to 49½.

To commence at the end of Years after pur- chasing.	½, or at Age 49	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1 ½	50	11	19	6	
1 ¾	51	11	0	10	
2 0	51	10	3	4	
2 ¼	52	9	5	11	
2 ½	53	8	9	10	
2 ¾	54	7	13	10	
3 0	55	6	7	7	
3 ¼	56	5	15	8	
3 ½	57	5	14	11	
3 ¾	58	4	14	11	
4 0	59	4	14	11	
4 ¼	60	4	14	11	
4 ½	61	3	17	5	
4 ¾	62	3	17	5	
5 0	63	3	17	5	
5 ¼	64	2	15	3	
5 ½	65	2	15	3	
5 ¾	66	2	15	3	
6 0	67	2	15	3	
6 ¼	68	1	18	4	
6 ½	69	1	18	4	
6 ¾	70	1	18	4	
7 0	71	1	18	4	
7 ¼	72	1	18	4	
7 ½	73	1	18	4	
7 ¾	74	1	18	4	
8 0	75	1	18	4	

TABLE

TABLE XXXVI.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 49½ to 50½.

To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.	£. s. d.
½, or at Age 50	50	11 15 6	6
1 ¼	51	10 16 9	9
2 ¼	52	9 18 10	10
3 ¼	53	8 1 0	0
4 ¼	54	7 5 4	4
5 ¼	55	6 9 8	8
6 ¼	56	5 16 5	5
7 ¼	57	4 12 1	1
8 ¼	58	3 11 1	1
9 ¼	59	2 11 0	0
10 ¼	60	1 11 9	9
11 ¼	61	0 11 4	4
12 ¼	62	0 14 4	4
13 ¼	63	0 19 6	6
14 ¼	64	0 19 12	12
15 ¼	65	0 12 6	6
16 ¼	66	0 7 7	7
17 ¼	67	0 0 9	9
18 ¼	68	0 16 1	1
19 ¼	69	0 11 4	4
20 ¼	70	0 7 7	7
21 ¼	71	0 4 0	0
22 ¼	72	0 1 11	11
23 ¼	73	0 17 10	10
24 ¼	74	0 15 4	4
25 ¼	75	0 13 0	0

Qqqq 2

TABLE XXXVII.

For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 50½ to 51½.

To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.	£. s. d.
½, or at Age 51	51	11 11 6	6
1 ½	52	10 12 8	8
2 ½	53	9 14 4	4
3 ½	54	8 16 0	0
4 ½	55	7 0 10	10
5 ½	56	6 5 7	7
6 ½	57	5 12 4	4
7 ½	58	4 19 0	0
8 ½	59	3 18 7	7
9 ½	60	2 18 1	1
10 ½	61	1 18 8	8
11 ½	62	0 19 7	7
12 ½	63	0 11 3	3
13 ½	64	0 16 6	6
14 ½	65	0 16 6	6
15 ½	66	0 9 6	6
16 ½	67	0 3 11	11
17 ½	68	0 1 18	18
18 ½	69	0 13 10	10
19 ½	70	0 9 4	4
20 ½	71	0 5 10	10
21 ½	72	0 2 3	3
22 ½	73	0 19 3	3
23 ½	74	0 16 3	3
24 ½	75	0 13 11	11

TABLE

TABLE XXXVIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $51\frac{1}{2}$ to $52\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment.		
	$\frac{1}{2}$, or at Age	£.	s.	d.
$1\frac{1}{2}$	52	11	7	6
$1\frac{3}{4}$	53	10	7	8
$2\frac{1}{4}$	54	9	9	9
$2\frac{3}{4}$	55	8	11	9
$3\frac{1}{4}$	56	7	16	7
$3\frac{3}{4}$	57	7	1	3
$4\frac{1}{4}$	58	6	8	8
$4\frac{3}{4}$	59	5	16	0
$5\frac{1}{4}$	60	5	5	3
$5\frac{3}{4}$	61	4	14	6
$6\frac{1}{4}$	62	4	5	3
$6\frac{3}{4}$	63	3	16	0
$7\frac{1}{4}$	64	3	8	1
$7\frac{3}{4}$	65	3	0	3
$8\frac{1}{4}$	66	2	13	6
$8\frac{3}{4}$	67	2	6	9
$9\frac{1}{4}$	68	2	1	5
$9\frac{3}{4}$	69	1	16	0
$10\frac{1}{4}$	70	1	11	9
$10\frac{3}{4}$	71	1	7	6
$11\frac{1}{4}$	72	1	4	0
$11\frac{3}{4}$	73	0	17	7
$12\frac{1}{4}$	74	0	14	5
$12\frac{3}{4}$	75			

TABLE XXXIX.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $52\frac{1}{2}$ to $53\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment.		
	$\frac{1}{2}$, or at Age	£.	s.	d.
$1\frac{1}{2}$	53	11	1	6
$1\frac{3}{4}$	54	10	2	8
$2\frac{1}{4}$	55	9	5	2
$2\frac{3}{4}$	56	8	7	7
$3\frac{1}{4}$	57	7	12	4
$3\frac{3}{4}$	58	6	17	0
$4\frac{1}{4}$	59	6	5	0
$4\frac{3}{4}$	60	5	12	11
$5\frac{1}{4}$	61	5	2	0
$5\frac{3}{4}$	62	4	11	0
$6\frac{1}{4}$	63	4	2	0
$6\frac{3}{4}$	64	3	13	1
$7\frac{1}{4}$	65	3	5	0
$7\frac{3}{4}$	66	2	17	0
$8\frac{1}{4}$	67	2	10	6
$8\frac{3}{4}$	68	2	4	0
$9\frac{1}{4}$	69	1	18	11
$9\frac{3}{4}$	70	1	13	10
$10\frac{1}{4}$	71	1	9	9
$10\frac{3}{4}$	72	1	5	8
$11\frac{1}{4}$	73	0	18	2
$11\frac{3}{4}$	74	0	16	9
$12\frac{1}{4}$	75			0

TABLE

TABLE XL.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 53½ to 54½.

To commence at the end of Years after pur- chasing.	1, or at Age 54	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
1 ½	54	10	17 6
2 ½	55	9	18 6
3 ½	56	9	0 11
4 ½	57	8	3 3
5 ½	58	7	8 7
6 ½	59	6	14 0
7 ½	60	6	1 7
8 ½	61	5	9 2
9 ½	62	4	18 7
10 ½	63	4	7 10
11 ½	64	3	18 8
12 ½	65	3	9 7
13 ½	66	3	1 9
14 ½	67	2	14 0
15 ½	68	2	7 9
16 ½	69	2	1 7
17 ½	70	1	16 8
18 ½	71	1	11 9
19 ½	72	1	7 9
20 ½	73	1	3 8
21 ½	74	0	0 4
	75	0	17 3

TABLE XLI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 54½ to 55½.

To commence at the end of Years after pur- chasing.	1, or at Age 55	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
1 ½	55	10	13 6
2 ½	56	9	14 5
3 ½	57	8	16 8
4 ½	58	7	18 11
5 ½	59	7	4 11
6 ½	60	6	10 11
7 ½	61	5	18 2
8 ½	62	5	5 6
9 ½	63	4	15 2
10 ½	64	4	4 9
11 ½	65	3	15 5
12 ½	66	3	6 1
13 ½	67	2	18 7
14 ½	68	2	11 0
15 ½	69	2	5 1
16 ½	70	1	19 2
17 ½	71	1	14 6
18 ½	72	1	9 6
19 ½	73	1	5 9
20 ½	74	0	1 8
	75	0	18 7

TABLE

TABLE XLII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 55½ to 56½.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1½	56	10	9	6	
2½	57	9	10	2	
3½	58	8	13	1	
4½	59	7	16	0	
5½	60	7	1	7	
6½	61	6	7	2	
7½	62	5	14	8	
8½	63	5	2	4	
9½	64	4	11	8	
10½	65	4	1	0	
11½	66	3	12	0	
12½	67	3	2	10	
13½	68	2	15	8	
14½	69	2	8	5	
15½	70	2	2	8	
16½	71	1	17	0	
17½	72	1	12	3	
18½	73	1	7	7	
19½	74	1	3	8	
	75	1	0	1	

TABLE XLIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 56½ to 57½.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1½	57	10	5	6	
2½	58	9	5	11	
3½	59	8	9	6	
4½	60	7	13	2	
5½	61	6	18	3	
6½	62	6	3	5	
7½	63	5	11	3	
8½	64	4	19	2	
9½	65	4	8	3	
10½	66	3	17	4	
11½	67	3	8	6	
12½	68	2	19	8	
13½	69	2	12	10	
14½	70	2	5	11	
15½	71	2	0	4	
16½	72	1	14	10	
17½	73	1	10	1	
18½	74	1	5	5	
	75	1	1	8	

TABLE XLIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 57½ to 58½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	£. s. d.
½ or at Age 58	9 19 6	
1 ½	9 1 9	
2 ½	8 5 8	
3 ½	7 7 6	
4 ½	6 13 13	
5 ½	5 19 2	
6 ½	4 14 9	
7 ½	4 3 10	
8 ½	3 13 3	
9 ½	3 4 10	
10 ½	2 16 5	
11 ½	2 9 9	
12 ½	2 3 1	
13 ½	1 17 7	
14 ½	1 12 2	
15 ½	1 7 7	
16 ½	1 3 5	
17 ½		

TABLE XLV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 58½ to 59½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	£. s. d.
½ or at Age 59	9 15 6	
1 ½	8 17 8	
2 ½	7 0 5	
3 ½	6 3 2	
4 ½	5 6 1	
5 ½	4 15 0	
6 ½	3 9 4	
7 ½	3 19 6	
8 ½	3 9 3	
9 ½	2 13 2	
10 ½	2 1 2	
11 ½	1 13 6	
12 ½	1 6 9	
13 ½	0 0 4	
14 ½	1 14 11	
15 ½	1 9 6	
16 ½	1 5 2	
17 ½		

TABLE XLVI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 59½ to 60½.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1, or at Age 60	9	11	6	
1½	8	12	7	
2½	7	15	9	
3½	6	18	10	
4½	6	4	5	
5½	5	10	0	
6½	4	17	8	
7½	4	5	4	
8½	3	15	7	
9½	3	5	9	
10½	2	18	0	
11½	2	10	2	
12½	2	3	10	
13½	1	17	5	
14½	1	12	1	
15½	1	7	0	

TABLE XLVII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 60½ to 61½.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1, or at Age 61	9	5	6	
1½	8	7	7	
2½	7	11	1	
3½	6	14	7	
4½	5	19	10	
5½	5	5	0	
6½	4	13	0	
7½	4	1	1	
8½	3	11	8	
9½	3	2	3	
10½	2	14	9	
11½	2	7	3	
12½	2	0	11	
13½	1	14	6	
14½	1	9	3	

T A B L E XLVIII.
For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 61½ to 62½.

To commence at the end of Years after pur- chasing.	To or at Age	Value of an an- nuity of £1. in one present pay- ment.
1 ½	62	9 1 6
2 ½	63	8 3 4
3 ½	64	7 6 4
4 ½	65	6 9 4
5 ½	66	5 14 11
6 ½	67	5 0 5
7 ½	68	4 8 11
8 ½	69	3 17 4
9 ½	70	3 8 3
10 ½	71	2 19 1
11 ½	72	2 11 6
12 ½	73	1 4 0
13 ½	74	1 17 10
	75	1 12 0

T A B L E XLIX.
For the Use of L O N D O N.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 62½ to 63½.

To commence at the end of Years after pur- chasing.	To or at Age	Value of an an- nuity of £1. in one present pay- ment.
1 ½	63	8 17 6
2 ½	64	7 19 2
3 ½	65	7 1 8
4 ½	66	6 4 2
5 ½	67	5 10 0
6 ½	68	4 15 11
7 ½	69	4 4 10
8 ½	70	3 13 8
9 ½	71	3 4 10
10 ½	72	2 15 11
11 ½	73	2 8 4
12 ½	74	1 0 10
	75	1 14 10

R r r r

T A B L E

TABLE L.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 63½ to 64½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
½, or at Age 64	£. s. d. 8 13 6
1 ¼	7 14 0
2 ¼	6 16 9
3 ¼	5 19 6
4 ¼	5 5 9
5 ¼	4 12 1
6 ¼	4 1 3
7 ¼	3 10 4
8 ¼	3 1 4
9 ¼	2 12 5
10 ¼	2 5 0
11 ¼	1 18 0

TABLE LI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75,

Age of the Purchaser from 64½ to 65½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
½, or at Age 65	£. s. d. 8 7 6
1 ¼	7 8 11
2 ¼	6 11 11
3 ¼	5 14 11
4 ¼	5 1 7
5 ¼	4 8 4
6 ¼	3 17 8
7 ¼	3 7 0
8 ¼	2 17 11
9 ¼	2 8 11
10 ¼	2 1 9

TABLE LII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 65½ to 66½.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.
1½	66	£. 8 s. 6 d.
2½	67	£. 7 s. 4 d.
3½	68	£. 6 s. 11 d.
4½	69	£. 5 s. 11 d.
5½	70	£. 4 s. 18 d.
6½	71	£. 4 s. 5 d.
7½	72	£. 3 s. 14 d.
8½	73	£. 3 s. 3 d.
9½	74	£. 2 s. 14 d.
	75	£. 2 s. 6 d.

TABLE LIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 66½ to 67½.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.
1½	67	£. 7 s. 6 d.
2½	68	£. 7 s. 19 d.
3½	69	£. 6 s. 0 d.
4½	70	£. 5 s. 11 d.
5½	71	£. 4 s. 7 d.
6½	72	£. 4 s. 14 d.
7½	73	£. 4 s. 1 d.
8½	74	£. 3 s. 10 d.
	75	£. 2 s. 19 d.
		£. 2 s. 11 d.

TABLE LIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 67 $\frac{1}{4}$ to 68 $\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment.		
$\frac{1}{4}$, or at Age 68		£.	s.	d.
$\frac{1}{4}$	68	7	15	6
$\frac{1}{2}$	69	6	16	1
$\frac{3}{4}$	70	6	0	0
$1\frac{1}{4}$	71	5	3	11
$1\frac{1}{2}$	72	4	10	7
$1\frac{3}{4}$	73	3	17	4
$2\frac{1}{4}$	74	3	6	6
$2\frac{1}{2}$	75	2	16	0

TABLE LV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 68 $\frac{1}{2}$ to 69 $\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment.		
$\frac{1}{4}$, or at Age 69		£.	s.	d.
$\frac{1}{4}$	69	7	9	6
$\frac{1}{2}$	70	6	12	0
$\frac{3}{4}$	71	5	16	0
$1\frac{1}{4}$	72	5	0	2
$1\frac{1}{2}$	73	4	6	7
$1\frac{3}{4}$	74	3	13	1
$2\frac{1}{4}$	75	3	2	4

TABLE LVI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 69½ to 70½.

To commence at the end of	Value of an annuity of £1. in one present payment.
Years after purchasing.	£. s. d.
½, or at Age 70	7 5 6
1 ½, —	6 7 7
2 ½, —	5 11 3
3 ½, —	4 15 0
4 ½, —	4 1 8
5 ½, —	3 9 0

TABLE LVII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 70½ to 71½.

To commence at the end of	Value of an annuity of £1. in one present payment.
Years after purchasing.	£. s. d.
½, or at Age 71	7 1 6
1 ½, —	6 3 3
2 ½, —	5 6 7
3 ½, —	4 9 11
4 ½, —	3 16 9

TABLE LVIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 71½ to 72½.

To commence at the end of	Value of an annuity of £1. in one present payment.
Years after purchasing.	£. s. d.
¼, Or at Age 72	6 17 3
1 ¼, ——— 73	5 18 1
2 ¼, ——— 74	5 1 9
3 ¼, ——— 75	4 6 6

TABLE LX.

For the Use of LONDON;

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 72½ to 73½.

To commence at the end of	Value of an annuity of £1. in one present payment.
Years after purchasing.	£. s. d.
¼, Or at Age 73	6 11 6
1 ¼, ——— 74	5 13 7
2 ¼, ——— 75	4 16 11

SECOND SET OF TABLES.

Intended for the Use of COUNTRY PARISHES.

T A B L E I.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 14 $\frac{1}{2}$ to 15 $\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
20 $\frac{1}{2}$, or at Age 35	41 $\frac{1}{2}$, or at Age 56	£. 6 s. 8 d.	£. 1 s. 4 d.
21 $\frac{1}{2}$, —	42 $\frac{1}{2}$, —	6 12 4	1 9 0
22 $\frac{1}{2}$, —	43 $\frac{1}{2}$, —	6 4 10	1 6 7
23 $\frac{1}{2}$, —	44 $\frac{1}{2}$, —	5 17 8	1 4 2
24 $\frac{1}{2}$, —	45 $\frac{1}{2}$, —	5 10 7	1 2 1
25 $\frac{1}{2}$, —	46 $\frac{1}{2}$, —	5 5 1	1 0 1
26 $\frac{1}{2}$, —	47 $\frac{1}{2}$, —	4 17 7	0 18 1
27 $\frac{1}{2}$, —	48 $\frac{1}{2}$, —	4 11 7	0 16 2
28 $\frac{1}{2}$, —	49 $\frac{1}{2}$, —	4 5 7	0 14 7
29 $\frac{1}{2}$, —	50 $\frac{1}{2}$, —	4 4 1	0 13 1
30 $\frac{1}{2}$, —	51 $\frac{1}{2}$, —	4 3 8	0 11 1
31 $\frac{1}{2}$, —	52 $\frac{1}{2}$, —	3 9 9	0 10 3
32 $\frac{1}{2}$, —	53 $\frac{1}{2}$, —	3 4 4	0 9 2
33 $\frac{1}{2}$, —	54 $\frac{1}{2}$, —	3 0 4	0 8 0
34 $\frac{1}{2}$, —	55 $\frac{1}{2}$, —	2 15 11	0 7 0
35 $\frac{1}{2}$, —	56 $\frac{1}{2}$, —	2 11 11	0 6 0
36 $\frac{1}{2}$, —	57 $\frac{1}{2}$, —	2 7 11	0 5 2
37 $\frac{1}{2}$, —	58 $\frac{1}{2}$, —	2 4 8	0 4 5
38 $\frac{1}{2}$, —	59 $\frac{1}{2}$, —	1 17 7	0 3 9
39 $\frac{1}{2}$, —	60 $\frac{1}{2}$, —	1 14 6	0 3 1
40 $\frac{1}{2}$, —		1 11 9	0 2 6

T A B L E

T A B L E II.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $15\frac{1}{2}$ to $16\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
19 $\frac{1}{2}$, or at Age 35	£. 6 s. 17 d. 5	40 $\frac{1}{2}$, or at Age 56	£. 1 s. 10 d. 3
20 $\frac{1}{2}$, —	6 9 10	41 $\frac{1}{2}$, —	1 7 7
21 $\frac{1}{2}$, —	6 2 2	42 $\frac{1}{2}$, —	1 5 3
22 $\frac{1}{2}$, —	5 15 1	43 $\frac{1}{2}$, —	1 2 11
23 $\frac{1}{2}$, —	5 8 1	44 $\frac{1}{2}$, —	0 0 9
24 $\frac{1}{2}$, —	5 1 6	45 $\frac{1}{2}$, —	0 18 9
25 $\frac{1}{2}$, —	4 15 0	46 $\frac{1}{2}$, —	0 16 11
26 $\frac{1}{2}$, —	4 9 0	47 $\frac{1}{2}$, —	0 15 2
27 $\frac{1}{2}$, —	4 3 1	48 $\frac{1}{2}$, —	0 13 8
28 $\frac{1}{2}$, —	3 17 3	49 $\frac{1}{2}$, —	0 12 1
29 $\frac{1}{2}$, —	3 12 3	50 $\frac{1}{2}$, —	0 10 10
30 $\frac{1}{2}$, —	3 7 6	51 $\frac{1}{2}$, —	0 9 4
31 $\frac{1}{2}$, —	3 2 8	52 $\frac{1}{2}$, —	0 8 4
32 $\frac{1}{2}$, —	2 18 3	53 $\frac{1}{2}$, —	0 7 3
33 $\frac{1}{2}$, —	2 13 10	54 $\frac{1}{2}$, —	0 6 3
34 $\frac{1}{2}$, —	2 9 11	55 $\frac{1}{2}$, —	0 5 5
35 $\frac{1}{2}$, —	2 6 0	56 $\frac{1}{2}$, —	0 4 7
36 $\frac{1}{2}$, —	2 2 5	57 $\frac{1}{2}$, —	0 3 10
37 $\frac{1}{2}$, —	1 19 0	58 $\frac{1}{2}$, —	0 3 3
38 $\frac{1}{2}$, —	1 16 0	59 $\frac{1}{2}$, —	0 2 8
39 $\frac{1}{2}$, —	1 12 11		

T A B L E

T A B L E III.

For the Life of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $16\frac{1}{2}$ to $17\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
18 $\frac{1}{2}$, Or at Age 35	£. 7. 3. d. 4	39 $\frac{1}{2}$, Or at Age 56	£. 11. 11. d. 4
19 $\frac{1}{2}$, —	6 14 10	40 $\frac{1}{2}$, —	1 8 9
20 $\frac{1}{2}$, —	6 7 1	41 $\frac{1}{2}$, —	1 6 2
21 $\frac{1}{2}$, —	5 19 5	42 $\frac{1}{2}$, —	1 3 11
22 $\frac{1}{2}$, —	5 12 6	43 $\frac{1}{2}$, —	1 1 7
23 $\frac{1}{2}$, —	5 5 7	44 $\frac{1}{2}$, —	0 19 7
24 $\frac{1}{2}$, —	4 19 0	45 $\frac{1}{2}$, —	0 17 7
25 $\frac{1}{2}$, —	4 12 5	46 $\frac{1}{2}$, —	0 15 10
26 $\frac{1}{2}$, —	4 6 6	47 $\frac{1}{2}$, —	0 14 1
27 $\frac{1}{2}$, —	4 0 7	48 $\frac{1}{2}$, —	0 12 8
28 $\frac{1}{2}$, —	3 15 4	49 $\frac{1}{2}$, —	0 11 2
29 $\frac{1}{2}$, —	3 10 0	50 $\frac{1}{2}$, —	0 9 11
30 $\frac{1}{2}$, —	3 5 3	51 $\frac{1}{2}$, —	0 8 7
31 $\frac{1}{2}$, —	3 0 6	52 $\frac{1}{2}$, —	0 7 7
32 $\frac{1}{2}$, —	2 16 1	53 $\frac{1}{2}$, —	0 6 6
33 $\frac{1}{2}$, —	2 11 9	54 $\frac{1}{2}$, —	0 5 7
34 $\frac{1}{2}$, —	2 7 11	55 $\frac{1}{2}$, —	0 4 9
35 $\frac{1}{2}$, —	2 4 0	56 $\frac{1}{2}$, —	0 4 0
36 $\frac{1}{2}$, —	2 0 8	57 $\frac{1}{2}$, —	0 3 3
37 $\frac{1}{2}$, —	1 17 4	58 $\frac{1}{2}$, —	0 2 8
38 $\frac{1}{2}$, —	1 14 4		

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T A B L E

T A B L E IV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $17\frac{1}{2}$ to $18\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
$17\frac{1}{2}$, or at Age 35	$38\frac{1}{2}$, or at Age 56	£. 7. 8. d. 11. 6.	£. 5. 12. 9. 1.
$18\frac{1}{2}$ —	$39\frac{1}{2}$ —	7. 0. 11. 6.	1. 12. 9. 10.
$19\frac{1}{2}$ —	$40\frac{1}{2}$ —	6. 12. 1. 6.	1. 9. 10. 10.
$20\frac{1}{2}$ —	$41\frac{1}{2}$ —	6. 4. 1. 6.	1. 7. 7. 4.
$21\frac{1}{2}$ —	$42\frac{1}{2}$ —	5. 16. 10. 10.	1. 4. 9. 7.
$22\frac{1}{2}$ —	$43\frac{1}{2}$ —	5. 9. 10. 10.	1. 2. 2. 7.
$23\frac{1}{2}$ —	$44\frac{1}{2}$ —	5. 2. 9. 4.	1. 0. 0. 3.
$24\frac{1}{2}$ —	$45\frac{1}{2}$ —	4. 16. 9. 4.	0. 18. 4. 4.
$25\frac{1}{2}$ —	$46\frac{1}{2}$ —	4. 9. 10. 1.	0. 16. 4. 4.
$26\frac{1}{2}$ —	$47\frac{1}{2}$ —	4. 4. 1. 2.	0. 14. 8. 8.
$27\frac{1}{2}$ —	$48\frac{1}{2}$ —	3. 18. 1. 2.	0. 13. 0. 8.
$28\frac{1}{2}$ —	$49\frac{1}{2}$ —	3. 12. 11. 1.	0. 11. 3. 3.
$29\frac{1}{2}$ —	$50\frac{1}{2}$ —	3. 7. 8. 0.	0. 10. 3. 0.
$30\frac{1}{2}$ —	$51\frac{1}{2}$ —	3. 3. 0. 9.	0. 9. 9. 0.
$31\frac{1}{2}$ —	$52\frac{1}{2}$ —	2. 18. 3. 10.	0. 7. 10. 9.
$32\frac{1}{2}$ —	$53\frac{1}{2}$ —	2. 13. 14. 8.	0. 6. 9. 10.
$33\frac{1}{2}$ —	$54\frac{1}{2}$ —	2. 9. 8. 0.	0. 5. 9. 11.
$34\frac{1}{2}$ —	$55\frac{1}{2}$ —	2. 6. 0. 2.	0. 4. 2. 2.
$35\frac{1}{2}$ —	$56\frac{1}{2}$ —	2. 2. 11. 8.	0. 4. 5. 5.
$36\frac{1}{2}$ —	$57\frac{1}{2}$ —	1. 18. 11. 8.	0. 3. 5. 9.
$37\frac{1}{2}$ —		1. 15. 8.	0. 2. 9. 9.

TABLE

TABLE V.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 18 $\frac{1}{2}$ to 19 $\frac{1}{2}$.

To commence at the end of	Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of	Years after purchasing.	Value of an annuity of £1. in one present payment.
16 $\frac{1}{2}$ or at Age 35	35	7 15 3	16 $\frac{1}{2}$ or at Age 56	56	1 14 0
17 $\frac{1}{2}$ — — —	36	7 7 6	17 $\frac{1}{2}$ — — —	57	1 11 2
18 $\frac{1}{2}$ — — —	37	6 17 9	18 $\frac{1}{2}$ — — —	58	1 8 4
19 $\frac{1}{2}$ — — —	38	6 6 1	19 $\frac{1}{2}$ — — —	59	1 5 10
20 $\frac{1}{2}$ — — —	39	5 14 11	20 $\frac{1}{2}$ — — —	60	1 3 4
21 $\frac{1}{2}$ — — —	40	5 7 4	21 $\frac{1}{2}$ — — —	61	1 1 2
22 $\frac{1}{2}$ — — —	41	5 0 2	22 $\frac{1}{2}$ — — —	62	0 19 0
23 $\frac{1}{2}$ — — —	42	4 13 8	23 $\frac{1}{2}$ — — —	63	0 17 1
24 $\frac{1}{2}$ — — —	43	4 7 4	24 $\frac{1}{2}$ — — —	64	0 15 2
25 $\frac{1}{2}$ — — —	44	4 1 7	25 $\frac{1}{2}$ — — —	65	0 13 7
26 $\frac{1}{2}$ — — —	45	3 15 10	26 $\frac{1}{2}$ — — —	66	0 12 0
27 $\frac{1}{2}$ — — —	46	3 10 7	27 $\frac{1}{2}$ — — —	67	0 10 8
28 $\frac{1}{2}$ — — —	47	3 5 5	28 $\frac{1}{2}$ — — —	68	0 9 4
29 $\frac{1}{2}$ — — —	48	3 0 9	29 $\frac{1}{2}$ — — —	69	0 8 2
30 $\frac{1}{2}$ — — —	49	2 16 1	30 $\frac{1}{2}$ — — —	70	0 7 0
31 $\frac{1}{2}$ — — —	50	2 11 10	31 $\frac{1}{2}$ — — —	71	0 6 1
32 $\frac{1}{2}$ — — —	51	2 7 7	32 $\frac{1}{2}$ — — —	72	0 5 1
33 $\frac{1}{2}$ — — —	52	2 4 0	33 $\frac{1}{2}$ — — —	73	0 4 4
34 $\frac{1}{2}$ — — —	53	2 0 5	34 $\frac{1}{2}$ — — —	74	0 3 7
35 $\frac{1}{2}$ — — —	54	1 17 2	35 $\frac{1}{2}$ — — —	75	0 2 11

S s s s 2

TABLE

TABLE VI.

For the Life of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $19\frac{1}{4}$ to $20\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
15 $\frac{1}{4}$, or at Age 35	36 $\frac{1}{4}$, or at Age 56	£. 8 s. 2 d.	£. 1 s. 15 d.
16 $\frac{1}{4}$ —	37 $\frac{1}{4}$ —	7 12 10	1 12 7
17 $\frac{1}{4}$ —	38 $\frac{1}{4}$ —	7 3 9	1 9 6
18 $\frac{1}{4}$ —	39 $\frac{1}{4}$ —	6 15 5	1 6 11
19 $\frac{1}{4}$ —	40 $\frac{1}{4}$ —	6 7 2	1 4 6
20 $\frac{1}{4}$ —	41 $\frac{1}{4}$ —	5 19 6	1 2 1
21 $\frac{1}{4}$ —	42 $\frac{1}{4}$ —	5 11 10	0 19 11
22 $\frac{1}{4}$ —	43 $\frac{1}{4}$ —	5 4 9	0 17 10
23 $\frac{1}{4}$ —	44 $\frac{1}{4}$ —	4 17 8	0 16 0
24 $\frac{1}{4}$ —	45 $\frac{1}{4}$ —	4 11 4	0 14 2
25 $\frac{1}{4}$ —	46 $\frac{1}{4}$ —	4 5 1	0 12 7
26 $\frac{1}{4}$ —	47 $\frac{1}{4}$ —	3 19 4	0 11 1
27 $\frac{1}{4}$ —	48 $\frac{1}{4}$ —	3 13 8	0 9 9
28 $\frac{1}{4}$ —	49 $\frac{1}{4}$ —	3 8 5	0 8 5
29 $\frac{1}{4}$ —	50 $\frac{1}{4}$ —	3 3 8	0 7 4
30 $\frac{1}{4}$ —	51 $\frac{1}{4}$ —	2 18 0	0 6 3
31 $\frac{1}{4}$ —	52 $\frac{1}{4}$ —	2 14 0	0 5 4
32 $\frac{1}{4}$ —	53 $\frac{1}{4}$ —	2 9 11	0 4 6
33 $\frac{1}{4}$ —	54 $\frac{1}{4}$ —	2 5 10	0 3 9
34 $\frac{1}{4}$ —	55 $\frac{1}{4}$ —	2 2 4	0 3 0
35 $\frac{1}{4}$ —	55 $\frac{1}{4}$ —	1 18 9	0 3 0

TABLE

TABLE VII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from $20\frac{1}{2}$ to $21\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
14 $\frac{1}{2}$, or at Age 35	35 $\frac{1}{2}$, or at Age 56	£. s. d.	£. s. d.
15 $\frac{1}{2}$, —	36 $\frac{1}{2}$, —	8 9 7	1 17 2
16 $\frac{1}{2}$, —	37 $\frac{1}{2}$, —	7 19 10	1 14 0
17 $\frac{1}{2}$, —	38 $\frac{1}{2}$, —	7 10 1	1 11 0
18 $\frac{1}{2}$, —	39 $\frac{1}{2}$, —	6 13 2	1 8 3
19 $\frac{1}{2}$, —	40 $\frac{1}{2}$, —	6 4 11	1 5 7
20 $\frac{1}{2}$, —	41 $\frac{1}{2}$, —	5 17 2	1 3 2
21 $\frac{1}{2}$, —	42 $\frac{1}{2}$, —	5 9 4	1 0 10
22 $\frac{1}{2}$, —	43 $\frac{1}{2}$, —	5 2 4	0 18 9
23 $\frac{1}{2}$, —	44 $\frac{1}{2}$, —	4 15 5	0 16 9
24 $\frac{1}{2}$, —	45 $\frac{1}{2}$, —	4 9 1	0 15 0
25 $\frac{1}{2}$, —	46 $\frac{1}{2}$, —	4 2 10	0 13 2
26 $\frac{1}{2}$, —	47 $\frac{1}{2}$, —	3 17 2	0 11 8
27 $\frac{1}{2}$, —	48 $\frac{1}{2}$, —	3 11 7	0 10 2
28 $\frac{1}{2}$, —	49 $\frac{1}{2}$, —	3 6 4	0 8 11
29 $\frac{1}{2}$, —	50 $\frac{1}{2}$, —	3 1 2	0 7 7
30 $\frac{1}{2}$, —	51 $\frac{1}{2}$, —	2 16 7	0 6 7
31 $\frac{1}{2}$, —	52 $\frac{1}{2}$, —	2 12 0	0 5 7
32 $\frac{1}{2}$, —	53 $\frac{1}{2}$, —	2 8 1	0 4 9
33 $\frac{1}{2}$, —	54 $\frac{1}{2}$, —	2 4 2	0 3 11
34 $\frac{1}{2}$, —	55 $\frac{1}{2}$, —	2 0 8	0 3 2

TABLE.

T A B L E VIII.

For the Life of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $21\frac{1}{2}$ to $22\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
13 $\frac{1}{4}$, or at Age 35	£. 8 s. 17 d. 2	34 $\frac{1}{4}$, or at Age 56	£. 1 s. 18 d. 11
14 $\frac{1}{4}$, —	8 7 2 2	35 $\frac{1}{4}$, —	1 15 6 6
15 $\frac{1}{4}$, —	7 17 2 2	36 $\frac{1}{4}$, —	1 12 6 6
16 $\frac{1}{4}$, —	7 8 2 2	37 $\frac{1}{4}$, —	1 9 6 6
17 $\frac{1}{4}$, —	6 19 1 1	38 $\frac{1}{4}$, —	1 6 9 9
18 $\frac{1}{4}$, —	6 10 9 2	39 $\frac{1}{4}$, —	1 4 10 2
19 $\frac{1}{4}$, —	6 2 4 4	40 $\frac{1}{4}$, —	1 1 10
20 $\frac{1}{4}$, —	5 14 7 7	41 $\frac{1}{4}$, —	0 19 7 7
21 $\frac{1}{4}$, —	5 6 10 10	42 $\frac{1}{4}$, —	0 17 7 7
22 $\frac{1}{4}$, —	4 19 11 7	43 $\frac{1}{4}$, —	0 15 7 7
23 $\frac{1}{4}$, —	4 13 0 11	44 $\frac{1}{4}$, —	0 13 11 11
24 $\frac{1}{4}$, —	4 6 10 7	45 $\frac{1}{4}$, —	0 12 9 9
25 $\frac{1}{4}$, —	4 4 7 7	46 $\frac{1}{4}$, —	0 10 9 9
26 $\frac{1}{4}$, —	3 14 11 3	47 $\frac{1}{4}$, —	0 8 11 3
27 $\frac{1}{4}$, —	3 9 4 4	48 $\frac{1}{4}$, —	0 6 11 1
28 $\frac{1}{4}$, —	3 4 2 2	49 $\frac{1}{4}$, —	0 5 10 10
29 $\frac{1}{4}$, —	2 19 1 8	50 $\frac{1}{4}$, —	0 4 11 11
30 $\frac{1}{4}$, —	2 14 3 3	51 $\frac{1}{4}$, —	0 4 4 4
31 $\frac{1}{4}$, —	2 10 3 3	52 $\frac{1}{4}$, —	0 4 1 1
32 $\frac{1}{4}$, —	2 6 3 3	53 $\frac{1}{4}$, —	0 3 4 4
33 $\frac{1}{4}$, —	2 2 5 5		

T A B L E

T A B L E IX.

For the Use of Country Parishes

Shewing the Payment due (reckoning Interest at 3 per Cent.) for
a Life-Annuity of One Pound, payable Quarterly. To com-
mence at any Age from 35 to 75.

Age of the Purchaser from $22\frac{1}{4}$ to $23\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.	
£. s. d.	£. s. d.	£. s. d.	£. s. d.	
12 $\frac{1}{4}$, or at Age 35	33 $\frac{1}{4}$, or at Age 56	2	0	8
13 $\frac{1}{4}$, —	34 $\frac{1}{4}$, —	1	17	3
14 $\frac{1}{4}$, —	35 $\frac{1}{4}$, —	1	13	11
15 $\frac{1}{4}$, —	36 $\frac{1}{4}$, —	1	11	0
16 $\frac{1}{4}$, —	37 $\frac{1}{4}$, —	1	8	0
17 $\frac{1}{4}$, —	38 $\frac{1}{4}$, —	1	5	5
18 $\frac{1}{4}$, —	39 $\frac{1}{4}$, —	1	2	10
19 $\frac{1}{4}$, —	40 $\frac{1}{4}$, —	1	0	18
20 $\frac{1}{4}$, —	41 $\frac{1}{4}$, —	0	18	4
21 $\frac{1}{4}$, —	42 $\frac{1}{4}$, —	0	16	5
22 $\frac{1}{4}$, —	43 $\frac{1}{4}$, —	0	14	5
23 $\frac{1}{4}$, —	44 $\frac{1}{4}$, —	0	12	10
24 $\frac{1}{4}$, —	45 $\frac{1}{4}$, —	0	11	2
25 $\frac{1}{4}$, —	46 $\frac{1}{4}$, —	0	9	10
26 $\frac{1}{4}$, —	47 $\frac{1}{4}$, —	0	8	5
27 $\frac{1}{4}$, —	48 $\frac{1}{4}$, —	0	7	3
28 $\frac{1}{4}$, —	49 $\frac{1}{4}$, —	0	6	2
29 $\frac{1}{4}$, —	50 $\frac{1}{4}$, —	0	5	2
30 $\frac{1}{4}$, —	51 $\frac{1}{4}$, —	0	4	3
31 $\frac{1}{4}$, —	52 $\frac{1}{4}$, —	0	3	6
32 $\frac{1}{4}$, —				

T A B L E

TABLE X.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $23\frac{1}{4}$ to $24\frac{1}{2}$.

To commence at the end of	Value of an annuity of £1. in one present payment.	To commence at the end of	Value of an annuity of £1. in one present payment.
Years after purchasing.	£. s. d.	Years after purchasing.	£. s. d.
11 $\frac{1}{4}$, or at Age 35	9 14 3	32 $\frac{1}{4}$, or at Age 56	2 2 10
12 $\frac{1}{4}$, —	9 3 3	33 $\frac{1}{4}$, —	1 18 11
13 $\frac{1}{4}$, —	8 12 4	34 $\frac{1}{4}$, —	1 15 8
14 $\frac{1}{4}$, —	8 2 4	35 $\frac{1}{4}$, —	1 12 6
15 $\frac{1}{4}$, —	7 12 5	36 $\frac{1}{4}$, —	1 9 6
16 $\frac{1}{4}$, —	7 3 3	37 $\frac{1}{4}$, —	1 6 6
17 $\frac{1}{4}$, —	6 14 1	38 $\frac{1}{4}$, —	1 4 0
18 $\frac{1}{4}$, —	6 5 8	39 $\frac{1}{4}$, —	1 1 7
19 $\frac{1}{4}$, —	5 17 2	40 $\frac{1}{4}$, —	0 19 4
20 $\frac{1}{4}$, —	5 9 6	41 $\frac{1}{4}$, —	0 17 1
21 $\frac{1}{4}$, —	5 1 11	42 $\frac{1}{4}$, —	0 15 3
22 $\frac{1}{4}$, —	4 15 3	43 $\frac{1}{4}$, —	0 13 6
23 $\frac{1}{4}$, —	4 8 7	44 $\frac{1}{4}$, —	0 11 10
24 $\frac{1}{4}$, —	4 2 3	45 $\frac{1}{4}$, —	0 10 2
25 $\frac{1}{4}$, —	3 16 0	46 $\frac{1}{4}$, —	0 8 11
26 $\frac{1}{4}$, —	3 10 7	47 $\frac{1}{4}$, —	0 7 8
27 $\frac{1}{4}$, —	3 5 2	48 $\frac{1}{4}$, —	0 6 8
28 $\frac{1}{4}$, —	3 0 1	49 $\frac{1}{4}$, —	0 5 7
29 $\frac{1}{4}$, —	2 15 0	50 $\frac{1}{4}$, —	0 4 6
30 $\frac{1}{4}$, —	2 10 10	51 $\frac{1}{4}$, —	0 3 8
31 $\frac{1}{4}$, —	2 6 9		

TABLE

TABLE XI.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 24 $\frac{1}{2}$ to 25 $\frac{1}{2}$.

To commence at the end of Years after purchasing.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.		Value of an annuity of £1. in one present payment.	
		£.	s. d.	£.	s. d.
10 $\frac{1}{2}$, or at Age 35	31 $\frac{1}{2}$, or at Age 56	10	3 9	2	5 1
11 $\frac{1}{2}$ — — —	32 $\frac{1}{2}$ — — —	9	11 9	2	5 2
12 $\frac{1}{2}$ — — —	33 $\frac{1}{2}$ — — —	9	0 9	1	17 2
13 $\frac{1}{2}$ — — —	34 $\frac{1}{2}$ — — —	8	9 10	1	14 1
14 $\frac{1}{2}$ — — —	35 $\frac{1}{2}$ — — —	7	19 11	1	11 1
15 $\frac{1}{2}$ — — —	36 $\frac{1}{2}$ — — —	7	10 0	1	8 0
16 $\frac{1}{2}$ — — —	37 $\frac{1}{2}$ — — —	7	0 8	1	5 0
17 $\frac{1}{2}$ — — —	38 $\frac{1}{2}$ — — —	6	11 5	1	2 8
18 $\frac{1}{2}$ — — —	39 $\frac{1}{2}$ — — —	6	3 0	1	0 5
19 $\frac{1}{2}$ — — —	40 $\frac{1}{2}$ — — —	5	14 7	0	18 2
20 $\frac{1}{2}$ — — —	41 $\frac{1}{2}$ — — —	5	7 0	0	15 1
21 $\frac{1}{2}$ — — —	42 $\frac{1}{2}$ — — —	4	19 5	0	14 2
22 $\frac{1}{2}$ — — —	43 $\frac{1}{2}$ — — —	4	12 11	0	12 6
23 $\frac{1}{2}$ — — —	44 $\frac{1}{2}$ — — —	4	6 6	0	10 10
24 $\frac{1}{2}$ — — —	45 $\frac{1}{2}$ — — —	4	0 0	0	9 2
25 $\frac{1}{2}$ — — —	46 $\frac{1}{2}$ — — —	3	13 7	0	8 1
26 $\frac{1}{2}$ — — —	47 $\frac{1}{2}$ — — —	3	8 5	0	7 0
27 $\frac{1}{2}$ — — —	48 $\frac{1}{2}$ — — —	3	3 3	0	6 0
28 $\frac{1}{2}$ — — —	49 $\frac{1}{2}$ — — —	2	18 2	0	4 11
29 $\frac{1}{2}$ — — —	50 $\frac{1}{2}$ — — —	2	13 0	0	3 10
30 $\frac{1}{2}$ — — —		2	9 0		

TABLE

TABLE XII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $25\frac{1}{2}$ to $26\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
9 $\frac{1}{4}$, or at Age 35	£. 10 s. 1 d.	30 $\frac{1}{4}$, or at Age 56	£. 2 s. 7 d.
10 $\frac{1}{4}$, —	10 13 1	31 $\frac{1}{4}$, —	2 3 2
11 $\frac{1}{4}$, —	9 9 1	32 $\frac{1}{4}$, —	1 19 3
12 $\frac{1}{4}$, —	8 18 2	33 $\frac{1}{4}$, —	1 15 4
13 $\frac{1}{4}$, —	8 7 3	34 $\frac{1}{4}$, —	1 12 4
14 $\frac{1}{4}$, —	7 17 2	35 $\frac{1}{4}$, —	1 9 5
15 $\frac{1}{4}$, —	7 7 1	36 $\frac{1}{4}$, —	1 6 5
16 $\frac{1}{4}$, —	6 17 10	37 $\frac{1}{4}$, —	1 3 6
17 $\frac{1}{4}$, —	6 8 7	38 $\frac{1}{4}$, —	1 1 3
18 $\frac{1}{4}$, —	6 0 3	39 $\frac{1}{4}$, —	0 19 0
19 $\frac{1}{4}$, —	5 11 10	40 $\frac{1}{4}$, —	0 16 10
20 $\frac{1}{4}$, —	5 4 4	41 $\frac{1}{4}$, —	0 14 8
21 $\frac{1}{4}$, —	4 16 10	42 $\frac{1}{4}$, —	0 13 0
22 $\frac{1}{4}$, —	4 10 5	43 $\frac{1}{4}$, —	0 11 5
23 $\frac{1}{4}$, —	4 4 0	44 $\frac{1}{4}$, —	0 9 10
24 $\frac{1}{4}$, —	3 17 3	45 $\frac{1}{4}$, —	0 8 3
25 $\frac{1}{4}$, —	3 11 2	46 $\frac{1}{4}$, —	0 7 2
26 $\frac{1}{4}$, —	3 6 1	47 $\frac{1}{4}$, —	0 6 1
27 $\frac{1}{4}$, —	3 1 0	48 $\frac{1}{4}$, —	0 5 0
28 $\frac{1}{4}$, —	2 16 0	49 $\frac{1}{4}$, —	0 4 0
29 $\frac{1}{4}$, —	2 11 0		

TABLE.

TABLE XIII.

For the Life of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $26\frac{1}{2}$ to $27\frac{1}{2}$.

To commence at the end of Years after purchasing.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	Value of an annuity of £1. in one present payment.
8 $\frac{1}{2}$, or at Age 35	29 $\frac{1}{2}$, or at Age 56	£. s. d.	£. s. d.
9 $\frac{1}{2}$, —	30 $\frac{1}{2}$, —	11 3 7	2 9 0
10 $\frac{1}{2}$, —	31 $\frac{1}{2}$, —	10 10 6	2 5 2
11 $\frac{1}{2}$, —	32 $\frac{1}{2}$, —	9 18 6	2 1 4
12 $\frac{1}{2}$, —	33 $\frac{1}{2}$, —	9 6 6	1 17 5
13 $\frac{1}{2}$, —	34 $\frac{1}{2}$, —	8 15 4	1 13 7
14 $\frac{1}{2}$, —	35 $\frac{1}{2}$, —	8 4 4	1 10 8
15 $\frac{1}{2}$, —	36 $\frac{1}{2}$, —	7 14 3	1 7 10
16 $\frac{1}{2}$, —	37 $\frac{1}{2}$, —	7 4 4	1 4 11
17 $\frac{1}{2}$, —	38 $\frac{1}{2}$, —	6 15 0	1 2 0
18 $\frac{1}{2}$, —	39 $\frac{1}{2}$, —	6 5 10	0 19 10
19 $\frac{1}{2}$, —	40 $\frac{1}{2}$, —	5 17 6	0 17 8
20 $\frac{1}{2}$, —	41 $\frac{1}{2}$, —	5 9 2	0 15 7
21 $\frac{1}{2}$, —	42 $\frac{1}{2}$, —	5 1 9	0 13 5
22 $\frac{1}{2}$, —	43 $\frac{1}{2}$, —	4 14 4	0 11 11
23 $\frac{1}{2}$, —	44 $\frac{1}{2}$, —	4 7 11	0 10 5
24 $\frac{1}{2}$, —	45 $\frac{1}{2}$, —	4 1 6	0 8 11
25 $\frac{1}{2}$, —	46 $\frac{1}{2}$, —	3 15 8	0 7 5
26 $\frac{1}{2}$, —	47 $\frac{1}{2}$, —	3 8 8	0 6 4
27 $\frac{1}{2}$, —	48 $\frac{1}{2}$, —	3 3 9	0 5 3
28 $\frac{1}{2}$, —	49 $\frac{1}{2}$, —	2 18 10	0 4 2
55	55	2 13 11	

T t t t 2

TABLE

TABLE XIV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $27\frac{1}{4}$ to $28\frac{1}{4}$.

To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.	
		£. s. d.				£. s. d.	
7 $\frac{1}{4}$, or at Age 35	11	14	1	28 $\frac{1}{4}$, or at Age 56	2	11	9
8 $\frac{1}{4}$, —	11	0	10	29 $\frac{1}{4}$, —	2	7	2
9 $\frac{1}{4}$, —	10	7	8	30 $\frac{1}{4}$, —	2	3	2
10 $\frac{1}{4}$, —	9	15	9	31 $\frac{1}{4}$, —	1	19	2
11 $\frac{1}{4}$, —	9	3	10	32 $\frac{1}{4}$, —	1	15	7
12 $\frac{1}{4}$, —	8	12	9	33 $\frac{1}{4}$, —	1	12	1
13 $\frac{1}{4}$, —	8	1	8	34 $\frac{1}{4}$, —	1	9	0
14 $\frac{1}{4}$, —	7	11	5	35 $\frac{1}{4}$, —	1	6	0
15 $\frac{1}{4}$, —	7	1	2	36 $\frac{1}{4}$, —	1	3	4
16 $\frac{1}{4}$, —	6	12	0	37 $\frac{1}{4}$, —	0	18	9
17 $\frac{1}{4}$, —	5	14	8	38 $\frac{1}{4}$, —	0	16	3
18 $\frac{1}{4}$, —	5	6	5	39 $\frac{1}{4}$, —	0	14	4
19 $\frac{1}{4}$, —	4	19	0	40 $\frac{1}{4}$, —	0	12	7
20 $\frac{1}{4}$, —	4	11	7	41 $\frac{1}{4}$, —	0	10	10
21 $\frac{1}{4}$, —	4	5	0	42 $\frac{1}{4}$, —	0	9	3
22 $\frac{1}{4}$, —	4	18	6	43 $\frac{1}{4}$, —	0	7	11
23 $\frac{1}{4}$, —	3	12	5	44 $\frac{1}{4}$, —	0	6	8
24 $\frac{1}{4}$, —	3	6	4	45 $\frac{1}{4}$, —	0	5	6
25 $\frac{1}{4}$, —	3	1	4	46 $\frac{1}{4}$, —	0	4	5
26 $\frac{1}{4}$, —	2	16	4	47 $\frac{1}{4}$, —	0		

TABLE

TABLE XV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 28 $\frac{1}{2}$ to 29 $\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
6 $\frac{1}{2}$, or at Age 35	27 $\frac{1}{2}$, or at Age 56	12	2
7 $\frac{1}{2}$ —	28 $\frac{1}{2}$ —	11	13
8 $\frac{1}{2}$ —	29 $\frac{1}{2}$ —	11	9
9 $\frac{1}{2}$ —	30 $\frac{1}{2}$ —	10	5
10 $\frac{1}{2}$ —	31 $\frac{1}{2}$ —	9	1
11 $\frac{1}{2}$ —	32 $\frac{1}{2}$ —	9	17
12 $\frac{1}{2}$ —	33 $\frac{1}{2}$ —	8	13
13 $\frac{1}{2}$ —	34 $\frac{1}{2}$ —	7	10
14 $\frac{1}{2}$ —	35 $\frac{1}{2}$ —	7	7
15 $\frac{1}{2}$ —	36 $\frac{1}{2}$ —	6	4
16 $\frac{1}{2}$ —	37 $\frac{1}{2}$ —	6	1
17 $\frac{1}{2}$ —	38 $\frac{1}{2}$ —	6	19
18 $\frac{1}{2}$ —	39 $\frac{1}{2}$ —	5	17
19 $\frac{1}{2}$ —	40 $\frac{1}{2}$ —	5	14
20 $\frac{1}{2}$ —	41 $\frac{1}{2}$ —	5	13
21 $\frac{1}{2}$ —	42 $\frac{1}{2}$ —	4	11
22 $\frac{1}{2}$ —	43 $\frac{1}{2}$ —	4	9
23 $\frac{1}{2}$ —	44 $\frac{1}{2}$ —	4	8
24 $\frac{1}{2}$ —	45 $\frac{1}{2}$ —	3	6
25 $\frac{1}{2}$ —	46 $\frac{1}{2}$ —	3	5
26 $\frac{1}{2}$ —		2	4

TABLE

TABLE XVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $29\frac{1}{2}$ to $30\frac{1}{2}$.

To commence at the end of Years after purchasing.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	Value of an annuity of £1. in one present payment.
5 $\frac{1}{2}$, or at Age 35	26 $\frac{1}{2}$, or at Age 56	£. s. d.	£. s. d.
6 $\frac{1}{4}$	27 $\frac{1}{4}$	2 11 6	2 11 6
7 $\frac{1}{4}$	28 $\frac{1}{4}$	2 7 3	2 7 3
8 $\frac{1}{4}$	29 $\frac{1}{4}$	2 3 0	2 3 0
9 $\frac{1}{4}$	30 $\frac{1}{4}$	1 19 1	1 19 1
10 $\frac{1}{4}$	31 $\frac{1}{4}$	1 15 3	1 15 3
11 $\frac{1}{4}$	32 $\frac{1}{4}$	1 11 11	1 11 11
12 $\frac{1}{4}$	33 $\frac{1}{4}$	1 8 7	1 8 7
13 $\frac{1}{4}$	34 $\frac{1}{4}$	1 5 5	1 5 5
14 $\frac{1}{4}$	35 $\frac{1}{4}$	1 2 9	1 2 9
15 $\frac{1}{4}$	36 $\frac{1}{4}$	17 4 4	17 4 4
16 $\frac{1}{4}$	37 $\frac{1}{4}$	16 10 10	16 10 10
17 $\frac{1}{4}$	38 $\frac{1}{4}$	15 15 8	15 15 8
18 $\frac{1}{4}$	39 $\frac{1}{4}$	14 13 8	14 13 8
19 $\frac{1}{4}$	40 $\frac{1}{4}$	13 11 10	13 11 10
20 $\frac{1}{4}$	41 $\frac{1}{4}$	12 10 2	12 10 2
21 $\frac{1}{4}$	42 $\frac{1}{4}$	11 8 8	11 8 8
22 $\frac{1}{4}$	43 $\frac{1}{4}$	10 7 2	10 7 2
23 $\frac{1}{4}$	44 $\frac{1}{4}$	9 5 11	9 5 11
24 $\frac{1}{4}$	45 $\frac{1}{4}$	8 4 10	8 4 10
25 $\frac{1}{4}$			

TABLE

T A B L E XVII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $30\frac{1}{2}$ to $31\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.
4 $\frac{1}{2}$	35	13 10 0	25 $\frac{1}{2}$	56	2 19 2
5 $\frac{1}{2}$	36	12 14 0	26 $\frac{1}{2}$	57	2 14 2
6 $\frac{1}{2}$	37	11 19 7	27 $\frac{1}{2}$	58	2 9 2
7 $\frac{1}{2}$	38	11 5 2	28 $\frac{1}{2}$	59	2 4 11
8 $\frac{1}{2}$	39	10 12 0	29 $\frac{1}{2}$	60	2 0 7
9 $\frac{1}{2}$	40	9 18 10	30 $\frac{1}{2}$	61	1 17 1
10 $\frac{1}{2}$	41	9 6 5	31 $\frac{1}{2}$	62	1 13 7
11 $\frac{1}{2}$	42	8 14 0	32 $\frac{1}{2}$	63	1 10 1
12 $\frac{1}{2}$	43	8 2 11	33 $\frac{1}{2}$	64	1 6 7
13 $\frac{1}{2}$	44	7 11 10	34 $\frac{1}{2}$	65	1 4 0
14 $\frac{1}{2}$	45	7 1 10	35 $\frac{1}{2}$	66	1 1 5
15 $\frac{1}{2}$	46	6 11 10	36 $\frac{1}{2}$	67	0 18 10
16 $\frac{1}{2}$	47	6 2 10	37 $\frac{1}{2}$	68	0 16 2
17 $\frac{1}{2}$	48	5 13 10	38 $\frac{1}{2}$	69	0 14 4
18 $\frac{1}{2}$	49	5 5 6	39 $\frac{1}{2}$	70	0 12 6
19 $\frac{1}{2}$	50	4 17 6	40 $\frac{1}{2}$	71	0 10 8
20 $\frac{1}{2}$	51	4 10 3	41 $\frac{1}{2}$	72	0 8 10
21 $\frac{1}{2}$	52	4 2 11	42 $\frac{1}{2}$	73	0 7 7
22 $\frac{1}{2}$	53	3 16 7	43 $\frac{1}{2}$	74	0 6 3
23 $\frac{1}{2}$	54	3 10 2	44 $\frac{1}{2}$	75	0 5 0
24 $\frac{1}{2}$	55	3 4 8			

T A B L E

TABLE XVIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $31\frac{1}{2}$ to $32\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	£. s. d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	£. s. d.
$3\frac{1}{2}$, or at Age 35	14	2	$24\frac{1}{2}$, or at Age 56	3	2
$4\frac{1}{2}$ — — —	13	6	$25\frac{1}{2}$ — — —	3	0
$5\frac{1}{2}$ — — —	12	10	$26\frac{1}{2}$ — — —	2	16
$6\frac{1}{2}$ — — —	11	16	$27\frac{1}{2}$ — — —	2	11
$7\frac{1}{2}$ — — —	11	1	$28\frac{1}{2}$ — — —	2	6
$8\frac{1}{2}$ — — —	10	8	$29\frac{1}{2}$ — — —	2	2
$9\frac{1}{2}$ — — —	9	14	$30\frac{1}{2}$ — — —	1	18
$10\frac{1}{2}$ — — —	9	2	$31\frac{1}{2}$ — — —	1	14
$11\frac{1}{2}$ — — —	8	10	$32\frac{1}{2}$ — — —	1	11
$12\frac{1}{2}$ — — —	7	19	$33\frac{1}{2}$ — — —	1	8
$13\frac{1}{2}$ — — —	7	8	$34\frac{1}{2}$ — — —	1	5
$14\frac{1}{2}$ — — —	6	18	$35\frac{1}{2}$ — — —	0	2
$15\frac{1}{2}$ — — —	6	8	$36\frac{1}{2}$ — — —	0	19
$16\frac{1}{2}$ — — —	5	9	$37\frac{1}{2}$ — — —	0	17
$17\frac{1}{2}$ — — —	5	10	$38\frac{1}{2}$ — — —	0	15
$18\frac{1}{2}$ — — —	5	2	$39\frac{1}{2}$ — — —	0	13
$19\frac{1}{2}$ — — —	4	14	$40\frac{1}{2}$ — — —	0	11
$20\frac{1}{2}$ — — —	4	7	$41\frac{1}{2}$ — — —	0	9
$21\frac{1}{2}$ — — —	3	19	$42\frac{1}{2}$ — — —	0	7
$22\frac{1}{2}$ — — —	3	13	$43\frac{1}{2}$ — — —	0	6
$23\frac{1}{2}$ — — —	3	7		0	5

TABLE

T A B L E XIX.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $32\frac{1}{2}$ to $33\frac{1}{2}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.	
		£. s. d.				£. s. d.	
2 $\frac{1}{2}$	35	14	16	1	56	3	4
3 $\frac{1}{2}$	36	13	18	1	57	2	10
4 $\frac{1}{2}$	37	13	2	7	58	2	5
5 $\frac{1}{2}$	38	12	6	8	59	2	0
6 $\frac{1}{2}$	39	11	12	10	60	2	3
7 $\frac{1}{2}$	40	10	18	5	61	2	7
8 $\frac{1}{2}$	41	10	4	6	62	2	6
9 $\frac{1}{2}$	42	9	11	0	63	1	4
10 $\frac{1}{2}$	43	8	18	10	64	1	9
11 $\frac{1}{2}$	44	8	6	7	65	1	2
12 $\frac{1}{2}$	45	7	15	7	66	1	6
13 $\frac{1}{2}$	46	7	4	7	67	1	4
14 $\frac{1}{2}$	47	6	14	9	68	0	8
15 $\frac{1}{2}$	48	6	4	10	69	0	10
16 $\frac{1}{2}$	49	5	15	11	70	0	10
17 $\frac{1}{2}$	50	5	7	0	71	0	10
18 $\frac{1}{2}$	51	4	18	11	72	0	10
19 $\frac{1}{2}$	52	4	10	10	73	0	5
20 $\frac{1}{2}$	53	4	3	11	74	0	0
21 $\frac{1}{2}$	54	3	17	0	75	0	7
22 $\frac{1}{2}$	55	3	10	11			

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T A B L E

TABLE XX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $33\frac{1}{2}$ to $34\frac{1}{2}$.

To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.			To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.		
		£.	s.	d.			£.	s.	d.
$1\frac{1}{4}$, or at Age 35	—	15	9	9	$22\frac{1}{4}$, or at Age 56	—	3	8	1
$2\frac{1}{4}$	—	14	12	3	$23\frac{1}{4}$	—	3	2	1
$3\frac{1}{4}$	—	13	14	9	$24\frac{1}{4}$	—	2	16	9
$4\frac{1}{4}$	—	12	18	11	$25\frac{1}{4}$	—	2	11	6
$5\frac{1}{4}$	—	12	3	1	$26\frac{1}{4}$	—	2	6	10
$6\frac{1}{4}$	—	11	8	5	$27\frac{1}{4}$	—	2	2	2
$7\frac{1}{4}$	—	10	13	9	$28\frac{1}{4}$	—	1	18	2
$8\frac{1}{4}$	—	10	0	4	$29\frac{1}{4}$	—	1	14	2
$9\frac{1}{4}$	—	9	6	11	$30\frac{1}{4}$	—	1	10	8
$10\frac{1}{4}$	—	8	14	10	$31\frac{1}{4}$	—	1	7	3
$11\frac{1}{4}$	—	8	2	8	$32\frac{1}{4}$	—	1	4	4
$12\frac{1}{4}$	—	7	11	9	$33\frac{1}{4}$	—	1	1	6
$13\frac{1}{4}$	—	7	0	10	$34\frac{1}{4}$	—	0	19	0
$14\frac{1}{4}$	—	6	11	0	$35\frac{1}{4}$	—	0	16	6
$15\frac{1}{4}$	—	6	1	1	$36\frac{1}{4}$	—	0	14	5
$16\frac{1}{4}$	—	5	12	3	$37\frac{1}{4}$	—	0	12	3
$17\frac{1}{4}$	—	5	3	5	$38\frac{1}{4}$	—	0	10	6
$18\frac{1}{4}$	—	4	15	7	$39\frac{1}{4}$	—	0	8	9
$19\frac{1}{4}$	—	4	7	9	$40\frac{1}{4}$	—	0	7	3
$20\frac{1}{4}$	—	4	0	11	$41\frac{1}{4}$	—	0	5	10
$21\frac{1}{4}$	—	3	14	2					

TABLE

T A B L E XXI.

For the Life of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $34\frac{1}{2}$ to $35\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
or at Age	£. s. d.	or at Age	£. s. d.
$35\frac{1}{2}$	16 5 0	$34\frac{1}{2}$	3 11 4
36	15 6 0	$35\frac{1}{2}$	3 5 4
$37\frac{1}{2}$	14 8 6	36	2 19 4
38	13 11 0	$37\frac{1}{2}$	2 14 2
$39\frac{1}{2}$	12 15 2	38	2 9 0
40	11 19 5	$39\frac{1}{2}$	2 4 5
$41\frac{1}{2}$	11 4 6	40	1 19 10
42	10 9 7	$41\frac{1}{2}$	1 15 11
$43\frac{1}{2}$	9 16 2	42	1 12 8
44	8 2 10	$43\frac{1}{2}$	1 8 8
$45\frac{1}{2}$	7 10 10	44	1 5 4
46	6 18 9	$45\frac{1}{2}$	1 2 5
$47\frac{1}{2}$	5 8 0	46	0 19 7
48	4 17 2	$47\frac{1}{2}$	0 17 5
$49\frac{1}{2}$	3 7 4	48	0 15 2
50	2 17 7	$49\frac{1}{2}$	0 13 0
$51\frac{1}{2}$	1 17 8	50	0 10 9
52	1 19 10	$51\frac{1}{2}$	0 9 2
$53\frac{1}{2}$	4 12 3	52	0 7 8
54	4 4 8	$53\frac{1}{2}$	0 7 4
$55\frac{1}{2}$	3 18 0	54	0 6 2
56		$55\frac{1}{2}$	

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T A B L E

TABLE XXII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $35\frac{1}{4}$ to $36\frac{1}{2}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.	
		£. s. d.				£. s. d.	
$1\frac{1}{4}$	36	16	0	10	$20\frac{1}{4}$	3	14
$1\frac{1}{2}$	37	15	2	0	$21\frac{1}{4}$	3	8
$2\frac{1}{4}$	38	14	4	6	$22\frac{1}{4}$	3	2
$2\frac{1}{2}$	39	13	7	2	$23\frac{1}{4}$	2	16
$3\frac{1}{4}$	40	12	11	0	$24\frac{1}{4}$	2	11
$3\frac{1}{2}$	41	11	15	0	$25\frac{1}{4}$	2	6
$4\frac{1}{4}$	42	11	0	2	$26\frac{1}{4}$	2	1
$4\frac{1}{2}$	43	10	5	3	$27\frac{1}{4}$	1	17
$5\frac{1}{4}$	44	9	12	0	$28\frac{1}{4}$	1	13
$5\frac{1}{2}$	45	8	18	8	$29\frac{1}{4}$	1	9
$6\frac{1}{4}$	46	8	6	9	$30\frac{1}{4}$	1	6
$6\frac{1}{2}$	47	7	14	9	$31\frac{1}{4}$	1	3
$7\frac{1}{4}$	48	7	3	11	$32\frac{1}{4}$	0	17
$7\frac{1}{2}$	49	6	13	4	$33\frac{1}{4}$	0	15
$8\frac{1}{4}$	50	6	3	7	$34\frac{1}{4}$	0	13
$8\frac{1}{2}$	51	5	13	0	$35\frac{1}{4}$	0	11
$9\frac{1}{4}$	52	5	5	5	$36\frac{1}{4}$	0	9
$9\frac{1}{2}$	53	4	16	11	$37\frac{1}{4}$	0	7
$10\frac{1}{4}$	54	4	8	5	$38\frac{1}{4}$	0	6
$10\frac{1}{2}$	55	4	1		$39\frac{1}{4}$	0	

TABLE

TABLE XXIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $36\frac{1}{4}$ to $37\frac{1}{4}$.

Years after pur- chasing.	To commence at the end of	or at Age	Value of an an- nuity of £1. in one present pay- ment.			Years after pur- chasing.	To commence at the end of	or at Age	Value of an an- nuity of £1. in one present pay- ment.		
			£.	s.	d.				£.	s.	d.
$\frac{1}{4}$	—	37	15	16	9	$20\frac{1}{4}$	—	57	3	11	8
$1\frac{1}{4}$	—	38	14	18	0	$21\frac{1}{4}$	—	58	3	5	2
$2\frac{1}{4}$	—	39	14	0	6	$22\frac{1}{4}$	—	59	2	19	6
$3\frac{1}{4}$	—	40	13	3	4	$23\frac{1}{4}$	—	60	2	13	10
$4\frac{1}{4}$	—	41	12	6	11	$24\frac{1}{4}$	—	61	2	8	10
$5\frac{1}{4}$	—	42	11	10	7	$25\frac{1}{4}$	—	62	2	3	10
$6\frac{1}{4}$	—	43	10	15	10	$26\frac{1}{4}$	—	63	1	19	6
$7\frac{1}{4}$	—	44	10	1	0	$27\frac{1}{4}$	—	64	1	15	2
$8\frac{1}{4}$	—	45	9	7	10	$28\frac{1}{4}$	—	65	1	11	6
$9\frac{1}{4}$	—	46	8	14	7	$29\frac{1}{4}$	—	66	1	7	10
$10\frac{1}{4}$	—	47	8	2	8	$30\frac{1}{4}$	—	67	1	4	7
$11\frac{1}{4}$	—	48	7	10	10	$31\frac{1}{4}$	—	68	1	1	5
$12\frac{1}{4}$	—	49	6	19	11	$32\frac{1}{4}$	—	69	0	18	10
$13\frac{1}{4}$	—	50	6	9	0	$33\frac{1}{4}$	—	70	0	16	2
$14\frac{1}{4}$	—	51	5	19	5	$34\frac{1}{4}$	—	71	0	14	0
$15\frac{1}{4}$	—	52	5	9	10	$35\frac{1}{4}$	—	72	0	11	10
$16\frac{1}{4}$	—	53	5	1	5	$36\frac{1}{4}$	—	73	0	10	1
$17\frac{1}{4}$	—	54	4	13	0	$37\frac{1}{4}$	—	74	0	8	5
$18\frac{1}{4}$	—	55	4	5	7						
$19\frac{1}{4}$	—	56	3	18	2				0	6	9

TABLE

TABLE XXIV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $37\frac{1}{4}$ to $38\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
$\frac{1}{4}$, or at Age 38	£. s. d.	$\frac{1}{4}$, or at Age 57	£. s. d.
1 $\frac{1}{4}$	39 15 6	19 $\frac{1}{4}$	3 15 0
2 $\frac{1}{4}$	40 13 11	20 $\frac{1}{4}$	3 8 7
3 $\frac{1}{4}$	41 13 0	21 $\frac{1}{4}$	3 2 0
4 $\frac{1}{4}$	42 12 7	22 $\frac{1}{4}$	2 16 7
5 $\frac{1}{4}$	43 11 11	23 $\frac{1}{4}$	2 11 0
6 $\frac{1}{4}$	44 11 3	24 $\frac{1}{4}$	2 6 2
7 $\frac{1}{4}$	45 10 16	25 $\frac{1}{4}$	2 1 4
8 $\frac{1}{4}$	46 9 5	26 $\frac{1}{4}$	1 17 1
9 $\frac{1}{4}$	47 8 3	27 $\frac{1}{4}$	1 12 10
10 $\frac{1}{4}$	48 7 4	28 $\frac{1}{4}$	1 9 3
11 $\frac{1}{4}$	49 6 5	29 $\frac{1}{4}$	1 5 8
12 $\frac{1}{4}$	50 5 8	30 $\frac{1}{4}$	1 2 7
13 $\frac{1}{4}$	51 4 11	31 $\frac{1}{4}$	0 19 7
14 $\frac{1}{4}$	52 3 6	32 $\frac{1}{4}$	0 17 1
15 $\frac{1}{4}$	53 2 1	33 $\frac{1}{4}$	0 14 7
16 $\frac{1}{4}$	54 1 10	34 $\frac{1}{4}$	0 12 6
17 $\frac{1}{4}$	55 9 7	35 $\frac{1}{4}$	0 10 5
18 $\frac{1}{4}$	56 7 3	36 $\frac{1}{4}$	0 8 9
		37 $\frac{1}{4}$	0 7 1

TABLE

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T A B L E XXV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 38 $\frac{1}{2}$ to 39 $\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.	Value of an an- nuity of £1. in one present pay- ment.
1, or at Age 39	1, or at Age 58	£. s. d.	£. s. d.
1 $\frac{1}{2}$	19 $\frac{1}{2}$	15 8 6	3 11 10
2 $\frac{1}{2}$	20 $\frac{1}{2}$	14 9 10	3 5 6
3 $\frac{1}{2}$	21 $\frac{1}{2}$	13 11 10	2 19 2
4 $\frac{1}{2}$	22 $\frac{1}{2}$	12 13 10	2 13 8
5 $\frac{1}{2}$	23 $\frac{1}{2}$	11 17 7	2 8 2
6 $\frac{1}{2}$	24 $\frac{1}{2}$	11 1 4	2 3 6
7 $\frac{1}{2}$	25 $\frac{1}{2}$	10 6 8	1 18 10
8 $\frac{1}{2}$	26 $\frac{1}{2}$	9 12 0	1 14 8
9 $\frac{1}{2}$	27 $\frac{1}{2}$	8 19 0	1 10 7
10 $\frac{1}{2}$	28 $\frac{1}{2}$	8 16 0	1 7 1
11 $\frac{1}{2}$	29 $\frac{1}{2}$	7 14 0	1 3 8
12 $\frac{1}{2}$	30 $\frac{1}{2}$	7 7 1	1 0 10
13 $\frac{1}{2}$	31 $\frac{1}{2}$	6 11 6	0 17 5
14 $\frac{1}{2}$	32 $\frac{1}{2}$	6 6 10	0 15 0
15 $\frac{1}{2}$	33 $\frac{1}{2}$	5 11 7	0 13 0
16 $\frac{1}{2}$	34 $\frac{1}{2}$	5 5 3	0 11 0
17 $\frac{1}{2}$	35 $\frac{1}{2}$	4 14 2	0 9 0
18 $\frac{1}{2}$	36 $\frac{1}{2}$	3 19 0	0 7 5

T A B L E

TABLE XXVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $39\frac{1}{2}$ to $40\frac{1}{2}$

To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.	To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
$1\frac{1}{2}$	40	15	4	4	18 $\frac{1}{2}$	58	3	15	3	15	8
$1\frac{3}{4}$	41	14	5	2	19 $\frac{1}{2}$	59	3	8	3	8	7
2 $\frac{1}{4}$	42	13	7	2	20 $\frac{1}{2}$	60	3	2	3	2	5
2 $\frac{3}{4}$	43	12	9	2	21 $\frac{1}{2}$	61	2	16	2	16	3
3 $\frac{1}{4}$	44	11	13	0	22 $\frac{1}{2}$	62	2	10	2	10	10
3 $\frac{3}{4}$	45	10	16	10	23 $\frac{1}{2}$	63	2	5	2	5	6
4 $\frac{1}{4}$	46	10	2	4	24 $\frac{1}{2}$	64	2	0	2	0	11
4 $\frac{3}{4}$	47	9	7	9	25 $\frac{1}{2}$	65	1	16	1	16	4
5 $\frac{1}{4}$	48	8	14	8	26 $\frac{1}{2}$	66	1	12	1	12	4
5 $\frac{3}{4}$	49	8	1	6	27 $\frac{1}{2}$	67	1	8	1	8	4
6 $\frac{1}{4}$	50	7	9	8	28 $\frac{1}{2}$	68	1	5	1	5	0
6 $\frac{3}{4}$	51	6	17	10	29 $\frac{1}{2}$	69	1	1	1	1	8
7 $\frac{1}{4}$	52	6	7	5	30 $\frac{1}{2}$	70	0	18	0	18	10
7 $\frac{3}{4}$	53	5	17	0	31 $\frac{1}{2}$	71	0	16	0	16	1
8 $\frac{1}{4}$	54	5	7	10	32 $\frac{1}{2}$	72	0	13	0	13	9
8 $\frac{3}{4}$	55	4	18	9	33 $\frac{1}{2}$	73	0	11	0	11	6
9 $\frac{1}{4}$	56	4	10	9	34 $\frac{1}{2}$	74	0	9	0	9	7
9 $\frac{3}{4}$	57	4	2	9	35 $\frac{1}{2}$	75	0	7	0	7	9

TABLE

TABLE XXVII.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $40\frac{1}{2}$ to $41\frac{1}{2}$.

Years after pur- chasing.	To commence at the end of	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.	Years after pur- chasing.	To commence at the end of	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1	Or at Age 41	41	15	0	0	18	Or at Age 59	3	12	5	5
2	—	42	14	0	7	19	—	3	12	5	5
3	—	43	13	0	7	20	—	3	12	5	5
4	—	44	12	2	7	21	—	2	19	4	4
5	—	45	11	4	6	22	—	2	13	8	1
6	—	46	10	8	5	23	—	2	8	10	10
7	—	47	9	12	0	24	—	1	18	4	4
8	—	48	9	18	7	25	—	1	13	10	0
9	—	49	8	3	4	26	—	1	10	2	2
10	—	50	7	10	1	27	—	1	6	11	11
11	—	51	7	17	4	28	—	0	19	9	9
12	—	52	6	5	7	29	—	0	17	1	1
13	—	53	6	13	5	30	—	0	14	5	5
14	—	54	5	3	2	31	—	0	12	2	2
15	—	55	5	13	2	32	—	0	10	0	0
16	—	56	4	4	2	33	—	0	8	0	2
17	—	57	4	15	3	34	—	0	8	0	2
18	—	58	3	7	5						
19	—		3	19							

XXXX

TABLE

TABLE XXVIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $41\frac{1}{2}$ to $42\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.
$4\frac{1}{2}$	42	£. 14. s. 6. d.	$17\frac{1}{2}$	59	£. 3. s. 16. d.
$1\frac{1}{2}$	43	13 15 2	$18\frac{1}{2}$	60	3 16 1
$2\frac{1}{2}$	44	12 16 2	$19\frac{1}{2}$	61	3 9 2
$3\frac{1}{2}$	45	12 18 2	$20\frac{1}{2}$	62	3 2 4
$4\frac{1}{2}$	46	11 0 3	$21\frac{1}{2}$	63	2 16 4
$5\frac{1}{2}$	47	11 4 2	$22\frac{1}{2}$	64	2 10 5
$6\frac{1}{2}$	48	10 8 1	$23\frac{1}{2}$	65	2 5 4
$7\frac{1}{2}$	49	9 13 7	$24\frac{1}{2}$	66	2 0 2
$8\frac{1}{2}$	50	8 19 0	$25\frac{1}{2}$	67	1 15 10
$9\frac{1}{2}$	51	8 5 11	$26\frac{1}{2}$	68	1 11 5
$10\frac{1}{2}$	52	7 12 9	$27\frac{1}{2}$	69	1 7 9
$11\frac{1}{2}$	53	7 1 2	$28\frac{1}{2}$	70	1 4 0
$12\frac{1}{2}$	54	6 9 8	$29\frac{1}{2}$	71	1 0 11
$13\frac{1}{2}$	55	5 19 7	$30\frac{1}{2}$	72	0 17 10
$14\frac{1}{2}$	56	5 9 4	$31\frac{1}{2}$	73	0 15 4
$15\frac{1}{2}$	57	5 0 7	$32\frac{1}{2}$	74	0 12 9
$16\frac{1}{2}$	58	4 11 8	$33\frac{1}{2}$	75	0 10 7

TABLE

T A B L E XXIX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $42\frac{2}{3}$ to $43\frac{1}{3}$.

To commence at the end of.	Value of an annuity of £1. in one present payment.	To commence at the end of.	Value of an annuity of £1. in one present payment.
Years after purchasing.	£. s. d.	Years after purchasing.	£. s. d.
1, or at Age 43	14 11 0	17 $\frac{1}{4}$, or at Age 60	3 12 9
1 $\frac{1}{4}$, —	13 11 9	18 $\frac{1}{4}$, —	3 6 0
2 $\frac{1}{4}$, —	12 13 10	19 $\frac{1}{4}$, —	2 19 3
3 $\frac{1}{4}$, —	11 15 11	20 $\frac{1}{4}$, —	2 13 5
4 $\frac{1}{4}$, —	10 19 10	21 $\frac{1}{4}$, —	2 7 7
5 $\frac{1}{4}$, —	9 3 10	22 $\frac{1}{4}$, —	2 2 7
6 $\frac{1}{4}$, —	8 9 2	23 $\frac{1}{4}$, —	1 17 7
7 $\frac{1}{4}$, —	8 14 6	24 $\frac{1}{4}$, —	1 13 4
8 $\frac{1}{4}$, —	7 1 6	25 $\frac{1}{4}$, —	1 9 1
9 $\frac{1}{4}$, —	6 8 5	26 $\frac{1}{4}$, —	1 5 6
10 $\frac{1}{4}$, —	6 17 1	27 $\frac{1}{4}$, —	1 1 11
11 $\frac{1}{4}$, —	5 5 10	28 $\frac{1}{4}$, —	0 18 11
12 $\frac{1}{4}$, —	5 15 10	29 $\frac{1}{4}$, —	0 16 0
13 $\frac{1}{4}$, —	5 5 10	30 $\frac{1}{4}$, —	0 13 7
14 $\frac{1}{4}$, —	4 17 0	31 $\frac{1}{4}$, —	0 11 2
15 $\frac{1}{4}$, —	4 8 2	32 $\frac{1}{4}$, —	0 9 2
16 $\frac{1}{4}$, —	4 0 6		
17 $\frac{1}{4}$, —			
18 $\frac{1}{4}$, —			
19 $\frac{1}{4}$, —			
20 $\frac{1}{4}$, —			
21 $\frac{1}{4}$, —			
22 $\frac{1}{4}$, —			
23 $\frac{1}{4}$, —			
24 $\frac{1}{4}$, —			
25 $\frac{1}{4}$, —			
26 $\frac{1}{4}$, —			
27 $\frac{1}{4}$, —			
28 $\frac{1}{4}$, —			
29 $\frac{1}{4}$, —			
30 $\frac{1}{4}$, —			
31 $\frac{1}{4}$, —			
32 $\frac{1}{4}$, —			

Xxxx 2

T A B L E

TABLE XXX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $43\frac{1}{4}$ to $44\frac{1}{4}$.

To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	To commence at the end of	Value of an annuity of £1. in one present payment.	
		£. s. d.				£. s. d.	
$4\frac{1}{4}$, or at Age 44	44	14	6	9	60	3	17
$1\frac{1}{4}$	45	13	7	2	61	3	9
$2\frac{1}{4}$	46	12	9	3	62	3	2
$3\frac{1}{4}$	47	11	11	4	63	2	16
$4\frac{1}{4}$	48	10	15	2	64	2	10
$5\frac{1}{4}$	49	9	19	0	65	2	4
$6\frac{1}{4}$	50	8	4	5	66	1	19
$7\frac{1}{4}$	51	7	9	10	67	1	14
$8\frac{1}{4}$	52	7	17	0	68	1	10
$9\frac{1}{4}$	53	6	4	2	69	1	6
$10\frac{1}{4}$	54	6	13	0	70	0	3
$11\frac{1}{4}$	55	5	1	10	71	0	19
$12\frac{1}{4}$	56	5	11	11	72	0	17
$13\frac{1}{4}$	57	5	2	0	73	0	14
$14\frac{1}{4}$	58	4	13	3	74	0	11
$15\frac{1}{4}$	59	4	4	7	75	0	9

TABLE XXXI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $44\frac{1}{2}$ to $45\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.	To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
$44\frac{1}{2}$	45	14	2 0	$44\frac{1}{2}$	61	3 13 6	6
$45\frac{1}{2}$	46	13	2 7	$45\frac{1}{2}$	62	3 6 0	6
$46\frac{1}{2}$	47	12	4 8	$46\frac{1}{2}$	63	2 19 0	6
$47\frac{1}{2}$	48	11	6 10	$47\frac{1}{2}$	64	2 13 0	5
$48\frac{1}{2}$	49	10	8 6	$48\frac{1}{2}$	65	2 7 11	5
$49\frac{1}{2}$	50	9	10 2	$49\frac{1}{2}$	66	2 1 11	4
$50\frac{1}{2}$	51	8	12 8	$50\frac{1}{2}$	67	1 17 1	4
$51\frac{1}{2}$	52	7	15 2	$51\frac{1}{2}$	68	1 12 4	5
$52\frac{1}{2}$	53	6	18 7	$52\frac{1}{2}$	69	1 8 5	5
$53\frac{1}{2}$	54	5	21 0	$53\frac{1}{2}$	70	1 4 1	1
$54\frac{1}{2}$	55	4	24 11	$54\frac{1}{2}$	71	1 17 10	1
$55\frac{1}{2}$	56	3	27 8	$55\frac{1}{2}$	72	1 15 1	5
$56\frac{1}{2}$	57	2	30 2	$56\frac{1}{2}$	73	1 12 5	2
$57\frac{1}{2}$	58	1	33 1	$57\frac{1}{2}$	74	1 10 2	2
$58\frac{1}{2}$	59	0	36 4	$58\frac{1}{2}$	75	1 8 2	2
$59\frac{1}{2}$	60	0	39 1				

TABLE

TABLE XXXII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $45\frac{1}{2}$ to $46\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.	To commence at the end of Years after pur- chasing.	Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
$1\frac{1}{4}$, or at	46	13	17	6		$15\frac{1}{4}$, or at	61	3	17	3	
$1\frac{1}{4}$	47	12	17	10		$16\frac{1}{4}$	62	3	9	11	
$2\frac{1}{4}$	48	11	19	9		$17\frac{1}{4}$	63	3	2	7	
$3\frac{1}{4}$	49	11	1	8		$18\frac{1}{4}$	64	2	16	2	
$4\frac{1}{4}$	50	10	5	6		$19\frac{1}{4}$	65	2	9	10	
$5\frac{1}{4}$	51	9	9	3		$20\frac{1}{4}$	66	2	4	5	
$6\frac{1}{4}$	52	8	15	0		$21\frac{1}{4}$	67	1	19	0	
$7\frac{1}{4}$	53	8	0	8		$22\frac{1}{4}$	68	1	14	4	
$8\frac{1}{4}$	54	7	8	2		$23\frac{1}{4}$	69	1	9	9	
$9\frac{1}{4}$	55	6	15	8		$24\frac{1}{4}$	70	1	5	11	
$10\frac{1}{4}$	56	6	4	8		$25\frac{1}{4}$	71	0	2	2	
$11\frac{1}{4}$	57	5	13	8		$26\frac{1}{4}$	72	0	18	11	
$12\frac{1}{4}$	58	5	4	0		$27\frac{1}{4}$	73	0	15	10	
$13\frac{1}{4}$	59	4	14	3		$28\frac{1}{4}$	74	0	13	2	
$14\frac{1}{4}$	60	4	5	9		$29\frac{1}{4}$	75	0	10	9	

TABLE

T A B L E XXXIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $46\frac{1}{2}$ to $47\frac{1}{2}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.			To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment.		
		£.	s.	d.			£.	s.	d.
1 $\frac{1}{4}$	47	13	12	6	15 $\frac{1}{4}$	62	3	13	7
1 $\frac{1}{2}$	48	12	13	2	16 $\frac{1}{4}$	63	3	6	5
2 $\frac{1}{4}$	49	11	14	11	17 $\frac{1}{4}$	64	2	19	2
2 $\frac{1}{2}$	50	10	16	7	18 $\frac{1}{4}$	65	2	12	11
3 $\frac{1}{4}$	51	10	0	6	19 $\frac{1}{4}$	66	2	6	9
3 $\frac{1}{2}$	52	9	4	4	20 $\frac{1}{4}$	67	2	1	5
4 $\frac{1}{4}$	53	8	10	4	21 $\frac{1}{4}$	68	1	16	2
4 $\frac{1}{2}$	54	7	16	3	22 $\frac{1}{4}$	69	1	11	8
5 $\frac{1}{4}$	55	7	3	10	23 $\frac{1}{4}$	70	1	7	2
5 $\frac{1}{2}$	56	6	11	5	24 $\frac{1}{4}$	71	1	3	6
6 $\frac{1}{4}$	57	6	0	6	25 $\frac{1}{4}$	72	0	19	10
6 $\frac{1}{2}$	58	5	9	7	26 $\frac{1}{4}$	73	0	16	10
7 $\frac{1}{4}$	59	5	0	4	27 $\frac{1}{4}$	74	0	13	10
7 $\frac{1}{2}$	60	4	10	4	28 $\frac{1}{4}$	75	0	11	4
8 $\frac{1}{4}$	61	4	2	0					

T A B L E XXXIV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $47\frac{3}{4}$ to $48\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.			To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.		
		£.	s.	d.			£.	s.	d.
$1\frac{1}{4}$	48	13	7	9	$14\frac{1}{4}$	62	3	18	1
$1\frac{1}{2}$	49	12	7	10	$15\frac{1}{4}$	63	3	9	10
$2\frac{1}{4}$	50	11	9	7	$16\frac{1}{4}$	64	3	2	10
$3\frac{1}{4}$	51	10	11	5	$17\frac{1}{4}$	65	2	15	8
$4\frac{1}{4}$	52	9	15	6	$18\frac{1}{4}$	66	2	9	7
$5\frac{1}{4}$	53	8	19	7	$19\frac{1}{4}$	67	2	3	7
$6\frac{1}{4}$	54	8	5	7	$20\frac{1}{4}$	68	1	18	5
$7\frac{1}{4}$	55	7	11	8	$21\frac{1}{4}$	69	1	13	3
$8\frac{1}{4}$	56	6	19	4	$22\frac{1}{4}$	70	1	8	11
$9\frac{1}{4}$	57	6	7	0	$23\frac{1}{4}$	71	1	4	8
$10\frac{1}{4}$	58	5	16	2	$24\frac{1}{4}$	72	1	1	2
$11\frac{1}{4}$	59	5	5	4	$25\frac{1}{4}$	73	0	17	8
$12\frac{1}{4}$	60	4	15	10	$26\frac{1}{4}$	74	0	14	9
$13\frac{1}{4}$	61	4	6	4	$27\frac{1}{4}$	75	0	12	0

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T A B L E XXXV.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $48\frac{1}{2}$ to $49\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.	To commence at the end of Years after pur- chasing.	Or at Age	Value of an an- nuity of £1. in one present pay- ment.	£.	s.	d.
1 $\frac{1}{2}$	49	13	2	6	14 $\frac{1}{2}$	63	3	14	3		
2 $\frac{1}{2}$	50	12	2	6	15 $\frac{1}{2}$	64	3	6	2		
3 $\frac{1}{2}$	51	11	4	5	16 $\frac{1}{2}$	65	2	19	3		
4 $\frac{1}{2}$	52	10	6	3	17 $\frac{1}{2}$	66	2	12	3		
5 $\frac{1}{2}$	53	9	10	6	18 $\frac{1}{2}$	67	2	6	4		
6 $\frac{1}{2}$	54	8	14	10	19 $\frac{1}{2}$	68	2	0	5		
7 $\frac{1}{2}$	55	8	0	11	20 $\frac{1}{2}$	69	1	15	5		
8 $\frac{1}{2}$	56	7	7	1	21 $\frac{1}{2}$	70	1	10	5		
9 $\frac{1}{2}$	57	6	14	10	22 $\frac{1}{2}$	71	1	6	3		
10 $\frac{1}{2}$	58	6	2	7	23 $\frac{1}{2}$	72	1	2	2		
11 $\frac{1}{2}$	59	5	11	10	24 $\frac{1}{2}$	73	0	18	10		
12 $\frac{1}{2}$	60	5	1	1	25 $\frac{1}{2}$	74	0	15	6		
13 $\frac{1}{2}$	61	4	11	9	26 $\frac{1}{2}$	75	0	12	8		

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T A B L E

TABLE XXXVI.

For the Life of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $49\frac{3}{4}$ to $50\frac{1}{2}$.

To commence at the end of Years after purchasing.	or at Age 50	Value of an annuity of £1. in one present payment.		To commence at the end of Years after purchasing.	or at Age 63	Value of an annuity of £1. in one present payment.	
		£. s. d.				£. s. d.	
$\frac{1}{4}$	—	12	17	$13\frac{1}{4}$	—	3	18
$\frac{1}{2}$	—	11	17	$14\frac{1}{4}$	—	3	10
$\frac{3}{4}$	—	10	19	$15\frac{1}{4}$	—	3	2
1	51	10	10	$16\frac{1}{4}$	—	2	15
$1\frac{1}{4}$	52	10	10	$17\frac{1}{4}$	—	2	8
$1\frac{1}{2}$	53	9	6	$18\frac{1}{4}$	—	2	3
$1\frac{3}{4}$	54	8	10	$19\frac{1}{4}$	—	1	17
2	55	7	16	$20\frac{1}{4}$	—	1	12
$2\frac{1}{4}$	56	7	10	$21\frac{1}{4}$	—	1	7
$2\frac{1}{2}$	57	6	2	$22\frac{1}{4}$	—	1	3
$2\frac{3}{4}$	58	5	18	$23\frac{1}{4}$	—	0	19
3	59	5	5	$24\frac{1}{4}$	—	0	16
$3\frac{1}{4}$	60	4	17	$25\frac{1}{4}$	—	0	13
$3\frac{1}{2}$	61	4	7				
$3\frac{3}{4}$	62	4					

TABLE

TABLE XXXVII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 50 $\frac{1}{2}$ to 51 $\frac{1}{2}$.

To commence at the end of	Years after purchasing.	Value of an annuity of £1. in one present payment.	To commence at the end of	Years after purchasing.	Value of an annuity of £1. in one present payment.
51	1 $\frac{1}{4}$	12 12 6	64	13 $\frac{1}{4}$	3 14 10
52	1 $\frac{3}{4}$	11 13 2	65	14 $\frac{1}{4}$	3 6 11
53	2 $\frac{1}{4}$	10 15 4	66	15 $\frac{1}{4}$	2 19 1
54	2 $\frac{3}{4}$	9 17 7	67	16 $\frac{1}{4}$	2 12 5
55	3 $\frac{1}{4}$	9 1 11	68	17 $\frac{1}{4}$	2 5 8
56	3 $\frac{3}{4}$	8 6 2	69	18 $\frac{1}{4}$	2 0 0
57	4 $\frac{1}{4}$	7 12 4	70	19 $\frac{1}{4}$	1 14 5
58	4 $\frac{3}{4}$	6 18 7	71	20 $\frac{1}{4}$	1 9 9
59	5 $\frac{1}{4}$	6 6 5	72	21 $\frac{1}{4}$	1 1 1
60	5 $\frac{3}{4}$	5 14 3	73	22 $\frac{1}{4}$	1 1 4
61	6 $\frac{1}{4}$	5 3 9	74	23 $\frac{1}{4}$	0 17 6
62	6 $\frac{3}{4}$	4 13 2	75	24 $\frac{1}{4}$	0 14 4
63	7 $\frac{1}{4}$	4 4 0			

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TABLE

TABLE XXXVIII.

For the Use of Country Parishes

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $51\frac{1}{2}$ to $52\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	1, or at Age 52	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
1 $\frac{1}{4}$	53	12	7 6
1 $\frac{1}{2}$	53	11	7 10
2 $\frac{1}{4}$	54	10	10 1
2 $\frac{1}{2}$	54	10	10 1
3 $\frac{1}{4}$	55	9	12 4
3 $\frac{1}{2}$	56	8	16 9
4 $\frac{1}{4}$	57	8	1 1
4 $\frac{1}{2}$	58	7	7 4
5 $\frac{1}{4}$	59	6	13 8
5 $\frac{1}{2}$	60	6	1 7
6 $\frac{1}{4}$	61	5	9 7
6 $\frac{1}{2}$	62	4	19 2
7 $\frac{1}{4}$	63	4	8 8
7 $\frac{1}{2}$	64	3	19 8
8 $\frac{1}{4}$	65	3	10 8
8 $\frac{1}{2}$	66	3	2 8
9 $\frac{1}{4}$	67	2	15 3
9 $\frac{1}{2}$	68	2	8 9
10 $\frac{1}{4}$	69	2	2 3
10 $\frac{1}{2}$	70	1	16 9
11 $\frac{1}{4}$	71	1	11 4
11 $\frac{1}{2}$	72	1	6 10
12 $\frac{1}{4}$	73	1	2 5
12 $\frac{1}{2}$	74	0	18 9
13 $\frac{1}{4}$	75	0	15 3

TABLE XXXIX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchaser from $52\frac{1}{2}$ to $53\frac{1}{2}$.

To commence at the end of Years after pur- chasing.	1, or at Age 53	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
1 $\frac{1}{4}$	54	12	2 0
1 $\frac{1}{2}$	54	11	2 7
2 $\frac{1}{4}$	55	10	4 11
2 $\frac{1}{2}$	55	9	7 2
3 $\frac{1}{4}$	56	8	11 8
3 $\frac{1}{2}$	57	7	16 1
4 $\frac{1}{4}$	58	7	2 5
4 $\frac{1}{2}$	59	6	8 9
5 $\frac{1}{4}$	60	5	16 10
5 $\frac{1}{2}$	61	5	4 11
6 $\frac{1}{4}$	62	4	14 7
6 $\frac{1}{2}$	63	4	4 3
7 $\frac{1}{4}$	64	3	15 3
7 $\frac{1}{2}$	65	3	6 5
8 $\frac{1}{4}$	66	2	19 7
8 $\frac{1}{2}$	67	2	11 0
9 $\frac{1}{4}$	68	2	5 6
9 $\frac{1}{2}$	69	2	2 2
10 $\frac{1}{4}$	70	1	18 10
10 $\frac{1}{2}$	71	1	13 7
11 $\frac{1}{4}$	72	1	8 3
11 $\frac{1}{2}$	73	1	4 0
12 $\frac{1}{4}$	74	0	19 9
12 $\frac{1}{2}$	75	0	16 2

TABLE

TABLE XL.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 53½ to 54½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment.		
	or at Age 54	£. s. d.		
1½	55	11 16 8		
2½	56	10 17 2		
3½	57	9 19 6		
4½	58	8 6 4		
5½	59	7 10 10		
6½	60	6 17 3		
7½	61	5 11 11		
8½	62	5 11 11		
9½	63	4 9 11		
10½	64	3 19 9		
11½	65	3 11 1		
12½	66	3 2 4		
13½	67	2 15 0		
14½	68	2 7 6		
15½	69	2 1 1		
16½	70	1 15 5		
17½	71	1 10 4		
18½	72	1 5 3		
19½	73	1 1 1		
20½	74	0 17 2		
21½	75			

TABLE XLI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 54½ to 55½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment.		
	or at Age 55	£. s. d.		
1½	56	11 11 3		
2½	57	10 11 9		
3½	58	9 14 6		
4½	59	8 1 0		
5½	60	7 5 7		
6½	61	6 12 1		
7½	62	5 18 7		
8½	63	5 7 0		
9½	64	4 15 4		
10½	65	4 5 4		
11½	66	3 15 3		
12½	67	3 6 9		
13½	68	2 18 2		
14½	69	2 11 0		
15½	70	2 3 11		
16½	71	1 17 11		
17½	72	1 12 0		
18½	73	1 7 2		
19½	74	1 2 4		
20½	75	0 18 2		

TABLE

TABLE XLII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 55½ to 56½.

To commence at the end of Years after pur- chasing.	½, or at Age 56	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
1 ½	57	11 5 6	
2 ½	58	10 5 10	
3 ½	59	9 8 6	
4 ½	60	8 10 11	
5 ½	61	7 15 2	
6 ½	62	6 7 9	
7 ½	63	5 13 4	
8 ½	64	5 1 11	
9 ½	65	4 10 5	
10 ½	66	4 0 7	
11 ½	67	3 10 8	
12 ½	68	3 2 4	
13 ½	69	2 14 0	
14 ½	70	2 7 0	
15 ½	71	2 0 1	
16 ½	72	1 14 4	
17 ½	73	1 8 8	
18 ½	74	1 3 11	
19 ½	75	0 19 6	

TABLE XLIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75,

Age of the Purchaser from 56½ to 57½.

To commence at the end of Years after pur- chasing.	½, or at Age 57	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
1 ½	58	11 0 0	
2 ½	59	10 0 5	
3 ½	60	9 2 11	
4 ½	61	8 5 4	
5 ½	62	7 10 0	
6 ½	63	6 14 9	
7 ½	64	6 1 2	
8 ½	65	5 8 10	
9 ½	66	4 16 6	
10 ½	67	4 5 10	
11 ½	68	3 15 1	
12 ½	69	3 6 1	
13 ½	70	2 18 0	
14 ½	71	2 9 10	
15 ½	72	2 3 1	
16 ½	73	1 16 4	
17 ½	74	1 10 10	
18 ½	75	1 5 4	
19 ½		1 0 9	

TABLE XLIV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 57½ to 58½.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
½	58	10	14 0
1 ¼	59	9	14 7
2 ¼	60	9	17 1
3 ¼	61	7	19 7
4 ¼	62	7	4 4
5 ¼	63	6	9 2
6 ¼	64	5	16 0
7 ¼	65	5	2 10
8 ¼	66	4	11 8
9 ¼	67	4	0 6
10 ¼	68	3	11 0
11 ¼	69	3	1 5
12 ¼	70	2	13 7
13 ¼	71	2	5 8
14 ¼	72	1	19 1
15 ¼	73	1	12 7
16 ¼	74	1	7 3
17 ¼	75	1	2 2

TABLE XLV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 58½ to 59½.

To commence at the end of Years after pur- chasing.	or at Age	Value of an an- nuity of £1. in one present pay- ment.	£. s. d.
½	59	10	8 0
1 ¼	60	9	8 9
2 ¼	61	8	11 3
3 ¼	62	7	13 10
4 ¼	63	6	18 8
5 ¼	64	6	3 7
6 ¼	65	5	10 7
7 ¼	66	4	17 6
8 ¼	67	4	6 6
9 ¼	68	3	15 2
10 ¼	69	3	6 10
11 ¼	70	2	16 2
12 ¼	71	2	9 6
13 ¼	72	2	1 2
14 ¼	73	1	15 2
15 ¼	74	1	8 11
16 ¼	75	1	3 8

TABLE

TABLE XLVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 59½ to 60½.

To commence at the end of Years after purchasing.	1, ½, or at Age 60	Value of an annuity of £1. in one present payment.	£. s. d.
1 ½	61	10	2 0
2 ½	61	9	2 8
3 ½	62	8	2 2
4 ½	62	7	2 10
5 ½	63	6	2 9
6 ½	63	5	2 9
7 ½	64	4	2 11
8 ½	64	3	2 11
9 ½	65	2	2 11
10 ½	65	1	2 11
11 ½	66	1	2 11
12 ½	66	1	2 11
13 ½	67	1	2 11
14 ½	67	1	2 11
15 ½	68	1	2 11

TABLE XLVII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 60½ to 61½.

To commence at the end of Years after purchasing.	1, ½, or at Age 61	Value of an annuity of £1. in one present payment.	£. s. d.
1 ½	61	9	16 0
2 ½	62	8	16 7
3 ½	63	7	19 2
4 ½	63	6	19 10
5 ½	64	5	12 0
6 ½	64	4	12 4
7 ½	65	4	12 7
8 ½	66	3	15 11
9 ½	66	3	15 3
10 ½	67	2	16 5
11 ½	67	2	16 7
12 ½	68	2	13 5
13 ½	68	1	13 4
14 ½	69	1	13 2

TABLE

TABLE XLVIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 61½ to 62½.

To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.
1½	62	£. 9 s. 8 d.
1¾	63	8 9 3
2	64	7 10 11
2½	65	6 12 8
3	66	5 15 0
3½	67	4 18 6
4	68	3 21 0
4½	69	2 24 0
5	70	1 27 0
5½	71	1 30 0
6	72	2 3 0
6½	73	1 15 0
7	74	1 3 0
7½	75	1 9 3

TABLE XLIX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 62½ to 63½.

To commence at the end of Years after purchasing.	Or at Age	Value of an annuity of £1. in one present payment.
1½	63	£. 9 s. 3 d.
1¾	64	8 3 11
2	65	7 6 8
2½	66	6 9 6
3	67	5 14 0
3½	68	4 17 0
4	69	3 20 0
4½	70	2 23 0
5	71	1 26 0
5½	72	1 29 0
6	73	2 2 0
6½	74	1 15 0
7	75	1 3 0

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TABLE

TABLE L.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 63½ to 64½.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
½, or at Age 64	64	£. 8 s. 8 d.
1 ½	65	7 17 4
2 ½	66	7 0 2
3 ½	67	6 3 0
4 ½	68	5 8 6
5 ½	69	4 13 11
6 ½	70	4 1 10
7 ½	71	3 9 9
8 ½	72	2 19 10
9 ½	73	2 9 10
10 ½	74	1 1 7
11 ½	75	1 14 0

TABLE LI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 64½ to 65½.

To commence at the end of Years after pur- chasing.	To commence at the end of Years after pur- chasing.	Value of an an- nuity of £1. in one present pay- ment.
½, or at Age 65	65	£. 8 s. 8 d.
1 ½	66	8 10 0
2 ½	67	7 10 10
3 ½	68	6 13 8
4 ½	69	5 16 7
5 ½	70	5 2 3
6 ½	71	4 7 11
7 ½	72	3 16 0
8 ½	73	3 4 1
9 ½	74	2 14 5
10 ½	75	2 4 9
		1 16 7

TABLE

TABLE LI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 65½ to 66½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
½, Or at Age 66	£. s. d.
1 ½	8 3 4
2 ½	7 4 0
3 ½	6 6 11
4 ½	5 10 0
5 ½	4 15 10
6 ½	4 1 8
7 ½	3 10 0
8 ½	2 18 4
9 ½	1 19 9

TABLE LII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 66½ to 67½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
½, Or at Age 67	£. s. d.
1 ½	7 16 6
2 ½	6 17 2
3 ½	6 0 3
4 ½	5 3 5
5 ½	4 9 5
6 ½	3 15 5
7 ½	3 3 0
8 ½	2 12 7
9 ½	2 3 9

TABLE LIV.

For the Use of Country Parishes

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 67½ to 68½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	£. s. d.
1, or at Age 68	7	9 6
1½	6	10 1
2	5	13 4
2½	4	16 7
3	4	2 10
3½	4	2 10
4	3	9 0
4½	3	17 6
5	2	17 6
5½	2	17 6
6	2	17 6
6½	2	17 6
7	2	17 6

TABLE LV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchaser from 68½ to 69½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.	£. s. d.
1, or at Age 69	7	2 4
1½	6	3 1
2	5	6 5
2½	4	9 9
3	4	9 9
3½	3	16 3
4	3	16 3
4½	3	16 3
5	3	16 3
5½	3	16 3
6	2	11 2
6½	2	11 2
7	2	11 2

TABLE

TABLE LVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 69½ to 70½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
½, or at Age 70	£. 6 s. 15 d.
1 ½	5 15 9
2 ½	4 19 3
3 ½	4 2 9
4 ½	3 9 0
5 ½	2 16 0

TABLE LVII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 70½ to 71½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
½, or at Age 71	£. 6 s. 7 d.
1 ½	5 8 9
2 ½	4 12 1
3 ½	3 15 9
4 ½	3 1 11
	75

TABLE

TABLE LVIII.

For the Life of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 71½ to 72½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
¼, or at Age 72	£. 6 s. 0 d.
1 ¼, — 73	5 0 11
2 ¼, — 74	4 4 2
3 ¼, — 75	3 8 6

TABLE LIX.

For the Life of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 72½ to 73½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment.
¼, or at Age 73	£. 5 s. 12 d.
1 ¼, — 74	4 13 4
2 ¼, — 75	3 16 3

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F I N I S.



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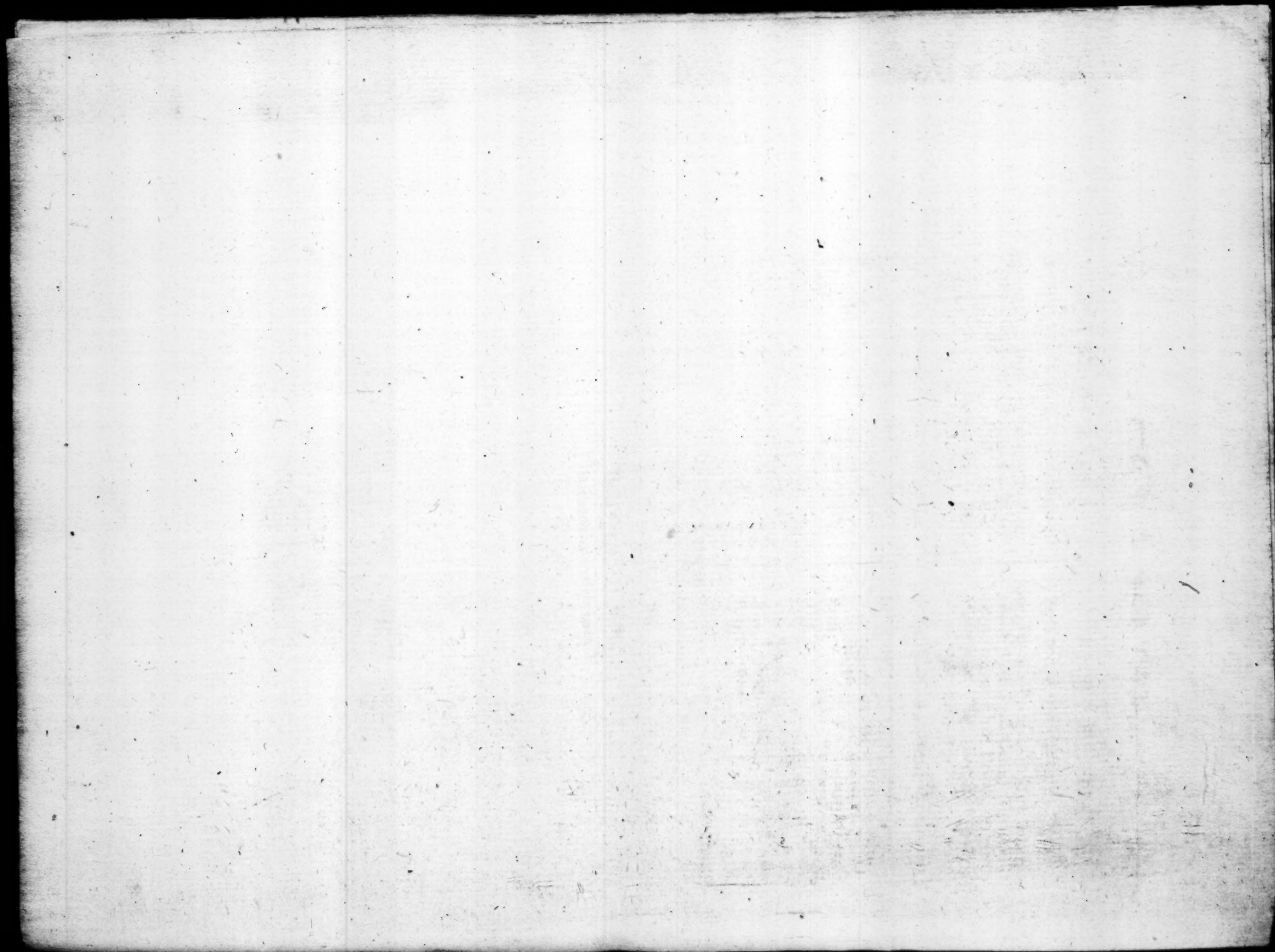
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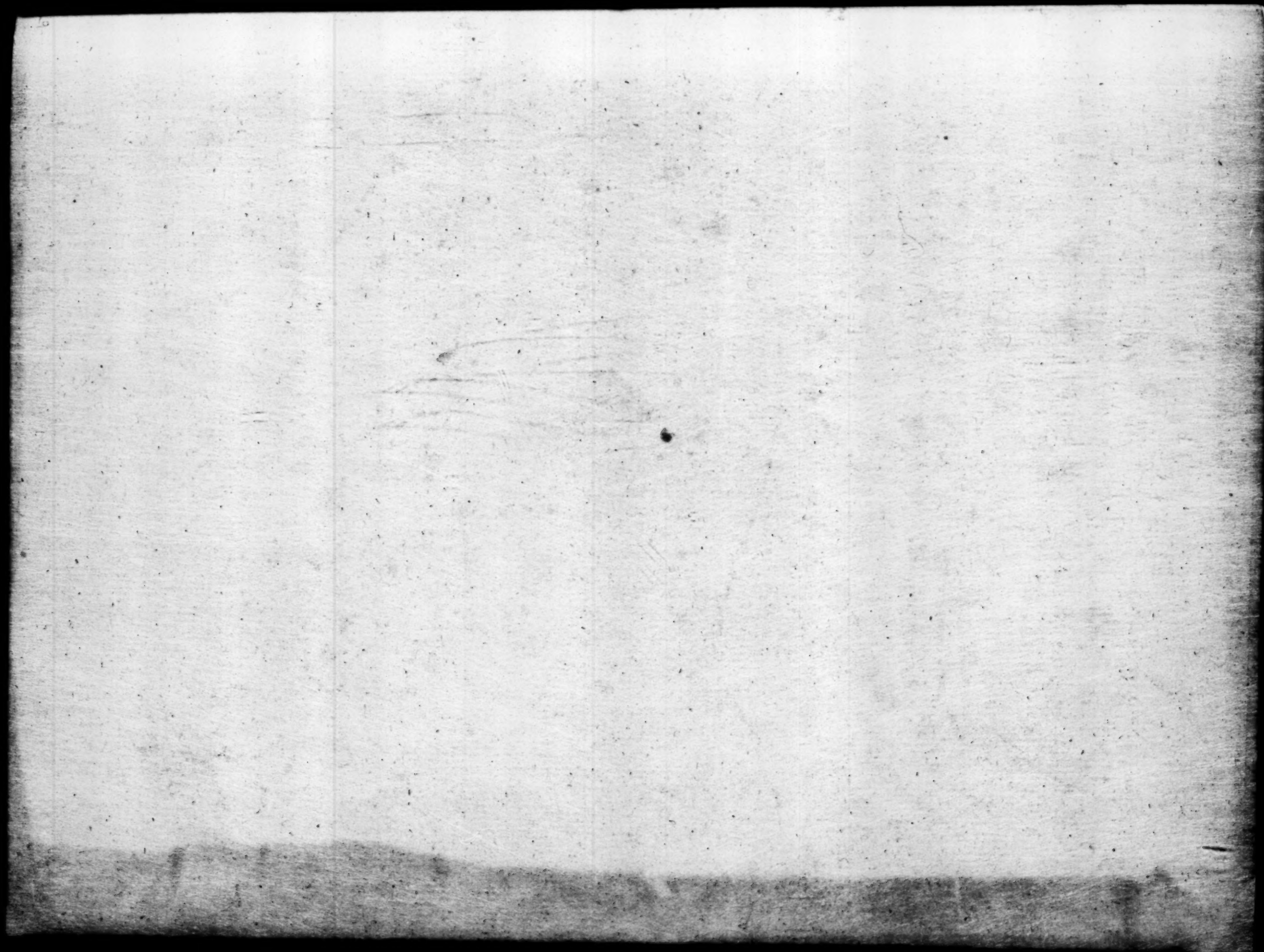
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Observations on the National Debt & the most likely method of
paying off a part of it, by Wm. Barron Munnings. 203p. 4s 3d.

An Essay on the Public Debt of this Kingdom, first published
in 1728 & supposed to have been written by Sir Mathew
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Expediency of an equal repayment of the Loan Tax &
of making the annual interest of all new loans of public
money subject to the same tax, by Wm. Barron Munnings.
449. to 459.